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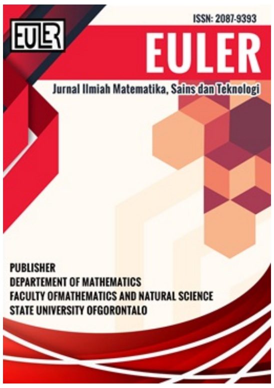

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Research Article

Reliability for Generalized Rayleigh of 1 Strength - 4 Stresses

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ABSTRACT. *In this paper, the reliability of a one-component model is found where this component is subjected to four stresses with random variables Y_1, Y_2, Y_3, Y_4 and this component resists these stresses with its strength with random variable X and it was assumed that these variables follow a generalized Rayleigh distribution. The model's reliability was estimated by three different estimation methods (Percentile method, the Regression method, and the Least Squares method). A Monte Carlo simulation was performed to compare the results obtained from the estimate using two statistical criteria: the mean squares error criterion and the mean absolute percentage error criterion. The comparison showed that the best estimator of the reliability of the model is the favorable Percentile estimator.*

1. Introduction

Reliability is one of the most important terms that has increased interest with the development of industrial machinery and its increasing complexity, because the term reliability means the period of work (life) of the machine or component, and this interest in reliability was accompanied by the emergence of other terms, including the term stress-strength [1–4], which means that when the component is working, it is subjected to stresses and resists these stresses with its strength the reliability of the component can be determined by knowing the probability that the strength overcomes the stresses or vice versa [5–7]. The increased interest of researchers in the term stress-strength led to the development of this concept, which positively reflected the increased reliability of industrial models in various fields. The stress that the component is subjected to can be expressed by the random variable Y and the component's strength can be expressed by the random variable X, so the reliability function can be found by the mathematical formula $R = P(Y < X)$ [8–10].

There are a lot of papers interested in this term, including Salman and Hamad [3] estimated the reliability model by several different estimation methods when stress and strength factors follow the Lomax distribution. Khaleel and Karam [11–13] studied one of the special cascade systems where the model has two basic components and a standby component. Khaleel [14] funded the mathematical formula for a reliability model consisting of three basic components and a backup component with an active standby mode that replaces the basic components in case one of them fails.

In a previous paper, a reliability function was found for a component model that has strength and is subjected to two stresses, (see Karaday, et al. [15]), in this paper, is to find a reliability function where the model will be one component that has robustness, expressed in the random variable X and subjected to four stresses, the random variables are Y_1, Y_2, Y_3, and Y_4.

This paper aims to find the mathematical formula for a component model that has strength and is subjected to four independent stresses, assuming that the strength and stress factors follow the generalized Raleigh distribution, the reliability of the model is also estimated by three different estimation methods (percentile, regression, least squares), as well as a simulation is done to find the best estimator of the reliability of the model.

2. Methods

In this subsection, the parameters of the generalized Raleigh distribution estimator will be found by three different estimation methods, which are (percentile, regression, and least squares) as follows:

Let the random variables (X, Y_1, Y_2, Y_3 and Y_4), follows the generalized Raleigh distribution where $X \sim GR(2, \mu, \eta)$ and $Y_\ell \sim GR(2, \mu, \eta_i)$; $i = 1, 2, 3, 4$ so that:

$$f(x) = 2\eta\mu x e^{-\eta\mu x^2} \tag{1}$$

$$F(x) = 1 - e^{-\eta\mu x^2} \tag{2}$$

$$F_i(y_i) = 1 - e^{-\eta_i\mu y_i^2} \quad ; i = 1, 2, 3, 4. \tag{3}$$

2.1. Percentile Method

In this method, the CDF function mentioned in Equation (2) is used as [12]:

$$F(x_{(i)}) = 1 - e^{-\eta\mu x_{(i)}^2}$$

$$ln(1 - F(x_{(i)})) = -\eta\mu x_{(i)}^2$$

$$x_{(i)} = \left(\frac{-ln(1 - F(x_{(i)}))}{\eta\mu}\right)^{\frac{1}{2}}.$$

Since P_i denotes some the estimate of $F(x_{(i)}; \mu, \eta)\, P_i$, where $P_i = \dfrac{i}{n+1}$; $i = 1, 2, \ldots, n$ then:

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$$x_{(i)} = \left(\frac{-ln\,(1-P_i)}{\eta\mu}\right)^{\frac{1}{2}}$$
$$\sum_{i=1}^{n}\left[x_{(i)} - \left(\frac{-ln\,(1-P_i)}{\eta\mu}\right)^{\frac{1}{2}}\right]^2 = 0. \tag{4}$$

Derivative the equation (4):

$$\sum_{i=1}^{n} 2\left[x_{(i)} - \eta^{\frac{-1}{2}}\left(\frac{-ln\,(1-P_i)}{\mu}\right)^{\frac{1}{2}}\right]$$
$$\left(\frac{1}{2}\eta^{\frac{-3}{2}}\right)\left(\frac{-ln\,(1-P_i)}{\mu}\right)^{\frac{1}{2}} = 0.$$

The percentile estimator of η is:

$$\widehat{\eta}_{Pr} = \left(\frac{\sum_{i=1}^{n}\left[\frac{-ln(1-P_i)}{\mu}\right]}{\sum_{i=1}^{n} x_{(i)}\left[\frac{-ln(1-P_i)}{\mu}\right]^{\frac{1}{2}}}\right)^2, \tag{5}$$

also the percentile estimators of parameters $(\eta_1, \eta_2, \eta_3, \eta_4)$ are:

$$\widehat{\eta}_{\zeta(Pr)} = \left(\frac{\sum_{i_\zeta=1}^{n_\zeta} y_{\zeta_{(i_\zeta)}}\left[\frac{-ln\left(1-P_{i_\zeta}\right)}{\mu}\right]^{\frac{-1}{2}}}{\sum_{i_\zeta=1}^{n_\zeta}\left[\frac{-ln\left(1-P_{i_\zeta}\right)}{\mu}\right]^{-1}}\right)^2, \; \zeta = 1,2,3,4. \tag{6}$$

2.2. Regression Method

The standard equation is used to start the estimate in this method and as follows [16]:

$$z_i = a + bu_i + e_i \tag{7}$$

where (z_i) is the dependent variable, (u_i) is the independent variable and (e_i) is error random variable independent.

Let $x_1, x_2, \ldots, x_n$ of $GR\,(2,\mu,\eta)$. Take the logarithm for equation (2):

$$F\left(x_{(i)}\right) = 1 - e^{-\eta\mu x_{(i)}^2}$$
$$\left(1 - F\left(x_{(i)}\right)\right)^{-1} = e^{-\eta\mu x_{(i)}^2}$$
$$Ln\left[\left(1 - F\left(x_{(i)}\right)\right)^{-1}\right] = \eta\mu x_{(i)}^2.$$

Changing $F\left(x_{(i)}\right)$ by the P_i, so

$$Ln\left[(1-P_i)^{-1}\right] = \eta\mu x_{(i)}^2 \tag{8}$$

By comparison between equations (7) and (8):

$$z_i = Ln\left[(1-P_i)^{-1}\right],$$
$$a = 0,\; b = \eta,\; u_i = \mu x_{(i)}^2; i = 1,2,\ldots,n. \tag{9}$$

where b can be estimated by minimizing summation of the squared error with respect to b:

$$\hat{b} = \frac{n\sum_{i=1}^{n} z_i u_i - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} u_i}{n\sum_{i=1}^{n}(u_i)^2 - \left(\sum_{i=1}^{n} u_i\right)^2}. \tag{10}$$

By substation (9) in (10), the estimator for η is :

$$\widehat{\eta}_{Rg} = \frac{A - B}{n\sum_{i=1}^{n}\mu x_{(i)}^4 - \left(\sum_{i=1}^{n}\mu x_{(i)}^2\right)^2}, \tag{11}$$

where

$A \;:\; n\sum_{i=1}^{n}\mu x_{(i)}^2\, Ln\left((1-P_i)^{-1}\right)$

$B \;:\; \sum_{i=1}^{n}\mu x_{(i)}^2 \sum_{i=1}^{n} Ln\left((1-P_i)^{-1}\right).$

Also the Rg estimators of the $(\eta_1, \eta_2, \eta_3, \eta_4)$ are:

$$\widehat{\eta}_{\zeta Rg} = \frac{C - D}{n_\zeta\sum_{i_\zeta=1}^{n_\zeta}\mu y_{\zeta_{(i_\zeta)}}^4 - \left(\sum_{i_\zeta=1}^{n_\zeta}\mu y_{\zeta_{(i_\zeta)}}^2\right)^2}, \tag{12}$$

where

$C \;:\; n_\zeta\sum_{i_\zeta=1}^{n_\zeta}\mu y_{\zeta_{(i_\zeta)}}^2\, ln\left(\left(1-P_{i_\zeta}\right)^{-1}\right)$

$D \;:\; \sum_{i_\zeta=1}^{n_\zeta}\mu y_{\zeta_{(i_\zeta)}}^2 \sum_{i_\zeta=1}^{n_\zeta} ln\left(\left(1-P_{i_\xi}\right)^{-1}\right).$

$\zeta \;=\; 1,2,3,4.$

2.3. Least Squares Method

The minimization equation is used to start with the least squares method and as follows [12]:

$$S = \sum_{i=1}^{n}\left[F(X_{(i)}) - E\left(F(X_{(i)})\right)\right]^2. \tag{13}$$

Equal $E\left(F(X_{(i)})\right)$ with P_i, so:

$$(1-P_i) = e^{-\eta\mu x_{(i)}^2},$$

then

$$\ln(1-P_i) + \eta\mu x_{(i)}^2 = 0. \tag{14}$$

The equation (13) is used in equation (14):

$$S = \sum_{i=1}^{n}\left[ln\,(1-P_i) + \eta\mu x_{(i)}^2\right]^2. \tag{15}$$

Derived equation (15), so:

$$\frac{\partial S}{\partial \delta} = \sum_{i=1}^{n} 2\left[ln\,(1-P_i) + \eta\mu x_{(i)}^2\right]\mu x_{(i)}^2$$
$$\sum_{i=1}^{n} 2\left[ln\,(1-P_i) + \eta\mu x_{(i)}^2\right]\mu x_{(i)}^2 = 0.$$

Then $\widehat{\eta}_{LS}$ is:

$$\widehat{\eta}_{LS} = \frac{\sum_{i=1}^{n}\mu x_{(i)}^2\, ln\,(1-P_i)}{-\sum_{i=1}^{n}\mu^2 x_{(i)}^4}, \tag{16}$$

and

$$\widehat{\eta}_{\zeta LS} = \frac{\sum_{i_\zeta=1}^{n_\zeta}\mu y_{\zeta_{(i_\zeta)}}^2\; ln\left(1-P_{i_\zeta}\right)}{-\sum_{i_\zeta=1}^{n_\zeta}\mu^2 y_{\zeta_{(i_\zeta)}}^4}, \zeta = 1,2,3,4. \tag{17}$$

3. Results and Discussion

The paper includes a development of a previous paper, see [15], where the reliability of the model will be derived in the next section, which includes several steps to reach the final formula, and then this formula will be estimated in three different estimations (percentile, regression, least squares). Methods and simulation work to discover the best way and obtain conclusions and rely on a group of approved sources related to the topic of the paper.

3.1. The mathematical form

In strength-stress models, the reliability of a component is determined based on its strength, which we denote by the random variable X, and the stress to which that component is exposed, which we denote by the random variable Y, where the component resists the stresses to which it is exposed with its strength and continues to work $(X > Y)$. when the stress increases and exceeds the strength $(Y > X)$, the component fails and stops working. Now the reliability of a model is expressed for one component that has one strength and is subjected to stress as follows:

$$R = pr\,(Y < X) = \int_{-\infty}^{\infty} f\,(x)\,F_y\,(x)\,dx.$$

If the component has one strength (X) and is subjected to four stresses $(Y_1, Y_2, Y\,3$ and $Y_4)$, the mathematical formula of this model is as follows:

$$R = \int_{-\infty}^{\infty} pr\,(Y_1 < X) \ldots pr\,(Y_4 < X)\, f_{x(x)} dx. \tag{18}$$

Suppose that the random variables of stress and strength are independent and identical, so the mathematical formula in Equation (19) can be written as follows:

$$\begin{aligned} R &= pr\,(Max\,(Y_1, Y_2, Y_3, Y_4) < X) \\ &= \int_0^{\infty}\int_0^{y_1}\int_0^{y_2}\int_0^{y_3}\int_0^{y_4} f(x,y) dy_4 dy_3 dy_2 dy_1 dx \\ &= \int_0^{\infty}\int_0^{y_1}\int_0^{y_2}\int_0^{y_3}\int_0^{y_4} f\,(y_1) \ldots f\,(y_4) f\,(x)\, dy_4 \ldots dx \\ &= \int_0^{\infty} F_{1y_1}\,(x)\, F_{2y_2} F_{3y_3}\,(x)\, F_{4y_4}\,(x)\, f\,(x)\, dx \end{aligned} \tag{19}$$

with

$$f(x,y) = f\,(y_1, y_2, y_3, y_4, x)\,.$$

Equations (1), (2) and (3) are used in Equation (19) to find the general formula for the reliability of the model as follows:

$$\begin{aligned} \mathcal{R} &= \int_0^{\infty} [[1-E_1][1-E_2][1-E_3][1-E_4]]\, 2\eta\mu x e^{-\eta\mu x^2} dx \\ &= \int_0^{\infty} 2\eta\mu x e^{-\eta\mu x^2} dx - \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1)\mu x^2} dx \\ &- \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_2)\mu x^2} dx - \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_3)\mu x^2} dx \\ &- \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_4)\mu x^2} dx + \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_2)\mu x^2} dx \\ &+ \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_3)\mu x^2} dx + \int_x^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_4)\mu x^2} dx \\ &+ \int_x^{\infty} 2\eta\mu x e^{-(\eta+\eta_2+\eta_3)\mu x^2} dx + \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_2+\eta_4)\mu x^2} dx \\ &+ \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_3+\eta_4)\mu x^2} dx - \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_2+\eta_3)\mu x^2} dx \\ &- \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_2+\eta_4)\mu x^2} dx - \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_3+\eta_4)\mu x^2} dx \\ &- \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_2+\eta_3+\eta_4)\mu x^2} dx + \int_0^{\infty} 2\eta\mu x e^{-(\eta+\eta_1+\eta_2+\eta_3+\eta_4)\mu x^2} d \end{aligned}$$

with

$$\begin{aligned} E_1 &= e^{-\eta_1\mu x^2} \quad & E_3 &= e^{-\eta_3\mu x^2} \\ E_2 &= e^{-\eta_2\mu x^2} \quad & E_4 &= e^{-\eta_4\mu x^2}. \end{aligned}$$

Now, the general formula for the final model reliability is as follows:

$$\begin{aligned} R = 1 &- \left[\frac{\eta}{\eta+\eta_1}\right] - \left[\frac{\eta}{\eta+\eta_2}\right] - \left[\frac{\eta}{\eta+\eta_3}\right] - \left[\frac{\eta}{\eta+\eta_4}\right] \\ &+ \left[\frac{\eta}{\eta+\eta_1+\eta_2}\right] + \left[\frac{\eta}{\eta+\eta_1+\eta_3}\right] + \left[\frac{\eta}{\eta+\eta_1+\eta_4}\right] \\ &+ \left[\frac{\eta}{\eta+\eta_2+\eta_3}\right] + \left[\frac{\eta}{\eta+\eta_2+\eta_4}\right] + \left[\frac{\eta}{\eta+\eta_3+\eta_4}\right] \\ &- \left[\frac{\eta}{\eta+\eta_1+\eta_2+\eta_3}\right] - \left[\frac{\eta}{\eta+\eta_1+\eta_2+\eta_4}\right] \\ &- \left[\frac{\eta}{\eta+\eta_1+\eta_3+\eta_4}\right] - \left[\frac{\eta}{\eta+\eta_2+\eta_3+\eta_4}\right] \\ &+ \left[\frac{\eta}{\eta+\eta_1+\eta_2+\eta_3+\eta_4}\right] \end{aligned} \tag{20}$$

3.2. Estimation

In this subsection, the reliability estimator will be found by three different estimation methods, which are (percentile, regression, and least squares). By substituting equations (5) and (6) into equation (20), a percentile estimator of the reliability function is obtained:

$$\begin{aligned} \hat{R}_{Pr} = 1 &- \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}}\right] - \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{2Pr}}\right] - \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{3Pr}}\right] \\ &- \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{4Pr}}\right] + \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{2Pr}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{3Pr}}\right] + \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{4Pr}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{2Pr}+\widehat{\eta}_{3Pr}}\right] + \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{2Pr}+\widehat{\eta}_{4Pr}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{3Pr}+\widehat{\eta}_{4Pr}}\right] - \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{2Pr}+\widehat{\eta}_{3Pr}}\right] \end{aligned} \tag{21}$$

$$- \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{2Pr}+\widehat{\eta}_{4Pr}}\right] - \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{3Pr}+\widehat{\eta}_{4Pr}}\right] - \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{2Pr}+\widehat{\eta}_{3Pr}+\widehat{\eta}_{4Pr}}\right] - \left[\frac{\widehat{\eta}_{Pr}}{\widehat{\eta}_{Pr}+\widehat{\eta}_{1Pr}+\widehat{\eta}_{2Pr}+\widehat{\eta}_{3Pr}}\right].$$

By substituting equations (11) and (12) into equation (20), a regression estimator of the reliability function is obtained:

$$\begin{aligned}\hat{R}_{Rg} =1 &- \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}}\right] - \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{2Rg}}\right] - \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{3Rg}}\right] \\ &- \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{4Rg}}\right] + \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{2Rg}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{3Rg}}\right] + \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{4Rg}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{2Rg}+\widehat{\eta}_{3Rg}}\right] + \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{2Rg}+\widehat{\eta}_{4Rg}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{3Rg}+\widehat{\eta}_{4Rg}}\right] - \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{2Rg}+\widehat{\eta}_{3Rg}}\right] \\ &- \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{2Rg}+\widehat{\eta}_{4Rg}}\right] \\ &- \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{3Rg}+\widehat{\eta}_{4Rg}}\right] \\ &- \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{2Rg}+\widehat{\eta}_{3Rg}+\widehat{\eta}_{4Rg}}\right] \\ &+ \left[\frac{\widehat{\eta}_{Rg}}{\widehat{\eta}_{Rg}+\widehat{\eta}_{1Rg}+\widehat{\eta}_{2Rg}+\widehat{\eta}_{3Rg}+\widehat{\eta}_{4Rg}}\right].\end{aligned} \tag{22}$$

By substituting equations (16) and (17) into equation (20), the least squares estimator of the reliability function is obtained:

$$\begin{aligned}\hat{R}_{LS} =1 &- \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}}\right] - \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{2LS}}\right] - \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{3LS}}\right] \\ &- \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{4LS}}\right] + \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{2LS}}\right] \\ &+ \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{3LS}}\right] + \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{4LS}}\right] \\ &+ \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{2LS}+\widehat{\eta}_{3LS}}\right] + \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{2LS}+\widehat{\eta}_{4LS}}\right] \\ &+ \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{3LS}+\widehat{\eta}_{4LS}}\right] - \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{2LS}+\widehat{\eta}_{3LS}}\right] \\ &- \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{2LS}+\widehat{\eta}_{4LS}}\right] \\ &- \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{3LS}+\widehat{\eta}_{4LS}}\right] \\ &- \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{2LS}+\widehat{\eta}_{3LS}+\widehat{\eta}_{4LS}}\right] \\ &+ \left[\frac{\widehat{\eta}_{LS}}{\widehat{\eta}_{LS}+\widehat{\eta}_{1LS}+\widehat{\eta}_{2LS}+\widehat{\eta}_{3LS}+\widehat{\eta}_{4LS}}\right].\end{aligned} \tag{23}$$

3.3. Simulation

The MATLAB program is used to conduct Monte Carlo simulation to compare different estimation methods using two statistical criteria, MSE and MAPE, to indicate which estimation methods are the best to estimate the reliability of the model, different parameter values and sample sizes were used as shown below [11, 12]. To make the simulation algorithm, the following steps were followed:

1. random samples $x_i,\ i = 1,\ldots,n;\ y_{i1},\ i1 = 1,\ldots,n_1;\ y_{i2},\ i2 = 1,\ldots,n_2;\ y_{i3},\ i3 = 1,\ldots,n_3$; and $y_{i4},\ i4 = 1,\ldots,n_4$ are generated.
2. The different sizes $(n, n_1, n_2, n_3, n_4) = A, B, C, D, E$ where $A = (15, 15, 15, 15, 15)$, $B = (25, 25, 25, 25, 25)$, $C = (45, 45, 45, 45, 45)$, $D = (80, 80, 80, 80, 80)$ and $E = (100, 100, 100, 100, 100)$ are used.
3. Ten experiments were conducted by giving values for parameters $\mu, \eta, \eta_1, \eta_2, \eta_3, \eta_4$ as shown in the Table 1:

Table 1. The values of the ten experiments

R	η_4	η_3	η_2	η_1	η	μ	Experiment
0.1108	0.2	0.2	0.2	0.2	0.3	0.8	1
0.5800	0.7	0.7	0.7	0.7	0.2	1.1	2
0.2350	0.9	0.9	0.9	1.9	0.9	0.9	3
0.2139	1.5	1.5	1.5	0.9	1.5	1.5	4
0.2488	0.3	0.3	1.8	0.3	0.3	0.3	5
0.1484	1	1	0.5	1	1	1	6
0.2500	0.1	1.9	0.1	0.1	0.1	0.1	7
0.1711	0.6	0.4	0.6	0.6	0.6	0.6	8
0.2469	1.7	0.4	0.4	0.4	0.4	0.4	9
0.2000	0.4	0.5	0.5	0.5	0.5	0.5	10

4. Equations (5), (6), (11), (12), (16) and (17) were used, which include the estimation of parameters $\eta, \eta_1, \eta_2, \eta_3$ and η_4.
5. Equations (21), (22) and (23) involved estimating the reliability of the model by estimation methods, respectively.
6. The results of different estimation methods were compared using two statistical criteria:
   (a) The mean squares error:
   $$MSE\left(\hat{R}\right) = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{R}_i - R\right)^2.$$
   (b) The mean absolute percentage error:
   $$MAPE(\hat{R}) = \frac{1}{L}\sum_{i=1}^{L}\left|\frac{\hat{R}_i - R}{R}\right|.$$

After carrying out the simulation, the results obtained are as in Table 2-11.

Table 2. The experiment (1), R = 0.1108

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0027	0.0081	0.0032	Pr
	MAPE	0.3670	0.6134	0.3985	Pr
B	MSE	0.0016	0.0052	0.0020	Pr
	MAPE	0.2898	0.4988	0.3172	Pr
C	MSE	0.0009	0.0032	0.0012	Pr
	MAPE	0.2137	0.3939	0.2399	Pr
D	MSE	0.0005	0.0019	0.0007	Pr
	MAPE	0.1638	0.3057	0.1844	Pr
E	MSE	0.0004	0.0016	0.0005	Pr
	MAPE	0.1466	0.2766	0.1652	Pr

Table 3. The experiment (2), R = 0.5800

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0076	0.0226	0.0090	Pr
	MAPE	0.1184	0.2065	0.1287	Pr
B	MSE	0.0043	0.0138	0.0053	Pr
	MAPE	0.0900	0.1612	0.0994	Pr
C	MSE	0.0024	0.0083	0.0030	Pr
	MAPE	0.0666	0.1250	0.0745	Pr
D	MSE	0.0014	0.0048	0.0017	Pr
	MAPE	0.0503	0.0954	0.0566	Pr
E	MSE	0.0011	0.0039	0.0014	Pr
	MAPE	0.0444	0.0856	0.0505	Pr

Table 4. The experiment (3), R = 0.2350

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0057	0.0155	0.0068	Pr
	MAPE	0.2598	0.4290	0.2805	Pr
B	MSE	0.0036	0.0109	0.0045	Pr
	MAPE	0.2048	0.3578	0.2270	Pr
C	MSE	0.0020	0.0069	0.0026	Pr
	MAPE	0.1533	0.2829	0.1732	Pr
D	MSE	0.0012	0.0040	0.0015	Pr
	MAPE	0.1171	0.2152	0.1315	Pr
E	MSE	0.0009	0.0034	0.0012	Pr
	MAPE	0.1040	0.1968	0.1177	Pr

Table 5. The experiment (4), R = 0.2139

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0054	0.0147	0.0064	Pr
	MAPE	0.2763	0.4557	0.3003	Pr
B	MSE	0.0033	0.0096	0.0040	Pr
	MAPE	0.2152	0.4557	0.2353	Pr
C	MSE	0.1629	0.0061	0.0024	Pr
	MAPE	0.1629	0.2924	0.1811	Pr
D	MSE	0.0011	0.0038	0.0014	Pr
	MAPE	0.1236	0.2293	0.1392	Pr
E	MSE	0.0009	0.0032	0.0011	Pr
	MAPE	0.1101	0.2082	0.1243	Pr

Table 6. The experiment (5), R = 0.2488

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0061	0.0161	0.0071	Pr
	MAPE	0.2528	0.4137	0.2724	Pr
B	MSE	0.0038	0.0111	0.0046	Pr
	MAPE	0.1980	0.3408	0.2168	Pr
C	MSE	0.0021	0.0070	0.0026	Pr
	MAPE	0.1465	0.2710	0.1657	Pr
D	MSE	0.0012	0.0042	0.0015	Pr
	MAPE	0.1113	0.2080	0.1250	Pr
E	MSE	0.0010	0.0034	0.0012	Pr
	MAPE	0.0989	0.1873	0.1119	Pr

Table 7. The experiment (6), R = 0.1484

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0037	0.0103	0.0043	Pr
	MAPE	0.3241	0.5334	0.3508	Pr
B	MSE	0.0023	0.0068	0.0028	Pr
	MAPE	0.2551	0.4344	0.2793	Pr
C	MSE	0.0012	0.0043	0.0016	Pr
	MAPE	0.1897	0.3472	0.2133	Pr
D	MSE	0.0007	0.0025	0.0009	Pr
	MAPE	0.1428	0.2651	0.1601	Pr
E	MSE	0.0006	0.0021	0.0007	Pr
	MAPE	0.1294	0.2420	0.1462	Pr

Table 8. The experiment (7), R = 0.2500

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0060	0.0163	0.0070	Pr
	MAPE	0.2481	0.4138	0.2691	Pr
B	MSE	0.0037	0.0111	0.0045	Pr
	MAPE	0.1946	0.3392	0.2148	Pr
C	MSE	0.0022	0.0069	0.0027	Pr
	MAPE	0.1490	0.2648	0.1660	Pr
D	MSE	0.0012	0.0042	0.0015	Pr
	MAPE	0.1109	0.2069	0.1249	Pr
E	MSE	0.0010	0.0034	0.0012	Pr
	MAPE	0.0995	0.1858	0.1129	Pr

Table 9. The experiment (8), R = 0.1711

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0043	0.0121	0.0051	Pr
	MAPE	0.3057	0.5088	0.3338	Pr
B	MSE	0.0026	0.0081	0.0032	Pr
	MAPE	0.2390	0.4168	0.2644	Pr
C	MSE	0.0015	0.0050	0.0019	Pr
	MAPE	0.1829	0.3244	0.2025	Pr
D	MSE	0.0008	0.0031	0.0011	Pr
	MAPE	0.1357	0.2557	0.1543	Pr
E	MSE	0.0007	0.0024	0.0009	Pr
	MAPE	0.1227	0.2285	0.1382	Pr

3.4. Discussion

By conducting the ten experiments in Table 1 and conducting the simulation, the following can be observed:

1. The reliability value is affected by different parameter values, where the reliability values decrease with the value of parameter η, while the reliability value increases with the values of parameters η_1, η_2, η_3 and η_4, and this is evident by looking at the reliability value of the model for the ten experiments in Table 1.
2. When looking at the results of the simulation procedure from Table 1 To Table 11, it turns out that the best estimate of the model's reliability is the estimate of the favorable Per-

Table 10. The experiment (9), R = 0.2469

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0061	0.0164	0.0072	Pr
	MAPE	0.2529	0.4196	0.2750	Pr
B	MSE	0.0037	0.0111	0.0045	Pr
	MAPE	0.1981	0.3419	0.2179	Pr
C	MSE	0.0021	0.0068	0.0026	Pr
	MAPE	0.1495	0.2683	0.1660	Pr
D	MSE	0.0012	0.0042	0.0015	Pr
	MAPE	0.1113	0.2080	0.1256	Pr
E	MSE	0.0010	0.0035	0.0013	Pr
	MAPE	0.1009	0.1902	0.1145	Pr

Table 11. The experiment (10), R = 0.2000

S.S.	criteria	Pr	Rg	LS	Best
A	MSE	0.0050	0.0135	0.0058	Pr
	MAPE	0.2829	0.4659	0.3052	Pr
B	MSE	0.0031	0.0095	0.0039	Pr
	MAPE	0.2237	0.4659	0.2468	Pr
C	MSE	0.0018	0.0058	0.0023	Pr
	MAPE	0.1703	0.3037	0.1900	Pr
D	MSE	0.0010	0.0035	0.0013	Pr
	MAPE	0.1255	0.2353	0.1411	Pr
E	MSE	0.0008	0.0029	0.0011	Pr
	MAPE	0.1146	0.2150	0.1296	Pr

centile method in all tables and for all different sample sizes.

4. Conclusion

The mathematical formula for the reliability of the model was found and this formula was estimated by three different estimation methods and a Monte Carlo simulation was conducted, where the results of the Monte Carlo simulation of the ten experiments, which were compared with MSE and MAPE, the results showed that the best estimator for the reliability of the model is percentile estimator compared to regression and least squares estimators.

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