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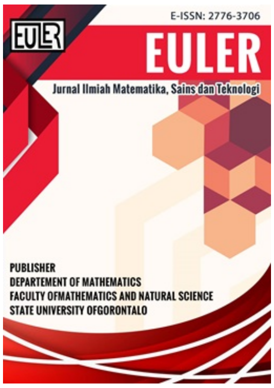

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Research Article

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Modeling the Health Service Queuing System Using Petri Net and Max-Plus Algebra at Integrated Health Service Post (Posyandu)

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ABSTRACT. *This study models the health service queue system at the Integrated Health Service Post (Posyandu) in Cilegon City, Banten, using the Petri Net and Max-Plus Algebra approaches to analyze the flow of participant arrivals and completion times. The data used are observational data from Posyandu activities simulated through a discrete event model, which includes several types of participants, namely: babies not standing yet, babies standing, pregnant mothers, and family planning programs. Petri Net modeling is used to represent the relationship between service transitions, while Max-Plus Algebra is used to calculate the process cycle time based on the critical path. The results of the study showed that the categories of non-standing babies, standing babies, and pregnant women/participants in the family planning program had identical service time patterns, namely a total duration of 17 minutes 19 seconds, with the main stages including measurement, midwife intervention, and provision of additional food. Max-Plus analysis confirms that the measurement and midwife intervention stages are the critical path that determines the length of service time. This study concludes that the combination of Petri Net and Max-Plus Algebra is effective in describing the dynamics of Posyandu queues and is able to provide quantitative information needed to identify bottleneck points and the basis for improving the service flow.*

1. Introduction

Integrated Health Service Post (Posyandu) are community-based activities under the auspices of government agencies to provide basic health services to the community [1, 2]. The target of the posyandu work program is babies not standing yet, babies standing, pregnant mothers, and family planning programs [1, 3, 4]. The presence of posyandu in Indonesia is currently almost evenly distributed. By 2024, there were 395 posyandu in 8 sub-districts in the Cilegon City area [5]. Each posyandu in Cilegon City is supervised by one village midwife. Several common problems encountered in Posyandu operations include limited human resources and facilities, which result in lengthy service processes and long queues [3, 4]. If left unchecked, this can lead to a lack of public interest in visiting Posyandus. This is also observed at Posyandus in Cilegon City. This research is important to conduct because if the problem of the long service process and long queues that occur at the Cilegon City Posyandu continues to be ignored, it will cause a lack of public interest in visiting the Posyandu, and public health services will also be disrupted.

In addressing the queuing system problem at Posyandu, previous researchers have used

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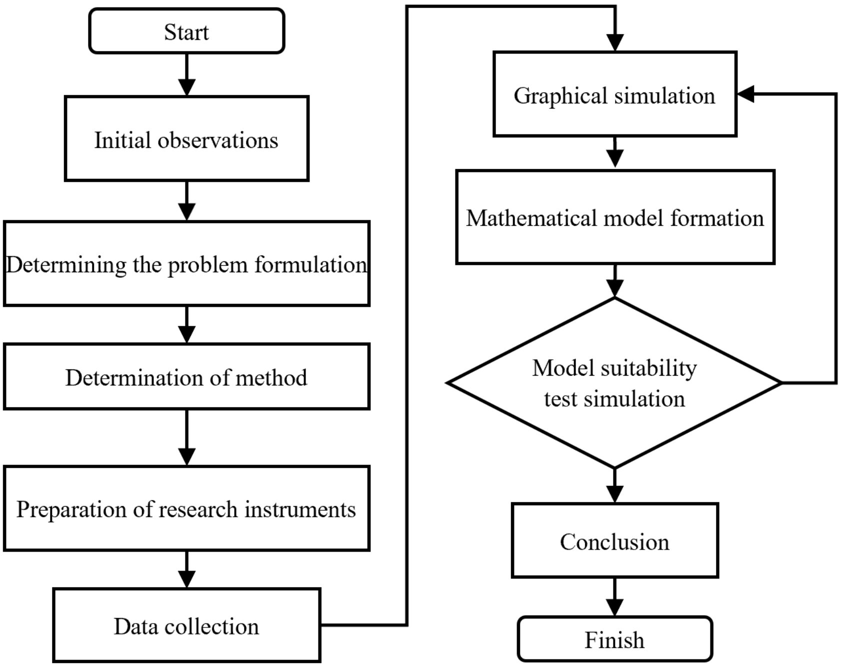

Figure 1. Flowchart of research steps

several approaches and solutions, such as using a web-based information system design method [1, 2, 6, 7] and the waiting line method [8]. One approach that can be used is the mathematical modeling of the Posyandu queuing system. Numerous methods exist for mathematical modeling of queuing systems, including Max-Plus Algebra [8–10]. Max-Plus Algebra, denoted $(R_\varepsilon, \oplus, \otimes)$, or $R_{\max}$, is a non-empty set $R_\varepsilon := R \cup \{\varepsilon\}$ with $\varepsilon = -\infty$, accompanied by two binary operations: $x \oplus y := \max(x, y)$ and $x \otimes y := x + y$ for each $x, y \in R_\varepsilon$ [11]. Furthermore, a tool called a Petri Net can be used to simulate mathematical modeling graphically. Petri Nets are widely used in queuing and scheduling system problems, such as in clinic queuing systems [12], bank customer services [13], air defense systems [14], and food production [15].

A system can be viewed as consisting of several services used by users. The start and end of a service in the system can be determined by mathematical modeling using Max-Plus Algebra, so that synchronization and concurrency of all services can occur properly [16]. Good service synchronization and concurrency can overcome queuing problems. Mathematical modeling of queuing systems using Petri Nets and Max-Plus Algebra in health services has been carried out on queuing systems in clinics [12], but this modeling has never been carried out at Posyandu. Based on these problems, an evaluation of the Posyandu queuing system in Cilegon City is necessary. Therefore, research will be conducted on the Posyandu queuing system in Cilegon City, Banten, using Petri Nets and Max-Plus Algebra.

To model the Posyandu queuing system in Cilegon City, it is necessary to collect data on the queues for Posyandu services that can represent 8 sub-districts in Cilegon City. From this data, mathematical modeling will be performed using Max-Plus Algebra. Next, graphical simulations will be conducted using Petri Nets. Simulations will also be conducted using data on the number of participants from the selected Posyandus. The results of this study are expected to produce an appropriate model related to the Posyandu queuing system in Cilegon City, thus improving the level of service to the community.

2. Methods

This study used a quantitative method with a population of 395 Integrated Health Service Post (Posyandu) in Cilegon City. The research sample was taken using a simple random sampling method, namely, simple random sampling of Posyandu queue data in 3 Districts in Cilegon City, Banten. The data collected included: (1) service type data, (2) duration of each service in a system, (3) initial or start time of the service, and (4) number of Posyandu participants, including babies not standing yet, babies standing, pregnant mothers, and family planning program participants.

The research steps are shown in Figure 1. Initial observations were conducted at Posyandu in Kotabumi District, Cilegon City. Observations at the Posyandu revealed a significant number of queues, with both pregnant women and toddlers gathering together, particularly at the midwife service post.

There are achievement indicators at each stage of the research, which are presented in Table 1.

Table 1. Research stage achievement indicators

No	Activity	Achievement indicator
1	Initial observation	Queue conditions at the Posyandu are identified
2	Problem formulation	Problem formulation is formulated
3	Method determination	Research methods for problem-solving are determined
4	Research instrument development	Research instruments are formulated, including: - Interview guidelines - Observation sheets
5	Data collection	Data collected, including: - Type of service - Duration of each service - Initial or start time of service - Number of Posyandu participants
6	Mathematical model formation	A mathematical model of Posyandu queues using Max-Plus Algebra is developed
7	Graphical simulation	A graphical simulation of Posyandu queues using Petri Nets is developed
8	Model suitability test simulation	Model conformity to real phenomena
9	Conclusion drawing	Conclusion obtained

The elements of a Max-Plus Algebra are real numbers and $\varepsilon = -\infty$. Furthermore, the set $R \cup \{\varepsilon\}$ is written as R_{max} where R is the set of real numbers. The basic operations of Max-Plus Algebra are maximization (denoted by the symbol $\oplus$) and addition (denoted by the symbol $\otimes$) [17, 18]. With these two operations, for any $x, y \in R_{\text{max}}$ we obtain:

$$x \oplus y = \max(x, y) \text{ dan } x \otimes y = x + y. \tag{1}$$

Furthermore, in the context of Max-Plus Algebra $a \otimes b := ab$. Note that: $x \oplus \varepsilon = x = \varepsilon \oplus x$ and $x \otimes 0 = x = 0 \otimes x$ for all $x \in R_{\text{max}}$. The operations $\oplus$ (max) and $\otimes$ (add) are extended to matrices as follows: for $A, B \in R_{\text{max}}^{m\times n}$,

$$[A \oplus B]_{i,j} = a_{i,j} \oplus b_{i,j} = \max\{a_{i,j}, b_{i,j}\}, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n, \tag{2}$$

and for $A \in R_{\max}^{m\times p}, B \in R_{\max}^{p\times n}$,

$$[A \otimes B]_{i,j} = \bigoplus_{k=1}^{p} a_{i,k} \otimes b_{k,j} = \max_{1\leq k\leq p} \{a_{i,k} + b_{k,j}\}, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n. \tag{3}$$

Next, consider a directed graph of a matrix $A \in R_{\max}^{n\times n}$. A directed graph of a matrix A is denoted by $G(A) = (E, V)$. Graph $G(A)$ has n vertices, and the set of all vertices of $G(A)$ is denoted by V. An edge from a vertex j to vertex i exists if $a_{i,j} \neq \varepsilon$, and this edge is denoted by (j, i). The set of all edges of a graph $G(A)$ is denoted by E. The weight of the edge (j, i) is the value of $a_{i,j}$. If $a_{i,j} = \varepsilon$, then edge (j, i) does not exist. A sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{l-1}, i_l)$ in a graph is called a path. A path is said to be elementary if no vertex occurs twice in the path. A circuit is a closed elementary path. The weight of a path $p = (i_1, i_2), (i_2, i_3), \ldots, (i_{l-1}, i_l)$ is given by $(a_{i_2,i_1} + a_{i_3,i_2} + \cdots + a_{i_l,i_{l-1}})$, while its average weight is the weight of p divided by the number of edges in p, that is, $(a_{i_2,i_1} + a_{i_3,i_2} + \cdots + a_{i_l,i_{l-1}})/(l-1)$ [16].

The mean circuit is the average weight of a circuit. Any circuit with the maximum mean circuit is called a critical circuit. A graph is said to be strongly connected if a path exists from every vertex i to every vertex j. In such a case, the matrix associated with the graph $G(A)$ is called an irreducible matrix. If the graph $G(A)$ is not strongly connected, then the matrix A is reducible.

A Petri Net, also known as a place/transition net (PT net), is one of several mathematical modeling languages for describing distributed systems [19]. It is a class of discrete-event dynamical systems. A Petri Net is a directed bipartite graph containing two types of elements: places and transitions. Place elements are depicted as white circles, and transition elements are depicted as rectangles. A place can contain any number of tokens, which are depicted as black circles. A transition is activated if all places connected to it as inputs contain at least one token.

A Petri Net is a net of the form $PN = (N, M, W)$, which extends elementary nets so that

1. $N = (P, T, F)$ is a net.
2. $M : P \to \mathbb{Z}$ is a set of places, where $\mathbb{Z}$ is a countable set. M extends the concept of configuration and is generally described by referring to a Petri Net diagram as a marker.
3. $W : F \to \mathbb{Z}$ is a set of arcs, so the count (or weight) for each arc is a measure of its multiplicity [16].

In a Petri Net diagram, places are conventionally depicted by circles, transitions by long, narrow rectangles, and arcs as one-way arrows indicating the relationship of places to transitions or transitions to places [20]. If the diagram were an elementary net, then the places in a configuration would be conventionally depicted as circles, where each circle encompasses a single point called a token. In a given Petri Net diagram, a place circle may encompass more than one token to indicate the number of times a place appears in a configuration. The configuration of tokens distributed throughout the Petri Net diagram is called the tagging or labelling.

3. Results and Discussion

Data collection in this study was carried out from August to September 2025. The data collection locations were in Purwakarta Village (Posyandu Wates Telu), in Citangkil Village (Posyandu Refflesia), and in Ketileng Village (Posyandu Flamboyan 1). The average attendance

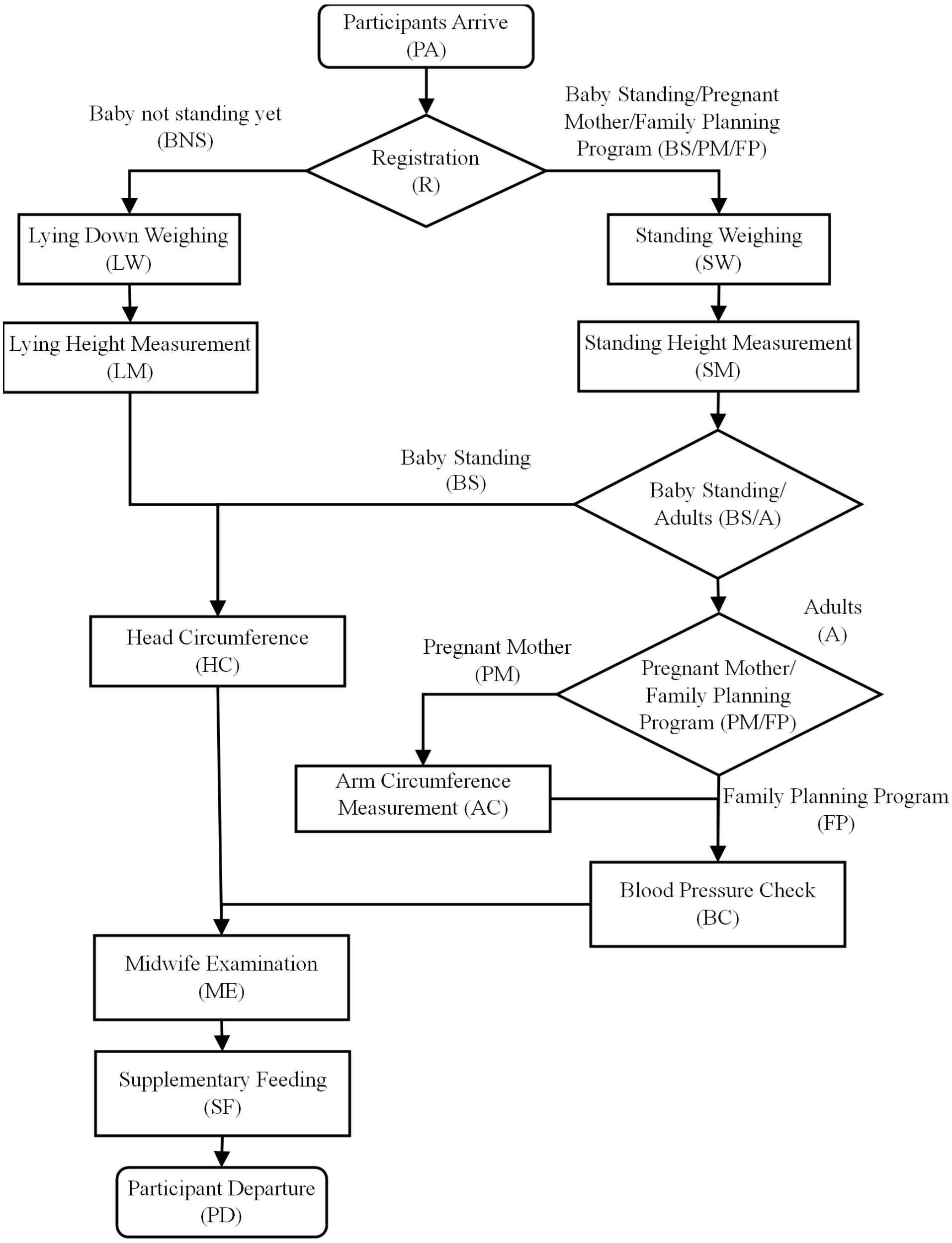

Figure 2. Flowchart of posyandu queue system

at each Posyandu was 30 participants per location. However, this study only collected data from 20 participants from each location.

Based on observations at the Cilegon City Posyandu, a queue model and service flow diagram were obtained. The queue model for health services at the Posyandu can be written as follows:

1. Participants arrive (PA)
2. Participants register. Participants are checked for completeness, including their Maternal and Child Health (KIA) Handbook. Participants are categorized by age and condition: Baby not Standing yet (BNS), Baby Standing (BS), Pregnant mothers (PM), and adults in Family Planning programs (FP).
3. Baby not Standing yet (BNS) undergo a Lying Down Weight (LW) measurement followed by a supine Lying Height Measurement (LM).
4. BS, PM, and FP undergo a Standing Weight (SW) measurement followed by a Standing

Height Measurement (SM).

5. After the measurements, BS joins BNS to wait in line for a Head Circumference (HC) measurement.
6. Adults in the PM category undergo an arm circumference (AC) measurement before joining FP for a Blood Pressure (BC) check.
7. After having their head circumferences measured, BS and BNS wait in line to be examined by the midwife (ME).
8. After the PM and FP had their blood pressure measured, the PM and FP waited in line to be examined by the midwife (ME).
9. After the midwife's examination, BS, BNS, PM, and FC waited to receive Supplementary Feeding (SF) before leaving the facility.

The queue model above can be described in the form of a flow diagram as in Figure 2.

Field observations also revealed the types of services and types of cadres. The types of services and number of cadres are presented in Table 2.

Table 2. The types of services and number of cadres

No	Service type	Number of cadres
1	Registration	1
2	Weighing and height measurement (baby not yet standing)	1
3	Weighing and height measurement (baby standing/pregnant women/women using family planning)	1
4	Head circumference measurement (baby not yet standing/baby standing)	1
5	Arm circumference measurement and blood pressure (pregnant women/women using family planning)	1
6	Midwife examination	1
7	Supplemental feeding	1

Based on the queue system obtained, it can be formed into a Petri Net model as in Figure 3.

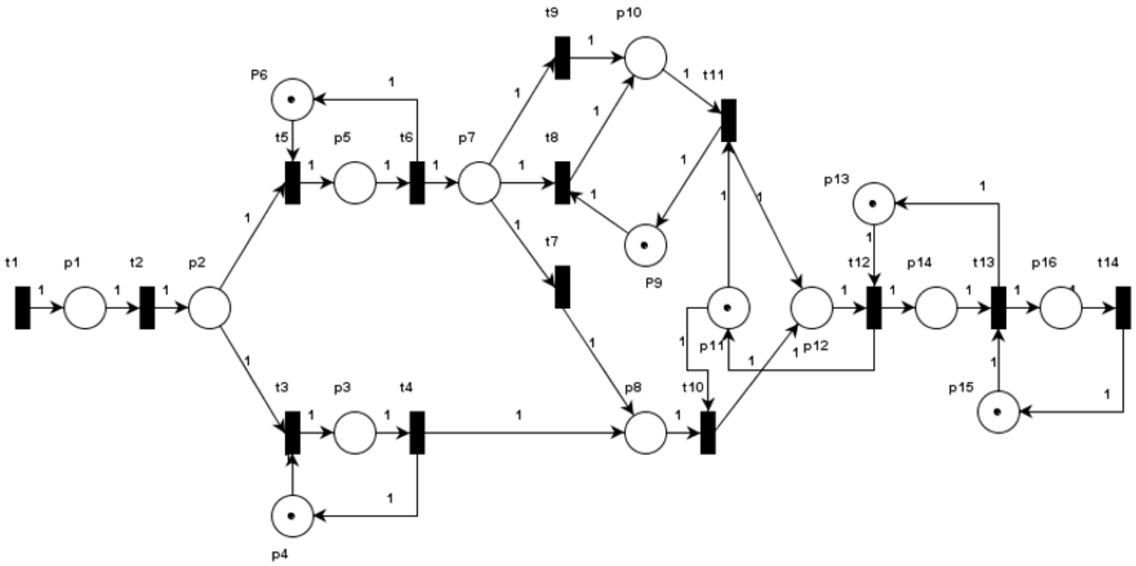

Figure 3. Petri net of posyandu queue system

The Petri Net in Figure 3 consists of two sets of points. The finite set of places is $P = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$ and the finite set of transitions in the Petri Net is $T = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14}\}$.

The transition information in the Petri Net is explained as follows:

t_1 : Participant arrives at the Posyandu

t_2 : Participant registers

t_3 : Participant (baby not yet standing) is weighed supine

t_4 : Participant (baby not yet standing) is measured supine

t_5 : Participant (baby standing/pregnant woman/family planning woman) is weighed standing

t_6 : Participant (baby standing/pregnant woman/family planning woman) is measured standing height

t_7 : Transition without time for participant (baby standing) before measuring head circumference

t_8 : Participant (pregnant woman) is measuring arm circumference

t_9 : Transition without time for the participant (family planning woman) before measuring blood pressure

t_{10} : Participant (baby not yet standing/baby standing) is measured for head circumference

t_{11} : Participant (pregnant woman/family planning woman) is measured blood pressure measured

t_{12} : Participant receives a midwife examination

t_{13} : Participant receives supplementary food

t_{14} : Participant leaves Posyandu

Meanwhile, the location information on the Petri Net is as follows:

p_1 : Participants waiting to register

p_2 : Participants waiting to be weighed by Posyandu cadres

p_3 : Participants (babies not yet standing) waiting to have their height measured lying down

p_4 : Idle Posyandu cadres weighing and measuring height (babies not yet standing)

p_5 : Participants (babies/pregnant women/family planning women) waiting to have their height measured standing

p_6 : Idle Posyandu cadres weighing and measuring height (baby standing/pregnant women/family planning women)

p_7 : Participants (pregnant women) waiting to have their arm circumference measured

p_8 : Participants (babies not yet standing/babies standing) waiting to have their head circumference measured

p_9 : Idle Posyandu cadres measuring arm circumference and blood pressure (pregnant women/family planning women)

p_{10} : Participants (pregnant women/family planning women) are waiting for blood pressure measurement

p_{11} : Idle Posyandu cadres measuring head circumference (babies not yet standing/babies standing)

p_{12} : Participants waiting to be examined by a midwife

p_{13} : Idle midwife

p_{14} : Participants waiting to be given supplementary food

p_{15} : Idle Posyandu cadres distributing supplementary food

p_{16} : Participants who have finished being given supplementary food

The variables used in the Max-Plus Algebra model with time are shown as follows:

- Variables that indicate time:

  $t_1(k)$: Participant arrival time at time k

  $t_2(k)$: Participant registration time at time k

  $t_3(k)$: Participant (baby not yet standing) weighed supine at time k

  $t_4(k)$: Participant (baby not yet standing) measured supine height at time k

  $t_5(k)$: Participant standing baby/pregnant woman/family planning woman) weighed standing at time k

  $t_6(k)$: Participant standing baby/pregnant woman/family planning woman) measured standing height at time k

  $t_8(k)$: Participant, pregnant woman, measured arm circumference at time k

  $t_{10}(k)$: Participant head circumference at time k

  $t_{11}(k)$: Participant, pregnant woman/family planning woman, had her blood pressure measured at time k

  $t_{12}(k)$: Participant midwife examination at time k

  $t_{13}(k)$: The time the participant received additional food at k

  $t_{14}(k)$: The time the participant left the Posyandu at k
- Variables that indicate the length of time:

  $v_{t_{1,k}}$: Time taken for participants to arrive at time k

  $v_{t_{2,k}}$: Time taken for participants to register at time k

  $v_{t_{3,k}}$: Time taken for participants (babies not yet standing) to weigh themselves on their backs at time k

  $v_{t_{4,k}}$: Time taken for participants (babies not yet standing) to measure their height on their backs at time k

  $v_{t_{5,k}}$: Time taken for participants (babies standing/pregnant women/family planning women) to weigh themselves standing at time k

  $v_{t_{6,k}}$: Time taken for participants (babies standing/pregnant women/family planning women) to measure their height standing at time k

  $v_{t_{8,k}}$: Time taken for participants (pregnant women) to measure their arm circumference at time k

  $v_{t_{10,k}}$: Time taken for participants (babies not yet standing/babies standing) to measure their head circumference at time k

  $v_{t_{11,k}}$: The time the participant (pregnant woman/family planning woman) took their blood pressure measurement at time k

  $v_{t_{12,k}}$: The time the participant received a midwife examination at time k

  $v_{t_{13,k}}$: The time the participant received additional food at time k

  $v_{t_{14,k}}$: The time the participant left the Posyandu at time k

Based on the Petri Net model shown in Figure 3, the Max-Plus Algebra model with time is as follows:

$$\begin{aligned}
t_1(k) &= v_{t_{1,k}} \otimes t_1(k-1) \\
t_2(k) &= v_{t_{2,k}} \otimes t_1(k) \\
&= v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1) \\
t_3(k) &= v_{t_{3,k}} \otimes (t_2(k) \oplus t_4(k-1)) \\
&= (v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{3,k}} \otimes t_4(k-1)) \\
t_4(k) &= v_{t_{4,k}} \otimes t_3(k) \\
&= v_{t_{4,k}} \otimes \big((v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{3,k}} \otimes t_4(k-1))\big)
\end{aligned}$$

$$
\begin{aligned}
&= (v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes t_4(k-1)) \\
t_5(k) &= v_{t_{5,k}} \otimes (t_2(k) \oplus t_6(k-1)) \\
&= (v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{5,k}} \otimes t_6(k-1)) \\
t_6(k) &= v_{t_{6,k}} \otimes t_5(k) \\
&= v_{t_{6,k}} \otimes \big((v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{5,k}} \otimes t_6(k-1))\big) \\
&= (v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
t_8(k) &= v_{t_{8,k}} \otimes (t_6(k) \oplus t_{11}(k-1)) \\
&= v_{t_{8,k}} \otimes \big((v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \oplus t_{11}(k-1)\big) \\
&= (v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \oplus (v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\quad \oplus (v_{t_{8,k}} \otimes t_{11}(k-1)) \\
t_{11}(k) &= v_{t_{11,k}} \otimes (t_8(k) \oplus t_{12}(k-1)) \\
&= v_{t_{11,k}} \otimes \big((v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \oplus (v_{t_{8,k}} \otimes t_{11}(k-1)) \oplus t_{12}(k-1)\big) \\
&= (v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \oplus (v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes t_{11}(k-1)) \\
&\quad \oplus (v_{t_{11,k}} \otimes t_{12}(k-1)) \\
t_{12}(k) &= v_{t_{12,k}} \otimes (t_{10}(k) \oplus t_{11}(k) \oplus t_{13}(k-1)) \\
&= v_{t_{12,k}} \otimes \big((v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes t_4(k-1)) \\
&\quad \oplus (v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \oplus (v_{t_{10,k}} \otimes t_{12}(k-1)) \\
&\quad \oplus (v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\quad \oplus (v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes t_{11}(k-1)) \oplus (v_{t_{11,k}} \otimes t_{12}(k-1)) \oplus t_{13}(k-1)\big) \\
t_{13}(k) &= v_{t_{13,k}} \otimes (t_{12}(k) \oplus t_{14}(k-1)) \\
&= v_{t_{13,k}} \otimes \big((v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes t_4(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes t_{12}(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes t_{11}(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes t_{12}(k-1)) \\
&\quad \oplus (v_{t_{12,k}} \otimes t_{13}(k-1)) \oplus t_{14}(k-1)\big) \\
t_{14}(k) &= v_{t_{14,k}} \otimes t_{13}(k) \\
&= v_{t_{14,k}} \otimes \big((v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\quad \oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes t_4(k-1)) \\
&\quad \oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1))
\end{aligned}
$$

$$
\begin{aligned}
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes t_{12}(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes t_{11}(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes t_{12}(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes t_{13}(k-1)) \\
&\oplus (v_{t_{13,k}} \otimes t_{14}(k-1))\big) \\
= &\,(v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes t_4(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes t_{12}(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \otimes t_1(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes t_6(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes t_{11}(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes t_{12}(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes t_{13}(k-1)) \\
&\oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes t_{14}(k-1))
\end{aligned}
$$

Suppose $j = \{1, 4, 6, 11, 12, 13, 14\}$, then only $t_j(k)$ is used in the Max-Plus Algebra matrix for each j, analogous to $t_j(k-1)$. Thus, the model of the Posyandu health service queue system in Cilegon City, Banten, is obtained in the form of a Max-Plus Algebra matrix with time, namely,

$$t_j(k) = A \otimes t_j(k-1), \tag{4}$$

for $k = 1, 2, 3, \ldots$ then,

$$
\begin{bmatrix} t_1(k) \\ t_4(k) \\ t_6(k) \\ t_{11}(k) \\ t_{12}(k) \\ t_{13}(k) \\ t_{14}(k) \end{bmatrix}
=
\begin{bmatrix}
v_{t_{1,k}} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
a & b & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
c & \varepsilon & d & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
e & \varepsilon & f & g & v_{t_{11,k}} & \varepsilon & \varepsilon \\
h & i & j & k & l & v_{t_{12,k}} & \varepsilon \\
m & n & o & p & q & r & v_{t_{13,k}} \\
s & t & u & v & w & x & y
\end{bmatrix}
\begin{bmatrix} t_1(k-1) \\ t_4(k-1) \\ t_6(k-1) \\ t_{11}(k-1) \\ t_{12}(k-1) \\ t_{13}(k-1) \\ t_{14}(k-1) \end{bmatrix}
$$

with the following information:

$$
\begin{aligned}
a &= v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \\
b &= v_{t_{4,k}} \otimes v_{t_{3,k}} \\
c &= v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \\
d &= v_{t_{6,k}} \otimes v_{t_{5,k}} \\
e &= v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}} \\
f &= v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \\
g &= v_{t_{11,k}} \otimes v_{t_{8,k}}
\end{aligned}
$$

$$
\begin{aligned}
h &= (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
i &= v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \\
j &= (v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}}) \\
&\quad \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}}) \\
k &= v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \\
l &= (v_{t_{12,k}} \otimes v_{t_{10,k}}) \oplus (v_{t_{12,k}} \otimes v_{t_{11,k}}) \\
m &= (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
&\quad \oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
&\quad \oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
n &= v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \\
o &= (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}}) \\
&\quad \oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}}) \\
p &= v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \\
q &= (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}}) \oplus (v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}}) \\
r &= v_{t_{13,k}} \otimes v_{t_{12,k}} \\
s &= (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
&\quad \oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
&\quad \oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}} \otimes v_{t_{2,k}} \otimes v_{t_{1,k}}) \\
t &= v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{4,k}} \otimes v_{t_{3,k}} \\
u &= (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}}) \\
&\quad \oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \otimes v_{t_{6,k}} \otimes v_{t_{5,k}}) \\
v &= v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}} \otimes v_{t_{8,k}} \\
w &= (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{10,k}}) \oplus (v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \otimes v_{t_{11,k}}) \\
x &= v_{t_{14,k}} \otimes v_{t_{13,k}} \otimes v_{t_{12,k}} \\
y &= v_{t_{14,k}} \otimes v_{t_{13,k}}
\end{aligned}
$$

Next, the arrival time and the time to the k-th health service (in seconds) were calculated based on direct observations at three Posyandu in Cilegon City. This value was taken from the average time from arrival, service, to participant departure. These values are presented in Table 3.

Table 3. Arrival and service time

	Arrival and service time											
	$v_{t_{1,k}}$	$v_{t_{2,k}}$	$v_{t_{3,k}}$	$v_{t_{4,k}}$	$v_{t_{5,k}}$	$v_{t_{6,k}}$	$v_{t_{8,k}}$	$v_{t_{10,k}}$	$v_{t_{11,k}}$	$v_{t_{12,k}}$	$v_{t_{13,k}}$	$v_{t_{14,k}}$
Time (seconds)	239	89	32	32	16	17	8	16	61	343	245	21

By substituting the values in Table 3 into the matrix of eq. (4), the following results are obtained:

$$\begin{bmatrix} t_1(k) \\ t_4(k) \\ t_6(k) \\ t_{11}(k) \\ t_{12}(k) \\ t_{13}(k) \\ t_{14}(k) \end{bmatrix} = \begin{bmatrix} 239 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 392 & 64 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 361 & \varepsilon & 33 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 430 & \varepsilon & 102 & 69 & 61 & \varepsilon & \varepsilon \\ 773 & 423 & 400 & 367 & 404 & 343 & \varepsilon \\ 1018 & 668 & 690 & 657 & 649 & 588 & 245 \\ 1039 & 689 & 711 & 678 & 670 & 609 & 266 \end{bmatrix} \otimes \begin{bmatrix} t_1(k-1) \\ t_4(k-1) \\ t_6(k-1) \\ t_{11}(k-1) \\ t_{12}(k-1) \\ t_{13}(k-1) \\ t_{14}(k-1) \end{bmatrix}$$

where

$$\begin{aligned}
a &= 32 \otimes 32 \otimes 89 \otimes 239 = 392 \\
b &= 32 \otimes 32 = 64 \\
c &= 17 \otimes 16 \otimes 89 \otimes 239 = 361 \\
d &= 17 \otimes 16 = 33 \\
e &= 61 \otimes 8 \otimes 17 \otimes 16 \otimes 89 \otimes 239 = 430 \\
f &= 61 \otimes 8 \otimes 17 \otimes 16 = 102 \\
g &= 61 \otimes 8 = 69 \\
h &= (343 \otimes 16 \otimes 32 \otimes 32 \otimes 89 \otimes 239) \oplus (343 \otimes 16 \otimes 17 \otimes 16 \otimes 89 \otimes 239) \\
&\quad \oplus (343 \otimes 61 \otimes 8 \otimes 17 \otimes 16 \otimes 89 \otimes 239) \\
&= 751 \oplus 720 \oplus 773 = 773 \\
i &= 343 \otimes 16 \otimes 32 \otimes 32 = 423 \\
j &= (343 \otimes 16 \otimes 17 \otimes 16) \oplus (343 \otimes 61 \otimes 8 \otimes 17 \otimes 16) \\
&= 392 \oplus 400 = 400 \\
k &= 343 \otimes 61 \otimes 8 = 367 \\
l &= (343 \otimes 16) \oplus (343 \otimes 61) = 359 \oplus 404 = 404 \\
m &= (245 \otimes 343 \otimes 16 \otimes 32 \otimes 32 \otimes 89 \otimes 239) \\
&\quad \oplus (245 \otimes 343 \otimes 16 \otimes 17 \otimes 16 \otimes 89 \otimes 239) \\
&\quad \oplus (245 \otimes 343 \otimes 61 \otimes 8 \otimes 17 \otimes 16 \otimes 89 \otimes 239) \\
&= 996 \oplus 965 \oplus 1018 = 1018 \\
n &= 245 \otimes 343 \otimes 16 \otimes 32 \otimes 32 = 668 \\
o &= (245 \otimes 343 \otimes 16 \otimes 17 \otimes 16) \oplus (245 \otimes 343 \otimes 61 \otimes 8 \otimes 17 \otimes 16) \\
&= 637 \oplus 690 = 690 \\
p &= 245 \otimes 343 \otimes 61 \otimes 8 = 657 \\
q &= (245 \otimes 343 \otimes 16) \oplus (245 \otimes 343 \otimes 61) = 604 \oplus 649 = 649 \\
r &= 245 \otimes 343 = 588 \\
s &= (21 \otimes 245 \otimes 343 \otimes 16 \otimes 32 \otimes 32 \otimes 89 \otimes 239) \\
&\quad \oplus (21 \otimes 245 \otimes 343 \otimes 16 \otimes 17 \otimes 16 \otimes 89 \otimes 239) \\
&\quad \oplus (21 \otimes 245 \otimes 343 \otimes 61 \otimes 8 \otimes 17 \otimes 16 \otimes 89 \otimes 239) \\
&= 1017 \oplus 986 \oplus 1039 = 1039 \\
t &= 21 \otimes 245 \otimes 343 \otimes 16 \otimes 32 \otimes 32 = 689 \\
u &= (21 \otimes 245 \otimes 343 \otimes 16 \otimes 17 \otimes 16) \oplus (21 \otimes 245 \otimes 343 \otimes 61 \otimes 8 \otimes 17 \otimes 16)
\end{aligned}$$

$$
\begin{aligned}
&= 658 \oplus 711 = 711 \\
v &= 21 \otimes 245 \otimes 343 \otimes 61 \otimes 8 = 678 \\
w &= (21 \otimes 245 \otimes 343 \otimes 16) \oplus (21 \otimes 245 \otimes 343 \otimes 61) = 625 \oplus 670 = 670 \\
x &= 21 \otimes 245 \otimes 343 = 609 \\
y &= 21 \otimes 245 = 266.
\end{aligned}
$$

Based on the results above, it can be concluded that:

1. A non-standing infant Posyandu participant arrives at 08:30:00 WIB, will have their weight and height measured at 08:36:32 WIB, will have their midwife's intervention completed at 08:42:53 WIB, will have their complementary feeding completed at 8:46:58 WIB, and will be discharged at 08:47:19 WIB. This means the entire process takes 17 minutes and 19 seconds.
2. A standing baby Posyandu participant arrives at 8:30:00 WIB, will have their weight and height measured at 08:36:01 WIB, will have their midwife's intervention completed at 08:42:53 WIB, will have their complementary feeding completed at 08:46:58 WIB, and will be discharged at 08:47:19 WIB. This means the entire process takes 17 minutes and 19 seconds.
3. Pregnant women/family planning participants arrive at 08:30:00 WIB, will be weighed and have their height measured at 08:36:01 WIB, their blood pressure measured at 08:37:10 WIB, their midwife's intervention will be completed at 08:42:53 WIB, they will receive complementary food at 08:46:58 WIB, and they will go home at 08:47:19 WIB. This means the entire process will take 17 minutes and 19 seconds.

The analysis of the health service queue system at the Cilegon Integrated Health Service Post (Posyandu), Banten, using mathematical approaches such as Petri Net and Max-Plus Algebra, shows that there are discrete, interrelated service patterns among the various stages. In other words, waiting times are influenced by the longest-duration activities. This study identified bottlenecks in the process that hamper the overall system. These bottlenecks occur at the weighing and recording stages, which require a longer time than other stages. This explains why queues accumulate at this stage, particularly during early arrivals when the number of babies is high. One cause is the limited number of available cadres, resulting in a low level of service. So the role and number of Posyandu cadres are important factor that needs attention [4]. This aligns with the freedom structure in the Petri Net model, which was then quantitatively evaluated using the Max-Plus equation, allowing for the identification of critical service paths.

This study found that the Petri Net can provide a clear and easily understood process overview for Posyandu managers. Meanwhile, Max-Plus Algebra offers a precise mathematical approach for analyzing time dynamics and service cycles. Another study examining healthcare queues using Petri Nets at community health centers (Posyandu) and clinics showed that bottlenecks typically occur during physical examinations or initial checkups, which require longer durations. This finding aligns with studies in the literature on discrete-event-based queuing systems, which confirm that long service times are a determining factor in overall waiting times. This study expands the scope of application of the Petri Net and Max-Plus Algebra methods in health services and the importance of optimizing the queuing system at Posyandu [12].

The results of modeling using Petri Nets and Max-Plus Algebra revealed the root causes of queues at Posyandu. These findings provide data-based recommendations that increasing capacity at the weighing stage, either through additional personnel or task allocation, can significantly reduce total waiting times. This information can serve as a basis for routine evaluation of Posyandu services, support scheduling and task distribution planning, and simulate flow changes before implementation in real-world settings. It is hoped that the results of this study will lead to improvements in the healthcare service management system at the Posyandu level.

4. Conclusion

This study demonstrates that the application of a Posyandu queuing system model in Posyandu Cilegon City, Banten, using Petri Net and Max-Plus Algebra can identify challenging service steps, particularly weighing and recording child growth. The Petri Net model clearly depicts the service flow, while Max-Plus Algebra quantitatively measures cycle length and the relationships between activities.

A shortage of cadres, longer service times, and uneven task distribution are the main causes of increased waiting times. The findings of this study are consistent with previous research showing that processes with the longest service times affect queuing system performance. The results of this study expand the use of Petri Net and Max-Plus in the context of public health services. This research suggests increasing capacity at the weighing stage to reduce waiting times and improve the effectiveness of Posyandu services.

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