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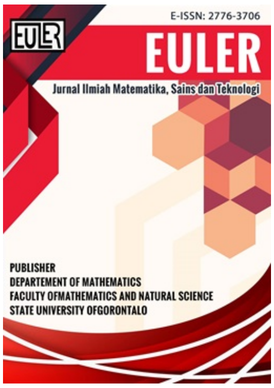

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Research Article

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Stability, Sensitivity, and Bifurcation Analysis of a Rice–Pest Interaction Model

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ABSTRACT. *This study develops a five-dimensional mathematical model describing the interaction between rice and four major pests: rice stem borer, rat, brown planthopper, and rice bug. The existence and local stability of several equilibrium points are analyzed using linearization and the Routh–Hurwitz criterion. In addition, the global stability of the pest-free equilibrium point is established using Lyapunov's direct method and LaSalle's Invariance Principle. The results show that the pest-free equilibrium is globally asymptotically stable under certain threshold conditions related to the interaction and mortality parameters. Furthermore, a transcritical bifurcation is identified, which determines the transition between pest extinction and coexistence. Sensitivity analysis indicates that the rice bug parameters significantly influence the rice population, with proportional effects on the equilibrium state. Numerical simulations are performed to support the analytical results, including the dynamic behavior around the bifurcation threshold and the sensitivity of the rice population with respect to key parameters. The results highlight the importance of controlling pest interaction rates and increasing natural mortality to maintain the stability of the rice population.*

1. Introduction

Rice production plays a crucial role in food security, particularly in developing countries. However, rice crops are highly vulnerable to multiple pest attacks, which can significantly reduce yield and threaten agricultural sustainability [1]. Mathematical modeling using systems of nonlinear differential equations is an important approach in understanding the dynamics of population interactions within an ecosystem [2]. Interaction models, particularly the classical Lotka–Volterra equations [3], have been widely used to explain long-term population behavior, equilibrium stability, and species coexistence conditions [4]. Notably, the nonlinear dynamics of complex ecosystems can often be effectively captured by relatively simple mathematical models [5]. This two-dimensional model serves as the foundation for the development of more realistic interaction models, one of which incorporates environmental carrying capacity [6]. This approach can be extended beyond two-dimensional systems to higher-dimensional systems involving one resource and multiple consumers [7].

In multispecies systems, the dynamics become significantly more complex due to indirect competition through shared resources. In addition, parameter variations may induce structural changes in system stability, such as transcritical bifurcation [8–10]. Moreover, nonlinear ecological systems may exhibit oscillatory behavior, fluctuations, and qualitative transitions under environmental perturbations, emphasizing the importance of rigorous analytical approaches, including stability theory based on invariant sets and Lyapunov's direct method

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[11]. In this context, sensitivity analysis is also essential to identify influential parameters and to evaluate the robustness of system behavior under uncertainty [12–15]. Bifurcation analysis, on the other hand, provides insight into critical thresholds separating extinction and persistence regimes [8, 16].

Several previous studies have developed population interaction models involving pest organisms (OPT) as consumers using the Lotka–Volterra equations approach. Study [7] developed a four-dimensional model involving three major rice pests, namely rice stem borer, rat, and brown planthopper. However, the analysis was limited to local stability around the equilibrium points based solely on the Jacobian matrix, which does not guarantee the behavior of solutions for all initial conditions. In addition, the effect of parameter variations on changes in the stability structure was not examined. Subsequent studies have introduced various extensions to population interaction models. [17] investigated a crop–pest–predator system incorporating harvesting strategies, while [18] examined pest dynamics by considering infection effects. In addition, [19] analyzed predator–prey interactions with cannibalism behavior, and [20] developed a plant–pest interaction model involving multiple pest species. Furthermore, [21] proposed a pest control model integrating sterile insect techniques and natural enemies. Nevertheless, most of these studies are limited to low-dimensional systems and primarily focus on local stability or specific mechanisms such as optimal control, predator behavior, or infection dynamics.

Mathematically, higher-dimensional systems involving one resource population and multiple consumers generate richer dynamics compared to lower-dimensional systems. Such systems allow the emergence of various equilibrium types, including total extinction, consumer-free equilibrium [22], partial coexistence, and potential full coexistence [8]. However, comprehensive analyses integrating global stability, parameter sensitivity, and bifurcation behavior in higher-dimensional multispecies systems remain limited, particularly in the context of rice–pest ecosystems. As a result, existing models are still insufficient to capture the complex interactions between rice and multiple pest species, especially in terms of global dynamics, parameter influence, and structural changes in system stability. Therefore, global stability analysis is necessary to ensure that system solutions converge to a particular equilibrium state for all initial conditions within the relevant region [23], while bifurcation analysis is important for determining threshold parameters that separate extinction conditions from population persistence [9]. This limitation highlights the need for more representative models that better reflect real field conditions, particularly by incorporating a greater number of pest species affecting rice plants.

Despite these developments, several important gaps remain. First, many previous rice–pest models involve fewer consumer populations, so the resulting dynamics may not fully represent field complexity. Second, higher-dimensional models often emphasize local stability, while global stability is rarely proven analytically. Third, sensitivity analysis and bifurcation analysis are commonly discussed separately, rather than integrated within a single modeling framework. Therefore, a more comprehensive mathematical framework is needed to simultaneously connect local behavior, global convergence, parameter influence, and critical transitions in rice–pest ecosystems.

This study aims to develop and analyze a five-dimensional population dynamics model describing the interaction between rice plants and four pest organisms, namely rice stem borer, rat, brown planthopper, and rice bug. The inclusion of rice bug is considered due to their role as major consumers during the generative phase of rice [24], which mathematically enriches the dynamical structure of the system. The model is formulated as a system of nonlinear differential equations in which rice growth follows a logistic model, while pest populations depend on resource availability. The model is subsequently analyzed to determine its equilibrium points and their stability in order to understand the long-term population dynamics and the influence of model parameters on the persistence of each population [25].

The main contribution of this study is the unified analytical framework integrating local stability, global stability, sensitivity, and transcritical bifurcation within a five-dimensional system. In addition, this study identifies critical threshold parameters governing extinction, persistence, and stability switching. The analytical results are validated through numerical simulations using MATLAB based on the fourth–fifth order Runge–Kutta method [26]. Such an integrated analysis within a five-dimensional rice–pest system has not been widely discussed in previous studies.

2. Methods

The stages of the research conducted are as follows.

a. Formulating the model assumptions used as constraints in the mathematical modeling of the interaction between rice plants and pest organisms (OPT). These assumptions aim to simplify the system so that the constructed model can be analyzed mathematically.
b. Determining the model variables and parameters representing the rice plant population as the resource, as well as rice stem borer, rat, brown planthopper, and rice bug as consumers.
c. Formulating the mathematical model by representing the interaction between the resource and the pest organisms (OPT) in the form of a system of nonlinear differential equations.
d. Determining the equilibrium points of the system along with their existence conditions.
e. Analyzing the local stability of the equilibrium points through eigenvalue analysis of the characteristic equation derived from the Jacobian matrix.
f. Conducting global stability analysis of a particular equilibrium point by constructing an appropriate Lyapunov function.
g. Performing sensitivity analysis of a particular equilibrium point to identify the parameters that most significantly influence the system dynamics and equilibrium populations.
h. Carrying out bifurcation analysis of the system, particularly transcritical bifurcation, to determine threshold parameters that cause changes in the stability structure of certain equilibrium points.
i. Performing numerical simulations using MATLAB to verify the consistency between the theoretical analysis and the numerical behavior of the model. The parameter values used are assumed values for simulation purposes. The numerical simulations include local stability, global stability, sensitivity analysis, and bifurcation analysis.
j. Providing interpretation and discussion of the theoretical analysis and numerical simulation results obtained.

3. Results and Discussion

3.1. Model Assumptions

The population groups in the five-dimensional population dynamics model describing the interaction between rice plants and four pest organisms are divided into five subpopulations: rice (P), rice stem borer (B), rat (T), brown planthopper (W), and rice bug (L). The assumptions used in this model are as follows:

1. The population is closed; therefore, no migration occurs into or out of the system.

2. The system consists of one resource population, namely rice plants, and four pest populations: rice stem borer, rat, brown planthopper, and rice bug.
3. In the absence of interaction with pest populations, rice plants grow according to the logistic growth model. Conversely, in the absence of resource availability, pest populations decline exponentially due to their natural mortality rates.
4. There is no direct interaction or competition among pest populations; thus, each pest population interacts directly only with the rice population as the resource.
5. No disease effects are considered for any population in the system.
6. The environmental carrying capacity for the rice population is assumed to be constant; therefore, environmental factors other than population interactions are not modeled explicitly.

3.2. Variables and Parameters

All model variables are defined on the set of nonnegative real numbers, while all model parameters are assumed to be positive. Thus, the system is defined on the phase space $\mathbb{R}^5_+$. The definitions of the model variables and parameters are presented in Table 1.

Table 1. List of Variables and Parameters

Symbol	Definition	Type	Unit	Value
P	Rice population	Variable	plants	$P \geq 0$
B	Rice stem borer population	Variable	individuals	$B \geq 0$
T	Rat population	Variable	individuals	$T \geq 0$
W	Brown planthopper population	Variable	individuals	$W \geq 0$
L	Rice bug population	Variable	individuals	$L \geq 0$
K	Environmental carrying capacity for rice	Parameter	plants	$K > 0$
r	Intrinsic growth rate of rice	Parameter	per unit time	$0 < r < 1$
α	Interaction rate of rice stem borer on rice	Parameter	per individual per unit time	$0 < \alpha < 1$
β	Interaction rate of rat on rice	Parameter	per individual per unit time	$0 < \beta < 1$
θ	Interaction rate of brown planthopper on rice	Parameter	per individual per unit time	$0 < \theta < 1$
η	Interaction rate of rice bug on rice	Parameter	per individual per unit time	$0 < \eta < 1$
c	Natural mortality rate of rice stem borer	Parameter	per unit time	$0 < c < 1$
b	Natural mortality rate of rat	Parameter	per unit time	$0 < b < 1$
m	Natural mortality rate of brown planthopper	Parameter	per unit time	$0 < m < 1$
h	Natural mortality rate of rice bug	Parameter	per unit time	$0 < h < 1$
γ	Interaction rate of rice on rice stem borer	Parameter	per plant per unit time	$0 < \gamma < 1$
δ	Interaction rate of rice on rat	Parameter	per plant per unit time	$0 < \delta < 1$
ω	Interaction rate of rice on brown planthopper	Parameter	per plant per unit time	$0 < \omega < 1$
ρ	Interaction rate of rice on rice bug	Parameter	per plant per unit time	$0 < \rho < 1$

3.3. Model

The five-dimensional population dynamics model describing the interaction between one resource (rice plants) and four pest populations is formulated by considering the factors that influence the rate of change of each population. The resulting model is expressed in the form of the following system of nonlinear differential equations:

$$\begin{aligned}
\frac{dP}{dt} &= rP\left(1-\frac{P}{K}\right) - \alpha BP - \beta TP - \theta WP - \eta LP, \\
\frac{dB}{dt} &= -cB + \gamma BP, \\
\frac{dT}{dt} &= -bT + \delta TP, \\
\frac{dW}{dt} &= -mW + \omega WP, \\
\frac{dL}{dt} &= -hL + \rho LP,
\end{aligned} \tag{1}$$

with the initial conditions $(P(0), B(0), T(0), W(0), L(0)) \in \mathbb{R}^5_+$.

Theorem 1. *If the initial condition of system (1) satisfies*

$$(P(0), B(0), T(0), W(0), L(0)) \in \mathbb{R}^5_+,$$

then the solution of the system remains in $\mathbb{R}^5_+$ for all $t \geq 0$.

Theorem 2. *Every solution of system (1) with initial condition*

$$(P(0), B(0), T(0), W(0), L(0)) \in \mathbb{R}^5_+$$

is bounded for all $t \geq 0$.

Corollary 1. *Based on Theorem 1 and Theorem 2, the system admits an invariant region*

$$\Omega = \left\{(P, B, T, W, L) \in \mathbb{R}^5_+ \mid 0 \leq P \leq K\right\}.$$

Every solution with an initial condition in Ω remains in Ω for all $t \geq 0$.

3.4. Equilibrium Points

The equilibrium points of system (1) are given in the following theorem.

Theorem 3. *The equilibrium point*

$$E_1 = (P^*, B^*, T^*, W^*, L^*) = (0, 0, 0, 0, 0)$$

is the trivial equilibrium point (total extinction) and

$$E_2 = (P^*, B^*, T^*, W^*, L^*) = (K, 0, 0, 0, 0)$$

is the pest-free equilibrium point for the system (1).

Proof. The equilibrium points of system (1) are obtained when

$$\frac{dP}{dt} = \frac{dB}{dt} = \frac{dT}{dt} = \frac{dW}{dt} = \frac{dL}{dt} = 0$$

such that

$$rP\left(1-\frac{P}{K}\right)-\alpha BP-\beta TP-\theta WP-\eta LP=0, \tag{2}$$

$$-cB+\gamma BP=0, \tag{3}$$

$$-bT+\delta TP=0, \tag{4}$$

$$-mW+\omega WP=0, \tag{5}$$

$$-hL+\rho LP=0. \tag{6}$$

If $B=0$, $T=0$, $W=0$, and $L=0$, then from eq. (2) we obtain $P(1-P/K)=0$ so that $P=0$ or $P=K$. Thus, two equilibrium points are obtained

$$E_1=(P^*,B^*,T^*,W^*,L^*)=(0,0,0,0,0),$$
$$E_2=(P^*,B^*,T^*,W^*,L^*)=(K,0,0,0,0).$$

Point E_1 is the trivial equilibrium point (total extinction), which occurs when all populations are zero. Point E_2 is the pest-free equilibrium point. □

Based on the model assumptions, there is no direct interaction or competition among the four pest populations. Each pest population is assumed to interact directly only with the rice plant population as the main resource. Nevertheless, since all pests utilize the same resource, indirect competition arises implicitly through the shared resource. This condition aligns with the Competitive Exclusion Principle in the context of mathematical modeling, where resource limitation can influence the long-term persistence of pest populations [7].

The next equilibrium points are the partial coexistence equilibrium (rice and one pest population).

Theorem 4. *The equilibrium points*

$$E_3=(P^*,B^*,T^*,W^*,L^*)=\left(\frac{c}{\gamma},\frac{r}{\alpha}\left(1-\frac{c}{\gamma K}\right),0,0,0\right),\ \textit{with}\ \frac{c}{\gamma}<K,$$

$$E_4=(P^*,B^*,T^*,W^*,L^*)=\left(\frac{b}{\delta},0,\frac{r}{\beta}\left(1-\frac{b}{\delta K}\right),0,0\right),\ \textit{with}\ \frac{b}{\delta}<K,$$

$$E_5=(P^*,B^*,T^*,W^*,L^*)=\left(\frac{m}{\omega},0,0,\frac{r}{\theta}\left(1-\frac{m}{\omega K}\right),0\right),\ \textit{with}\ \frac{m}{\omega}<K,$$

$$E_6=(P^*,B^*,T^*,W^*,L^*)=\left(\frac{h}{\rho},0,0,0,\frac{r}{\eta}\left(1-\frac{h}{\rho K}\right)\right),\ \textit{with}\ \frac{h}{\rho}<K$$

are the partial coexistence equilibrium for the system (1).

Proof. Based on eq. (3), if $B>0$ we obtain

$$P=\frac{c}{\gamma}. \tag{7}$$

Based on eq. (4), if $T>0$ we obtain

$$P=\frac{b}{\delta}. \tag{8}$$

Based on eq. (5), if $W>0$ we obtain

$$P=\frac{m}{\omega}. \tag{9}$$

Based on eq. (6), if $L > 0$ we obtain

$$P = \frac{h}{\rho}. \tag{10}$$

The remaining equilibrium points are obtained by substituting eq. (7) to (10) into eq. (2) to (6):

$$E_3 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{c}{\gamma}, \frac{r}{\alpha}\left(1 - \frac{c}{\gamma K}\right), 0, 0, 0\right), \text{ with } \frac{c}{\gamma} < K.$$

$$E_4 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{b}{\delta}, 0, \frac{r}{\beta}\left(1 - \frac{b}{\delta K}\right), 0, 0\right), \text{ with } \frac{b}{\delta} < K.$$

$$E_5 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{m}{\omega}, 0, 0, \frac{r}{\theta}\left(1 - \frac{m}{\omega K}\right), 0\right), \text{ with } \frac{m}{\omega} < K.$$

$$E_6 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{h}{\rho}, 0, 0, 0, \frac{r}{\eta}\left(1 - \frac{h}{\rho K}\right)\right), \text{ with } \frac{h}{\rho} < K.$$

□

3.5. *Local Stability Analysis of Equilibrium Points*

The Jacobian matrix for system eq. (1) is given by

$$J(P^*, B^*, T^*, W^*, L^*) = \begin{bmatrix} J_{11} & -\alpha P^* & -\beta P^* & -\theta P^* & -\eta P^* \\ \gamma B^* & -c + \gamma P^* & 0 & 0 & 0 \\ \delta T^* & 0 & -b + \delta P^* & 0 & 0 \\ \omega W^* & 0 & 0 & -m + \omega P^* & 0 \\ \rho L^* & 0 & 0 & 0 & -h + \rho P^* \end{bmatrix} \tag{11}$$

with

$$J_{11} = r - \frac{2rP^*}{K} - \alpha B^* - \beta T^* - \theta W^* - \eta L^*.$$

3.5.1. Local Stability Analysis of Point E_1: Total Extinction

Local stability of point E_1 are given in the following theorem.

Theorem 5. *The equilibrium point E_1 is unstable.*

Proof. The stability analysis is carried out at the equilibrium point E_1 to determine the behavior of the system around the total extinction state. Based on eq. (11), the Jacobian matrix for E_1 is given by

$$J(E_1) = \begin{bmatrix} r & 0 & 0 & 0 & 0 \\ 0 & -c & 0 & 0 & 0 \\ 0 & 0 & -b & 0 & 0 \\ 0 & 0 & 0 & -m & 0 \\ 0 & 0 & 0 & 0 & -h \end{bmatrix}. \tag{12}$$

Based on eq. (12), the eigenvalues are obtained as follows:

$$\lambda_1 = r, \lambda_2 = -c, \lambda_3 = -b, \lambda_4 = -m, \text{ and } \lambda_5 = -h.$$

The eigenvalue $\lambda_1 = r > 0$ indicates that the equilibrium point E_1 is unstable. □

This means that the rice population tends to grow away from extinction in the absence of pests. The total extinction equilibrium represents a state in which all populations, both rice and pests, are extinct. Biologically, this condition reflects the failure of the agricultural system and is undesirable in cultivation practice.

3.5.2. Local Stability Analysis of Point E_2: Pest-Free Equilibrium

Local stability of point E_2 is given in the following theorem.

Theorem 6. *The equilibrium point E_2 is locally asymptotically stable if all eigenvalues have negative real parts, that is,*

$$K < \frac{c}{\gamma}, \quad K < \frac{b}{\delta}, \quad K < \frac{m}{\omega}, \quad \text{and} \quad K < \frac{h}{\rho}.$$

Proof. The stability analysis of the equilibrium point E_2 aims to determine the persistence of the rice population. Based on eq. (11), the Jacobian matrix for E_2 is given by

$$J(E_2) = \begin{bmatrix} -r & -\alpha K & -\beta K & -\theta K & -\eta K \\ 0 & -c+\gamma K & 0 & 0 & 0 \\ 0 & 0 & -b+\delta K & 0 & 0 \\ 0 & 0 & 0 & -m+\omega K & 0 \\ 0 & 0 & 0 & 0 & -h+\rho K \end{bmatrix}. \tag{13}$$

Based on eq. (13), the eigenvalues are

$$\lambda_1 = -r, \lambda_2 = -c+\gamma K, \lambda_3 = -b+\delta K, \lambda_4 = -m+\omega K, \text{ and } \lambda_5 = -h+\rho K.$$

The equilibrium point E_2 is locally asymptotically stable if all eigenvalues have negative real parts, that is,

$$K < \frac{c}{\gamma}, \quad K < \frac{b}{\delta}, \quad K < \frac{m}{\omega}, \quad \text{and} \quad K < \frac{h}{\rho}.$$

□

If these conditions are satisfied, it means that the environmental carrying capacity (K) is sufficiently small so that it cannot support any pest populations, resulting in the extinction of all pests. This condition can occur if the environment or pest management efforts prevent pests from surviving. Conversely, if the environmental carrying capacity (rice) is abundant, the pest populations will grow, causing the equilibrium point E_2 to become unstable.

Next, at the partial coexistence points (rice and one active pest), the Jacobian matrix automatically takes the form of a 2×2 block for the rice–active pest interaction and diagonal blocks for the other pest populations. For each partial coexistence point, the equilibrium is locally asymptotically stable if the other pests cannot invade.

3.5.3. Local Stability Analysis of Point E_3: Rice and Rice Stem Borer Equilibrium

Local stability of point E_3 is given in the following theorem.

Theorem 7. *The equilibrium point $E_3 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{c}{\gamma}, \frac{r}{\alpha}\left(1 - \frac{c}{\gamma K}\right), 0, 0, 0\right)$ is locally asymptotically stable if*

$$\delta\frac{c}{\gamma} < b, \quad \omega\frac{c}{\gamma} < m, \quad \text{and} \quad \rho\frac{c}{\gamma} < h.$$

Proof. The local stability analysis of the equilibrium point E_3 (rice and rice stem borer) is conducted to determine the conditions for locally stable coexistence between rice and the rice stem borer, while preventing invasion by the other pest populations. Based on eq. (11), the Jacobian matrix for E_3 is given by

$$J(E_3) = \begin{bmatrix} J_{11_B} & -\alpha P^* & -\beta P^* & -\theta P^* & -\eta P^* \\ \gamma B^* & 0 & 0 & 0 & 0 \\ 0 & 0 & -b+\delta P^* & 0 & 0 \\ 0 & 0 & 0 & -m+\omega P^* & 0 \\ 0 & 0 & 0 & 0 & -h+\rho P^* \end{bmatrix} \tag{14}$$

with

$$J_{11_B} = r\left(1 - \frac{2P^*}{K}\right) - \alpha B^*.$$

For the rice–rice stem borer subsystem in eq. (14), based on the matrix

$$\begin{bmatrix} J_{11_B} & -\alpha P^* \\ \gamma B^* & 0 \end{bmatrix},$$

the characteristic equation is

$$\lambda^2 - J_{11_B}\lambda + \alpha\gamma P^* B^* = 0.$$

Note that $\alpha > 0, \gamma > 0, P^* > 0$, and $B^* > 0$, so $\alpha\gamma P^* B^* > 0$. Substituting

$$P^* = \frac{c}{\gamma} \quad \text{and} \quad B^* = \frac{r}{\alpha}\left(1 - \frac{c}{\gamma K}\right)$$

into J_{11_B} gives

$$J_{11_B} = -\frac{rc}{\gamma K}.$$

Since $r > 0, c > 0, \gamma > 0$, and $K > 0$, it follows that $J_{11_B} < 0$. Based on the Routh–Hurwitz criterion [27], the characteristic equation

$$\lambda^2 - J_{11_B}\lambda + \alpha\gamma P^* B^* = 0$$

with $J_{11_B} < 0$ and $\alpha\gamma P^* B^* > 0$ has both eigenvalues with negative real parts. For the directions of the other pest populations, the remaining eigenvalues are given by

$$\lambda_3 = -b + \delta\frac{c}{\gamma}, \quad \lambda_4 = -m + \omega\frac{c}{\gamma}, \quad \lambda_5 = -h + \rho\frac{c}{\gamma}.$$

The real parts of the three remaining eigenvalues will be negative if

$$\delta\frac{c}{\gamma} < b, \quad \omega\frac{c}{\gamma} < m, \quad \text{and} \quad \rho\frac{c}{\gamma} < h.$$

Thus, the equilibrium point E_3 is locally asymptotically stable if

$$\delta\frac{c}{\gamma} < b, \quad \omega\frac{c}{\gamma} < m, \quad \text{and} \quad \rho\frac{c}{\gamma} < h.$$

□

This means that no other pest populations can invade the system when the rice–rice stem borer subsystem is at equilibrium.

3.5.4. Local Stability Analysis of Point E_4: Rice and Rat Equilibrium

Local stability of point E_4 is given in the following theorem.

Theorem 8. *The equilibrium point*

$$E_4 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{b}{\delta}, 0, \frac{r}{\beta}\left(1 - \frac{b}{\delta K}\right), 0, 0\right)$$

is locally asymptotically stable if

$$\gamma\frac{b}{\delta} < c, \quad \omega\frac{b}{\delta} < m, \quad \text{and} \quad \rho\frac{b}{\delta} < h.$$

Proof. The local stability analysis of the equilibrium point E_4 (rice and rat) is conducted to determine the conditions for locally stable coexistence between rice and the rat, while preventing invasion by the other pest populations. The characteristic equation for the rice–rat subsystem is

$$\lambda^2 - J_{11_T}\lambda + \beta\delta P^*T^* = 0$$

with

$$J_{11_T} = r\left(1 - \frac{2P^*}{K}\right) - \beta T^*.$$

Note that $\beta > 0$, $\delta > 0$, $P^* > 0$, and $T^* > 0$, so $\beta\delta P^*T^* > 0$. Substituting

$$P^* = \frac{b}{\delta} \quad \text{and} \quad T^* = \frac{r}{\beta}\left(1 - \frac{b}{\delta K}\right)$$

into J_{11_T} gives

$$J_{11_T} = -\frac{rb}{\delta K}.$$

Since $r > 0$, $b > 0$, $\delta > 0$, and $K > 0$, it follows that $J_{11_T} < 0$. Based on the Routh–Hurwitz criterion [27], the characteristic equation

$$\lambda^2 - J_{11_T}\lambda + \beta\delta P^*T^* = 0$$

with $J_{11_T} < 0$ and $\beta\delta P^*T^* > 0$ has both eigenvalues with negative real parts. For the directions of the other pest populations, the remaining eigenvalues are given by

$$\lambda_3 = -c + \gamma\frac{b}{\delta}, \quad \lambda_4 = -m + \omega\frac{b}{\delta}, \quad \lambda_5 = -h + \rho\frac{b}{\delta}.$$

The real parts of the three remaining eigenvalues will be negative if

$$\gamma\frac{b}{\delta} < c, \quad \omega\frac{b}{\delta} < m, \quad \text{and} \quad \rho\frac{b}{\delta} < h.$$

Thus, the equilibrium point E_4 is locally asymptotically stable if

$$\gamma\frac{b}{\delta} < c, \quad \omega\frac{b}{\delta} < m, \quad \text{and} \quad \rho\frac{b}{\delta} < h.$$

□

This means that no other pest populations can invade the system when the rice–rat subsystem is at equilibrium.

3.5.5. Local Stability Analysis of Point E_5: Rice and Brown Planthopper Equilibrium

Local stability of point E_5 is given in the following theorem.

Theorem 9. *The equilibrium point*

$$E_5 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{m}{\omega}, 0, 0, \frac{r}{\theta}\left(1 - \frac{m}{\omega K}\right), 0\right)$$

is locally asymptotically stable if

$$\gamma\frac{m}{\omega} < c, \quad \delta\frac{m}{\omega} < b, \quad \text{and} \quad \rho\frac{m}{\omega} < h.$$

Proof. The local stability analysis of the equilibrium point E_5 (rice and brown planthopper) is conducted to determine the conditions for locally stable coexistence between rice and the brown planthopper, while preventing invasion by the other pest populations. The characteristic equation for the rice–brown planthopper subsystem is

$$\lambda^2 - J_{11_W}\lambda + \theta\omega P^* W^* = 0$$

with

$$J_{11_W} = r\left(1 - \frac{2P^*}{K}\right) - \theta W^*.$$

Note that $\theta > 0$, $\omega > 0$, $P^* > 0$, and $W^* > 0$, so $\theta\omega P^* W^* > 0$. Substituting

$$P^* = \frac{m}{\omega} \quad \text{and} \quad W^* = \frac{r}{\theta}\left(1 - \frac{m}{\omega K}\right)$$

into J_{11_W} gives

$$J_{11_W} = -\frac{rm}{\omega K}.$$

Since $r > 0$, $m > 0$, $\omega > 0$, and $K > 0$, it follows that $J_{11_W} < 0$. Based on the Routh–Hurwitz criterion [27], the characteristic equation

$$\lambda^2 - J_{11_W}\lambda + \theta\omega P^* W^* = 0$$

with $J_{11_W} < 0$ and $\theta\omega P^* W^* > 0$ has both eigenvalues with negative real parts. For the directions of the other pest populations, the remaining eigenvalues are given by

$$\lambda_3 = -c + \gamma\frac{m}{\omega}, \quad \lambda_4 = -b + \delta\frac{m}{\omega}, \quad \lambda_5 = -h + \rho\frac{m}{\omega}.$$

The real parts of the three remaining eigenvalues will be negative if

$$\gamma\frac{m}{\omega} < c, \quad \delta\frac{m}{\omega} < b, \quad \text{and} \quad \rho\frac{m}{\omega} < h.$$

Thus, the equilibrium point E_5 is locally asymptotically stable if

$$\gamma\frac{m}{\omega} < c, \quad \delta\frac{m}{\omega} < b, \quad \text{and} \quad \rho\frac{m}{\omega} < h.$$

□

This means that no other pest populations can invade the system when the rice–brown planthopper subsystem is at equilibrium.

3.5.6. Local Stability Analysis of Point E_6: Rice and Rice Bug Equilibrium

Local stability of point E_6 is given in the following theorem.

Theorem 10. *The equilibrium point*

$$E_6 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{h}{\rho}, 0, 0, 0, \frac{r}{\eta}\left(1 - \frac{h}{\rho K}\right)\right)$$

is locally asymptotically stable if

$$\gamma\frac{h}{\rho} < c, \quad \delta\frac{h}{\rho} < b, \quad \text{and} \quad \omega\frac{h}{\rho} < m.$$

Proof. The local stability analysis of the equilibrium point E_6 (rice and rice bug) is conducted to determine the conditions for locally stable coexistence between rice and the rice bug, while preventing invasion by the other pest populations. The characteristic equation for the rice–rice bug subsystem is

$$\lambda^2 - J_{11_L}\lambda + \eta\rho P^* L^* = 0$$

with

$$J_{11_L} = r\left(1 - \frac{2P^*}{K}\right) - \eta L^*.$$

Note that $\eta > 0$, $\rho > 0$, $P^* > 0$, and $L^* > 0$, so $\eta\rho P^* L^* > 0$. Substituting

$$P^* = \frac{h}{\rho} \quad \text{and} \quad L^* = \frac{r}{\eta}\left(1 - \frac{h}{\rho K}\right)$$

into J_{11_L} gives

$$J_{11_L} = -\frac{rh}{\rho K}.$$

Since $r > 0$, $h > 0$, $\rho > 0$, and $K > 0$, it follows that $J_{11_L} < 0$. Based on the Routh–Hurwitz criterion [27], the characteristic equation

$$\lambda^2 - J_{11_L}\lambda + \eta\rho P^* L^* = 0$$

with $J_{11_L} < 0$ and $\eta\rho P^* L^* > 0$ has both eigenvalues with negative real parts. For the directions of the other pest populations, the remaining eigenvalues are given by

$$\lambda_3 = -c + \gamma\frac{h}{\rho}, \quad \lambda_4 = -b + \delta\frac{h}{\rho}, \quad \lambda_5 = -m + \omega\frac{h}{\rho}.$$

The real parts of the three remaining eigenvalues will be negative if

$$\gamma\frac{h}{\rho} < c, \quad \delta\frac{h}{\rho} < b, \quad \text{and} \quad \omega\frac{h}{\rho} < m.$$

Thus, the equilibrium point E_6 is locally asymptotically stable if

$$\gamma\frac{h}{\rho} < c, \quad \delta\frac{h}{\rho} < b, \quad \text{and} \quad \omega\frac{h}{\rho} < m.$$

□

This means that no other pest populations can invade the system when the rice–rice bug subsystem is at equilibrium.

The partial coexistence equilibrium points indicate stable conditions in which the rice plants coexist with a single pest population, while the other pest populations cannot persist. Biologically, this reflects the dominance of one pest species that has a competitive advantage through the utilization of the shared food resource.

In the studied system, full coexistence equilibrium involving rice and all four pest populations does not occur. This is a consequence of the model structure, which involves a single resource exploited collectively by all pest populations. The limitation of the shared resource allows only partial coexistence or the extinction of some pest populations. Mathematically, this condition is associated with threshold parameters that determine the persistence of each pest population, as demonstrated through the stability and bifurcation analysis.

3.6. Global Stability

Global stability is analyzed at the pest-free equilibrium point $E_2 = (K, 0, 0, 0, 0)$. since the single-resource structure of the system allows the construction of a Lyapunov function that satisfies LaSalle's conditions [28].

Theorem 11. *The pest-free equilibrium point*

$$E_2 = (K, 0, 0, 0, 0)$$

is globally asymptotically stable in the invariant region

$$\Omega = \left\{(P, B, T, W, L) \in \mathbb{R}^5_+ \mid 0 \le P \le K\right\},$$

provided that

$$K < \frac{c}{\gamma}, \quad K < \frac{b}{\delta}, \quad K < \frac{m}{\omega}, \quad \text{and} \quad K < \frac{h}{\rho}.$$

Proof. To establish the global asymptotic stability of E_2, we employ Lyapunov's direct method combined with LaSalle's Invariance Principle. Consider the Lyapunov function defined by

$$V(B, T, W, L) = B + T + W + L.$$

This function is continuous and positive definite in Ω, and satisfies $V = 0$ if and only if $B = T = W = L = 0$. The derivative along the system trajectories is

$$\dot{V} = \dot{B} + \dot{T} + \dot{W} + \dot{L}$$

and it is obtained that

$$\dot{V} = B(\gamma P - c) + T(\delta P - b) + W(\omega P - m) + L(\rho P - h).$$

Since $0 \le P \le K$, it follows that

$$\dot{V} \le B(\gamma K - c) + T(\delta K - b) + W(\omega K - m) + L(\rho K - h).$$

If the conditions

$$K < \frac{c}{\gamma}, \quad K < \frac{b}{\delta}, \quad K < \frac{m}{\omega}, \quad \text{and} \quad K < \frac{h}{\rho}$$

are satisfied, then all coefficients in the inequality are negative. Therefore, there exists a constant $\varepsilon > 0$ such that

$$\dot{V} \leq -\varepsilon(B + T + W + L) = -\varepsilon V$$

which implies that $V(t) \to 0$ as $t \to \infty$. Consequently,

$$(B, T, W, L) \to (0, 0, 0, 0).$$

The set where $\dot{V} = 0$ is given by

$$\Gamma = \{(P, B, T, W, L) \in \Omega \mid B = T = W = L = 0\}.$$

On the set Γ, all pest populations vanish, and the system reduces to the scalar equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

which is the logistic equation. The logistic equation admits two equilibrium points, $P = 0$ and $P = K$, where $P = K$ is globally asymptotically stable for all $P(0) > 0$. Hence,

$$P(t) \to K \quad \text{as } t \to \infty.$$

Since

$$(B, T, W, L) \to (0, 0, 0, 0) \quad \text{and} \quad P \to K,$$

it follows that

$$(P, B, T, W, L) \to (K, 0, 0, 0, 0) = E_2.$$

Therefore, by Lyapunov's direct method and LaSalle's Invariance Principle, the equilibrium point E_2 is globally asymptotically stable in Ω. □

3.7. Sensitivity Analysis

The results of the sensitivity analysis are given in the following theorem.

Theorem 12. *Consider the partial coexistence equilibrium point between rice and rice bug, denoted by*

$$E_6 = (P^*, B^*, T^*, W^*, L^*) = \left(\frac{h}{\rho}, 0, 0, 0, \frac{r}{\eta}\left(1 - \frac{h}{\rho K}\right)\right).$$

The normalized sensitivity indices of the rice population P^* with respect to the interaction parameter ρ and the natural mortality rate h of the rice bug are given by

$$S^{\rho}_{P^*} = -1, \qquad S^{h}_{P^*} = 1.$$

Proof. Sensitivity analysis is conducted at the partial coexistence equilibrium point of rice and rice bug E_6. The normalized forward sensitivity index of a variable P^* with respect to a parameter ξ, where ξ denotes any model parameter, is defined as

$$S^{\xi}_{P^*} = \frac{\partial P^*}{\partial \xi}\frac{\xi}{P^*}. \tag{15}$$

a. Sensitivity with respect to the interaction parameter ρ

Differentiating P^* with respect to ρ, we obtain $\frac{\partial P^*}{\partial \rho} = -\frac{h}{\rho^2}$. Substituting into the sensitivity formula (15) yields

$$S^{\rho}_{P*} = \frac{\partial P^*}{\partial \rho}\frac{\rho}{P^*} = \left(-\frac{h}{\rho^2}\right)\left(\frac{\rho}{h/\rho}\right) = -1.$$

Ths value indicates that a 1% increase in ρ leads to a 1% decrease in the rice population at the equilibrium point E_6. Thus, the interaction parameter of the rice bug has a negative effect on the rice population. In practical terms, a higher value of ρ represents more intense pest attacks on rice plants, which may result from unfavorable environmental conditions or ineffective pest management. Therefore, reducing ρ can be achieved through the use of pest-resistant rice varieties, improved cultivation practices, or biological control methods to minimize pest–plant interactions.

b. Sensitivity with respect to the natural mortality rate parameter h

Differentiating P^* with respect to h, we obtain $\frac{\partial P^*}{\partial h} = \frac{1}{\rho}$. Substituting into the sensitivity formula (15) yields

$$S^{h}_{P*} = \frac{\partial P^*}{\partial h}\frac{h}{P^*} = \left(\frac{1}{\rho}\right)\left(\frac{h}{h/\rho}\right) = 1.$$

This means that a 1% increase in h increases the rice population by 1% at the equilibrium point E_6. Therefore, the natural mortality parameter of the rice bug has a positive effect in maintaining the stability of the rice population. In real-world applications, increasing h can be associated with enhanced pest mortality due to biological control agents, natural predators, or environmentally friendly pesticides. Hence, strategies that increase the mortality rate of pests are beneficial for sustaining rice production. □

3.8. Bifurcation Analysis with Respect to the Interaction Parameter

Bifurcation analysis is conducted to examine changes in the stability structure of the system due to parameter variation. In this study, the bifurcation parameter is chosen as the interaction rate of the rice bug with rice, since the local stability analysis shows that this parameter affects the sign of one eigenvalue at the pest-free equilibrium point.

Theorem 13. *Consider system (1) and let the interaction parameter of the rice bug be denoted by ρ. Define the threshold parameter*

$$\rho^* = \frac{h}{K}.$$

Then, the system undergoes a transcritical bifurcation at the pest-free equilibrium point $E_2 = (K, 0, 0, 0, 0)$ when $\rho = \rho^*$. Specifically:

1. *If $\rho < \rho^*$, then E_2 is locally asymptotically stable.*
2. *If $\rho > \rho^*$, then E_2 is unstable and a coexistence equilibrium between rice and rice bug emerges.*

Proof. At the pest-free equilibrium point $E_2 = (K, 0, 0, 0, 0)$, the eigenvalue associated with the rice bug population is obtained from the diagonal element of the Jacobian matrix (13), namely $\lambda_5 = -h + \rho K$. Observe that the sign of λ_5 is determined by the expression $\rho K - h$. Define the threshold parameter

$$\rho^* = \frac{h}{K}.$$

Then:

1. If $\rho < \frac{h}{K}$, then $\lambda_5 < 0$, so the rice bug population cannot invade the system and the pest-free equilibrium is locally asymptotically stable against small perturbations of the rice bug population.
2. If $\rho > \frac{h}{K}$, then $\lambda_5 > 0$, so the pest-free equilibrium becomes unstable and the rice bug population can grow from a small initial condition.

At the critical condition $\rho = \frac{h}{K}$, the eigenvalue changes sign from negative to positive. At the same time, a partial coexistence equilibrium between rice and the rice bug emerges. This indicates the occurrence of a transcritical bifurcation. □

These results indicate the existence of a threshold parameter that determines whether the rice bug population can persist in the system. When the interaction capability between the rice bug and the rice population is relatively weak compared to its natural mortality rate, the pest cannot sustain its population and eventually dies out. In this case, the system remains in the pest-free state.

Conversely, when the interaction parameter is sufficiently large, the rice bug is able to utilize the rice population as a resource more effectively, allowing it to survive and grow. As a result, the system transitions from the pest-free equilibrium to a partial coexistence state where both rice and the rice bug persist.

3.9. Numerical Simulation

Numerical simulations are conducted to verify the analytical results of stability, sensitivity, and transcritical bifurcation using MATLAB with the fourth–fifth order Runge–Kutta method. The parameter values used in the numerical simulations are summarized in Table 2. These values are selected to satisfy the conditions for both local and global stability derived in the previous sections, thereby ensuring that the numerical results are fully consistent with the theoretical analysis.

Table 2. Parameter Values Used in the Numerical Simulations Satisfying the Stability Conditions

Parameter	E_2 (Figure 1)	E_3 (Figure 2)	E_4 (Figure 2)	E_5 (Figure 2)	E_6 (Figure 2)	E_2 (Global (Figure 3))
r	0.8	0.8	0.8	0.8	0.8	0.8
K	100	100	100	100	100	100
α	0.002	0.002	0.002	0.002	0.002	0.002
β	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015
θ	0.001	0.001	0.001	0.001	0.001	0.001
η	0.001	0.001	0.001	0.001	0.001	0.001
γ	0.0005	0.004	0.0005	0.0005	0.0005	0.0005
δ	0.0005	0.0005	0.004	0.0005	0.0005	0.0005
ω	0.0005	0.0005	0.0005	0.004	0.0005	0.0005
ρ	0.0005	0.0005	0.0005	0.0005	0.004	0.0005
c	0.5	0.2	0.5	0.5	0.5	0.5
b	0.4	0.4	0.3	0.4	0.4	0.4
m	0.3	0.3	0.3	0.3	0.3	0.3
h	0.25	0.25	0.25	0.25	0.25	0.25

3.9.1. Numerical Simulation of the Local Stability of the Pest-Free Equilibrium Point

The results of the numerical simulation of the local stability of the equilibrium point E_2 are presented in Figure 1.

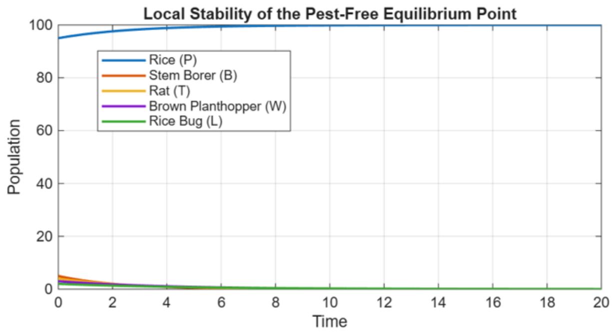
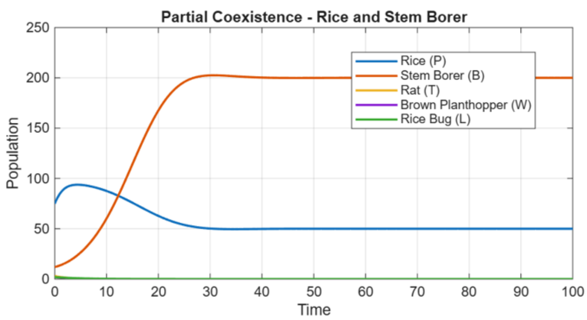


Figure 1. Numerical Simulation of the Local Stability of the Pest-Free Equilibrium Point

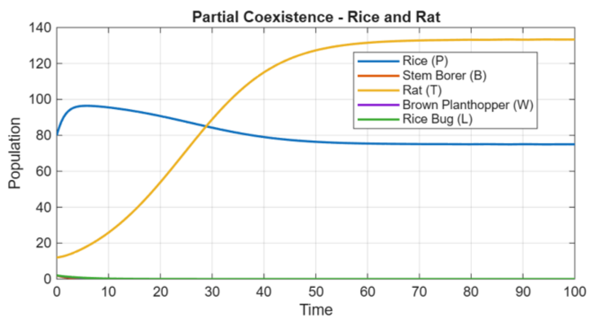
Figure 1 shows that the solutions converge to the pest-free equilibrium point $E_2 = (K, 0, 0, 0, 0)$. The rice population approaches the carrying capacity K, while all pest populations (rice stem borer, rat, brown planthopper, and rice bug) decay to zero. This result is consistent with the analytical finding that E_2 is locally asymptotically stable under the given parameter conditions.

3.9.2. Numerical Simulation of the Local Stability of the Partial Coexistence Equilibrium (Rice and One Pest)

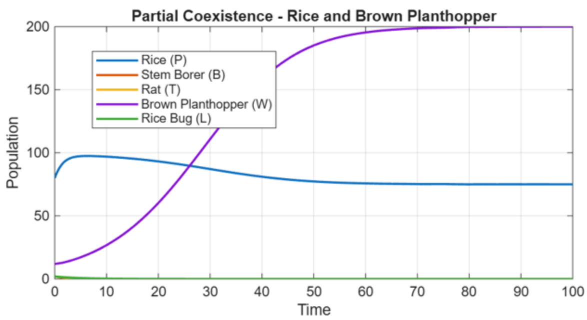
The simulations of the partial coexistence equilibrium points E_3 (rice and stem borer), E_4 (rice and rat), E_5 (rice and brown planthopper), and E_6 (rice and rice bug) are shown in Figure 2.



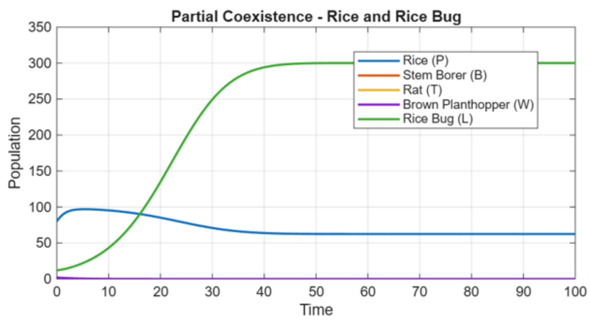
(a) E_3: Rice and stem borer



(b) E_4: Rice and rat



(c) E_5: Rice and brown planthopper



(d) E_6: Rice and rice bug

Figure 2. Numerical Simulation of the Local Stability of the Partial Coexistence Equilibrium Point

Figure 2 shows that the solutions converge to partial coexistence equilibrium points E_3, E_4, E_5, and E_6, depending on the initial conditions and parameter values. In each case, the rice population stabilizes at a positive level below the carrying capacity, while only one pest population persists and the others decay to zero. This indicates that a single pest species successfully exploits the shared resource, whereas the remaining pest populations cannot invade the system. The dominance of one pest reflects indirect competition through the shared resource, leading to competitive exclusion. These results are consistent with the analytical findings that only one pest population can coexist with rice under the given conditions, while the others become extinct.

3.9.3. Numerical Simulation of Global Stability

The results of the numerical simulation of the global stability of the equilibrium point E_2 are presented in Figure 3.

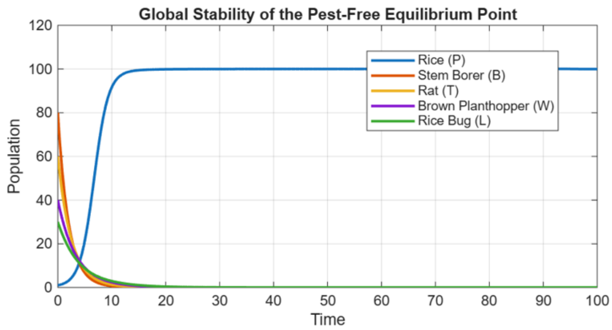


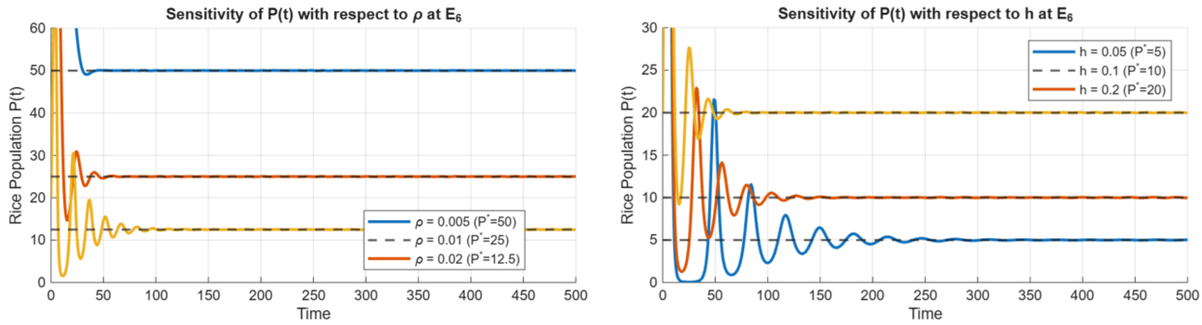
Figure 3. Numerical Simulation of the Global Stability of the Pest-Free Equilibrium Point

Figure 3 shows that for different initial conditions, all solutions converge to the pest-free equilibrium point E_2. The rice population approaches the carrying capacity K, while all pest populations decay to zero over time. This indicates that the system is globally asymptotically stable at E_2, meaning that the long-term behavior of the system is independent of the initial conditions. Biologically, this reflects a condition in which the natural mortality rates of the pests dominate their interaction with the resource, leading to the extinction of all pest populations. These results are consistent with the analytical proof of global stability using Lyapunov's direct method and LaSalle's Invariance Principle.

3.9.4. Numerical Simulation of the Sensitivity Analysis at the Partial Coexistence Equilibrium of Rice and the Rice Bug

The parameters varied in the simulation are the interaction rate of rice and the rice bug (ρ) and the natural mortality rate of the rice bug (h). The results of the sensitivity analysis are presented in Figure 4.

Figure 4 illustrates the sensitivity of the rice population at the partial coexistence equilibrium point E_6. The simulation shows that the solution $P(t)$ converges to the equilibrium value $P^* = \dfrac{h}{\rho}$ for different parameter values. It can be observed that increasing ρ leads to a decrease in the equilibrium level of the rice population, whereas increasing h results in an increase in the rice population. This behavior reflects the negative effect of the interaction parameter ρ, which represents the intensity of pest attacks, and the positive effect of the natural mortality rate h, which reduces pest pressure. These results are consistent with the analytical sensitivity indices $S^{\rho}_{P*} = -1$ and $S^{h}_{P*} = 1$, indicating proportional changes in the rice population with respect to both parameters.

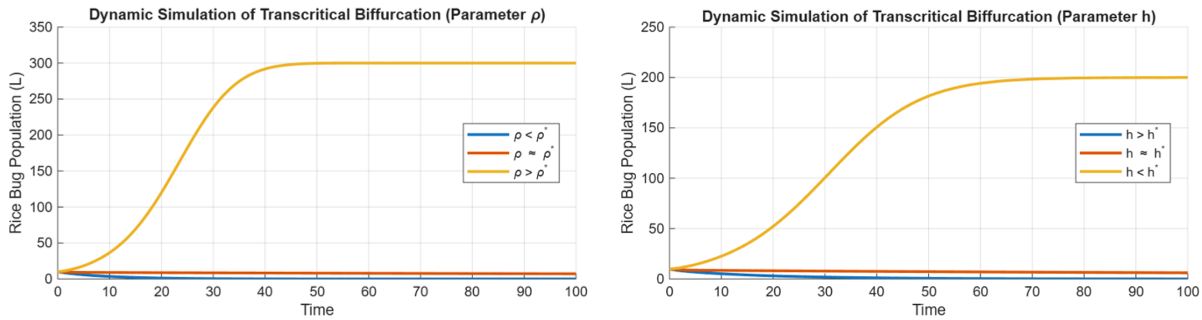


(a) Variation of parameter ρ **(b)** Variation of parameter h

Figure 4. Simulation of the Sensitivity to Parameters ρ and h at Equilibrium Point E_6

3.9.5. Numerical Simulation of the Dynamic Behavior around the Transcritical Bifurcation

The dynamic behavior of the system around the bifurcation threshold is illustrated in Figure 5.



(a) $\rho < \rho^*$ **(b)** $\rho = \rho^*$

Figure 5. Dynamic Behavior of the System Around the Transcritical Bifurcation Point for Different Values of the Parameters ρ and h

Figure 5 illustrates the dynamic behavior around a transcritical bifurcation between the pest-free equilibrium E_2 as the parameter ρ crosses the critical threshold $\rho^* = \dfrac{h}{K}$. When $\rho < \rho^* = \frac{h}{K}$, the pest-free equilibrium is stable and the rice bug population tends to zero. As ρ increases and passes the threshold ρ^*, the equilibrium becomes unstable and a new stable state with a positive rice bug population appears. This indicates a qualitative change in the system dynamics. The results are consistent with the analytical bifurcation analysis.

3.10. Discussion

The results of this study are consistent with the theory of multispecies population dynamics. The absence of full coexistence agrees with the competitive exclusion principle, where multiple consumers competing for a single resource cannot persist simultaneously [7]. The stability of the pest-free equilibrium is determined by threshold parameters related to interaction rates and natural mortality, which is in line with previous studies indicating that pest extinction occurs when mortality dominates interaction strength [12]. The global stability result also supports earlier findings obtained using Lyapunov-based approaches [10]. The sensitivity analysis shows that the interaction and natural mortality parameters of the rice bug are the most influential factors, consistent with other studies highlighting the importance of these parameters in population dynamics [11]. In addition, the occurrence of a transcritical bifurcation confirms previous results where stability exchange is governed by threshold conditions [9]. Overall, these findings are consistent with existing studies while extending the analysis to a higher-dimensional system, providing a more comprehensive understanding of the dynamics of rice–pest interactions.

4. Conclusion

This study develops a five–dimensional model of rice–pest interactions and investigates the equilibrium points and their stability. The results show that pest-free equilibrium can be locally and globally stable under certain conditions, while full coexistence is not achieved due to resource limitations. The system dynamics are governed by key threshold parameters related to interaction rates and natural mortality. Sensitivity analysis identifies rice bug-related parameters as the most influential factors, while bifurcation analysis reveals a transcritical bifurcation that governs the transition between pest extinction and persistence. Overall, this study highlights the critical conditions for pest control and provides a mathematical framework for maintaining a stable pest-free agricultural system, particularly in designing effective pest control strategies. Future work may extend this model by incorporating additional ecological factors or control strategies.

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