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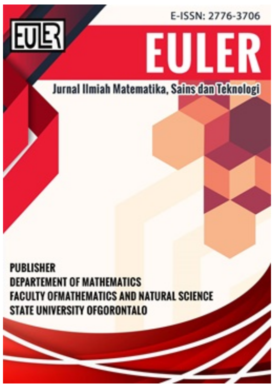

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Research Article

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(α, β)-derivation on Matrix Ring $M_n(R)$

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ABSTRACT. *In ring theory, a derivation is an additive mapping $d : R \to R$ satisfying Leibniz's rule. A well-known generalization of this notion is the (α, β)-derivation, defined with respect to two ring endomorphisms α and β. In this paper we study (α, β)-derivations on the matrix ring $M_n(R)$ and several of its subrings, including scalar matrices, diagonal matrices, and upper and lower triangular matrix rings. It is shown that an (α, β)-derivation on the base ring R induces an (α', β')-derivation on these matrix subrings via entrywise extension, preserving their structural properties. Furthermore, we examine certain properties of (α, β)-derivations on the direct product ring $R \times R$. In particular, we show that the sum of two (α, β)-derivations does not necessarily form an (α, β)-derivation, which is demonstrated through a counterexample.*

1. Introduction

The concept of derivation originated from the work of Newton and Leibniz in differential calculus and was later extended to various branches of mathematics, including abstract algebra. In ring theory, a derivation on a ring R is an additive mapping $d : R \to R$ satisfying Leibniz's rule $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$ [1, 2]. Derivations play an important role in the structural study of rings and algebras and have been widely investigated in several algebraic contexts, including semiprime rings, $*$-algebras, and polynomial rings [3–9].

Over the years, various generalizations of derivations have been introduced in order to better understand algebraic structures. Examples include Jordan derivations, generalized derivations, higher derivations, and several mixed or nonlinear variants [10–14]. These generalizations have been studied in different classes of rings and algebras and have provided useful tools for investigating the structural properties of algebraic systems [15–18].

One important generalization is the notion of an (α, β)-derivation, which depends on two ring endomorphisms $\alpha, \beta : R \to R$. An additive mapping $d : R \to R$ is called an (α, β)-derivation if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$, for all $x, y \in R$ [19]. This concept extends the classical notion of derivation and has been studied in various algebraic settings, including prime and semiprime rings and polynomial rings [20, 21].

Several recent studies have investigated properties and applications of derivations and their generalizations in different algebraic structures. For instance, derivations on polynomial modules and generalized power series modules were studied in [22, 23], while other works considered Jordan-type derivations and related mappings in prime rings and algebras [12, 13]. These results indicate that derivations and their variants provide an effective framework for analyzing structural properties of rings and modules.

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Despite these developments, the study of (α, β)-derivations in matrix rings remains relatively limited. Matrix rings $M_n(R)$ play a central role in algebra and its applications, and their subrings, such as diagonal, scalar, and triangular matrix rings, frequently appear in various mathematical contexts.

Motivated by these considerations, this paper studies (α, β)-derivations on the matrix ring $M_n(R)$ and several of its subrings, including diagonal, scalar, and triangular matrix rings. In particular, we construct (α, β)-derivations on $M_n(R)$ induced by (α, β)-derivations on the base ring R and investigate their structural properties. Furthermore, we study the behavior of the composition of (α, β)-derivations on the direct product ring $R \times R$.

2. Preliminaries

In this section we recall several basic definitions that will be used throughout the paper, including rings, derivations, matrix rings, and (α, β)-derivations.

Definition 1. [24] A non-empty set R equipped with two binary operations, addition and multiplication, is called a ring if the following conditions hold:
1. $(R, +)$ is an abelian group;
2. multiplication is associative, that is, $(ab)c = a(bc)$ for all $a, b, c \in R$;
3. the distributive laws hold:
$$a(b + c) = ab + ac, \qquad (a + b)c = ac + bc$$
for all $a, b, c \in R$.

Derivations on rings generalize the notion of differentiation from calculus to algebraic structures and play an important role in the structural study of rings and algebras.

Definition 2. [1] Let R be a ring. A mapping $d : R \to R$ is called a *derivation* if the following conditions hold for all $a, b \in R$:
1. $d(a + b) = d(a) + d(b)$;
2. $d(ab) = d(a)b + ad(b)$.

One important generalization of derivation is the (α, β)-derivation.

Definition 3. [19] Let R be a ring and let $\alpha, \beta : R \to R$ be ring endomorphisms. An additive mapping $d : R \to R$ is called an (α, β)-derivation if
$$d(xy) = d(x)\alpha(y) + \beta(x)d(y) \tag{1}$$
for all $x, y \in R$.

Since this paper investigates derivations on matrix rings, we recall the definition of a matrix ring.

Definition 4. Let R be a ring. The set of all $n \times n$ matrices with entries in R is denoted by $M_n(R)$. Equipped with the usual matrix addition and multiplication, $M_n(R)$ forms a ring called the *matrix ring* over R.

Some important subrings of $M_n(R)$ are the diagonal matrices, scalar matrices, and triangular matrices. These subrings will be used in the construction of (α, β)-derivations in the next section. Finally, we recall the direct product of rings.

Definition 5. Let R be a ring. The set

$$R \times R = \{(a, b) \mid a, b \in R\}$$

with componentwise addition and multiplication

$$(a, b) + (c, d) = (a + c, b + d), \qquad (a, b)(c, d) = (ac, bd)$$

forms a ring called the *direct product ring*.

3. Results and Discussion

In this section, we construct (α, β)-derivations on various matrix rings over a base ring R using the definitions introduced in the previous section. We first extend a given (α, β)-derivation on R to the full matrix ring $M_n(R)$, and then restrict it to subrings such as diagonal, scalar, upper, and lower triangular matrices. Finally, we examine properties of (α, β)-derivations on the direct product ring $R \times R$.

3.1. (α, β)-Derivation on the Matrix Ring $M_n(R)$

In this subsection, we show that an (α, β)-derivation on a ring R can be naturally extended to the matrix ring $M_n(R)$.

Theorem 1. *Let R be a ring and let $d : R \to R$ be an (α, β)-derivation on R. Then there exists a mapping $d' : M_n(R) \to M_n(R)$ which is an (α', β')-derivation on the matrix ring $M_n(R)$.*

Proof. Let $d : R \to R$ be an (α, β)-derivation on R, where α and β are endomorphisms of R. Define mappings

$$d'([a_{ij}]) = [d(a_{ij})], \qquad \alpha'([a_{ij}]) = [\alpha(a_{ij})], \qquad \beta'([a_{ij}]) = [\beta(a_{ij})].$$

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be arbitrary matrices in $M_n(R)$, where $a_{ij}, b_{ij} \in R$ for all $i, j = 1, 2, \ldots, n$. We prove that d' is an (α', β')-derivation on $M_n(R)$.
Additivity

$$\begin{aligned} d'(A + B) &= d'([a_{ij} + b_{ij}]) \\ &= [d(a_{ij} + b_{ij})] \\ &= [d(a_{ij}) + d(b_{ij})] \\ &= [d(a_{ij})] + [d(b_{ij})] \end{aligned}$$

$$= d'(A) + d'(B).$$

Thus, d' is additive.
Derivation property
Recall that matrix multiplication is defined by

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

Then

$$\begin{aligned} d'(AB) &= d'\left(\left[\sum_{k=1}^{n} a_{ik} b_{kj}\right]\right) \\ &= \left[d\left(\sum_{k=1}^{n} a_{ik} b_{kj}\right)\right]. \end{aligned}$$

Since d is an (α, β)-derivation on R, based on Equation (1) we have

$$d(xy) = d(x)\alpha(y) + \beta(x)d(y)$$

for all $x, y \in R$. Hence

$$d'(AB) = \left[\sum_{k=1}^{n} (d(a_{ik})\alpha(b_{kj}) + \beta(a_{ik})d(b_{kj}))\right].$$

Next we compute

$$d'(A)\alpha'(B) = [d(a_{ij})][\alpha(b_{ij})] = \left[\sum_{k=1}^{n} d(a_{ik})\alpha(b_{kj})\right],$$

and

$$\beta'(A)d'(B) = [\beta(a_{ij})][d(b_{ij})] = \left[\sum_{k=1}^{n} \beta(a_{ik})d(b_{kj})\right].$$

Therefore,

$$d'(A)\alpha'(B) + \beta'(A)d'(B) = \left[\sum_{k=1}^{n} (d(a_{ik})\alpha(b_{kj}) + \beta(a_{ik})d(b_{kj}))\right] = d'(AB).$$

Hence, d' satisfies additivity and the (α', β')-derivation identity

$$d'(AB) = d'(A)\alpha'(B) + \beta'(A)d'(B).$$

Therefore, $d' : M_n(R) \to M_n(R)$ is an (α', β')-derivation on the matrix ring $M_n(R)$. □

3.2. (α, β)-Derivation on Diagonal Matrix Rings

In this subsection, we study the behavior of (α, β)-derivations on the ring of diagonal matrices.

Theorem 2. *Let*

$$D = \left\{ \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \middle| \, a_{ii} \in R \right\}$$

be the diagonal matrix ring of $M_n(R)$. If $d : R \to R$ is an (α, β)-derivation on R, then there exists a mapping $d' : D \to D$ which is an (α', β')-derivation on D.

Proof. Define

$$d'([a_{ii}]) = [d(a_{ii})], \qquad \alpha'([a_{ii}]) = [\alpha(a_{ii})], \qquad \beta'([a_{ii}]) = [\beta(a_{ii})].$$

Let

$$A = \text{diag}(a_{11}, a_{22}, \ldots, a_{nn}), \quad B = \text{diag}(b_{11}, b_{22}, \ldots, b_{nn}).$$

Additivity

$$\begin{aligned} d'(A + B) &= d'(\text{diag}(a_{11} + b_{11}, \ldots, a_{nn} + b_{nn})) \\ &= \text{diag}(d(a_{11} + b_{11}), \ldots, d(a_{nn} + b_{nn})) \\ &= \text{diag}(d(a_{11}), \ldots, d(a_{nn})) + \text{diag}(d(b_{11}), \ldots, d(b_{nn})) \\ &= d'(A) + d'(B). \end{aligned}$$

Multiplicative property

Since

$$AB = \text{diag}(a_{11}b_{11}, \ldots, a_{nn}b_{nn}),$$

we obtain

$$\begin{aligned} d'(AB) &= \text{diag}(d(a_{11}b_{11}), \ldots, d(a_{nn}b_{nn})) \\ &= \text{diag}(d(a_{11})\alpha(b_{11}) + \beta(a_{11})d(b_{11}), \ldots). \end{aligned}$$

Moreover,

$$d'(A)\alpha'(B) = \text{diag}(d(a_{11})\alpha(b_{11}), \ldots, d(a_{nn})\alpha(b_{nn}))$$

and

$$\beta'(A)d'(B) = \text{diag}(\beta(a_{11})d(b_{11}), \ldots, \beta(a_{nn})d(b_{nn})).$$

Hence

$$d'(AB) = d'(A)\alpha'(B) + \beta'(A)d'(B).$$

Therefore, $d' : D \to D$ is an (α', β')-derivation on the diagonal matrix ring D. □

Corollary 1. *Let*

$$S = \left\{ \begin{bmatrix} a & 0 & \cdots & 0 \\ 0 & a & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a \end{bmatrix} \middle| \, a \in R \right\}$$

be the scalar matrix ring of $M_n(R)$. If $d : R \to R$ is an (α, β)-derivation on R, then there exists a mapping $d' : S \to S$ which is an (α', β')-derivation on S.

Proof. Let $d : R \to R$ be an (α, β)-derivation on R. Define the mappings

$$d'(aI_n) = d(a)I_n, \qquad \alpha'(aI_n) = \alpha(a)I_n, \qquad \beta'(aI_n) = \beta(a)I_n,$$

for all $a \in R$, where I_n denotes the identity matrix of order n.

Let $A = aI_n$ and $B = bI_n$ be arbitrary elements of S. Since the scalar matrix ring S is a special case of the diagonal matrix ring, the result follows directly from the previous theorem. Hence,

$$d'(AB) = d'(A)\alpha'(B) + \beta'(A)d'(B),$$

fulfills eq. (1) and d' is additive. Therefore, $d' : S \to S$ is an (α', β')-derivation on the scalar matrix ring S. □

3.3. (α, β)-Derivation on Upper and Lower Triangular Matrix Rings

In this subsection, we investigate the behavior of (α, β)-derivations on certain subrings of the matrix ring $M_n(R)$, namely the rings of upper and lower triangular matrices. Specifically, we show that an (α, β)-derivation defined on the ring R can be naturally extended to these triangular matrix rings. The following theorems establish the existence of such induced (α', β')-derivations.

Theorem 3. *Let $M_n(R)$ be the matrix ring over a ring R, and let*

$$S_a = \left\{ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \middle| \; a_{ij} \in R \right\}$$

be the ring of upper triangular matrices over R. If $d : R \to R$ is an (α, β)-derivation on R, then there exists a mapping $d' : S_a \to S_a$ which is an (α', β')-derivation on S_a.

Proof. For $A = [a_{ij}], B = [b_{ij}] \in S_a$, define

$$d'(A) = [d(a_{ij})], \quad \alpha'(A) = [\alpha(a_{ij})], \quad \beta'(A) = [\beta(a_{ij})].$$

Clearly d' is additive since d is additive. For the product AB, the (i, j)-entry is

$$(AB)_{ij} = \sum_{k=i}^{j} a_{ik}b_{kj}.$$

Hence

$$d'((AB)_{ij}) = d\left(\sum_{k=i}^{j} a_{ik}b_{kj}\right) = \sum_{k=i}^{j} \left(d(a_{ik})\alpha(b_{kj}) + \beta(a_{ik})d(b_{kj})\right).$$

This coincides with the (i, j)-entry of

$$d'(A)\alpha'(B) + \beta'(A)d'(B).$$

Therefore
$$d'(AB) = d'(A)\alpha'(B) + \beta'(A)d'(B),$$
fulfills eq. (1). Thus d' is an (α', β')-derivation on S_a. □

Besides the upper triangular matrix S_a, another subset of $M_n(R)$ is the ring of lower triangular matrices S_b. The following theorem describes the (α, β)-derivation on S_b.

Theorem 4. *Let $M_n(R)$ be a matrix ring and let*
$$S_b = \left\{ \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \middle| \; a_{ij} \in R \right\}$$
be the ring of lower triangular matrices. If $d : R \to R$ is an (α, β)-derivation on R, then there exists a mapping $d' : S_b \to S_b$ which is an (α', β')-derivation on S_b.

Proof. Let $d : R \to R$ be an (α, β)-derivation on R. Define the mappings
$$d'(A) = (d(a_{ij})), \qquad \alpha'(A) = (\alpha(a_{ij})), \qquad \beta'(A) = (\beta(a_{ij}))$$
for every $A = (a_{ij}) \in S_b$. First, we show that d' is additive. For $A = (a_{ij})$ and $B = (b_{ij})$ in S_b, we have
$$d'(A + B) = \big(d(a_{ij} + b_{ij})\big) = \big(d(a_{ij}) + d(b_{ij})\big) = d'(A) + d'(B),$$
since d is additive. Next, consider the product AB. The (i, j)-entry of AB is
$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}.$$
Thus
$$d'((AB)_{ij}) = d\left(\sum_{k=1}^{n} a_{ik}b_{kj}\right) = \sum_{k=1}^{n} d(a_{ik}b_{kj}).$$
Since d is an (α, β)-derivation on R, we obtain
$$d(a_{ik}b_{kj}) = d(a_{ik})\alpha(b_{kj}) + \beta(a_{ik})d(b_{kj}).$$
Therefore,
$$d'((AB)_{ij}) = \sum_{k=1}^{n} d(a_{ik})\alpha(b_{kj}) + \sum_{k=1}^{n} \beta(a_{ik})d(b_{kj}),$$
which corresponds to the (i, j)-entry of
$$d'(A)\alpha'(B) + \beta'(A)d'(B).$$
Hence
$$d'(AB) = d'(A)\alpha'(B) + \beta'(A)d'(B),$$
fulfills eq. (1). Thus d' is an (α', β')-derivation on S_b. □

3.4. Properties of (α, β)-Derivations on the Ring $R \times R$

In this section, we discuss several properties of (α, β)-derivations on the direct product ring $R \times R$. In particular, we analyze the behavior of compositions and sums of (α, β)-derivations. It will be shown that the composition of two (α, β)-derivations does not necessarily preserve the (α, β)-derivation property, and similarly the sum of two such derivations may fail to satisfy the defining condition.

Remark 1. Let d_1, d_2 be (α, β)-derivations on the ring $R \times R$. In general, the composition $d = d_1 \circ d_2$ is not necessarily an (α, β)-derivation.

Remark 2. Let d_1, d_2 be (α, β)-derivations on $R \times R$. Then the sum $d = d_1 + d_2$ is not necessarily an (α, β)-derivation.

The following example illustrates the failure of the (α, β)-derivation property for the sum of two (α, β)-derivations.

Example 1. Consider the ring $R \times R = \mathbb{Z}_{11} \times \mathbb{Z}_{11}$. Define two mappings

$$d_1(a, b) = (0, b), \qquad d_2(a, b) = (a, b),$$

with endomorphisms

$$\alpha(a, b) = (b, 0), \qquad \beta(a, b) = (0, b)$$

for every $(a, b) \in R \times R$. Then

$$d = d_1 + d_2$$

is given by

$$d(a, b) = (a, 2b).$$

Take $(a, b) = (\overline{6}, \overline{4})$ and $(c, d) = (\overline{3}, \overline{10})$. First,

$$d((a, b)(c, d)) = d((\overline{6}, \overline{4}) \cdot (\overline{3}, \overline{10})) = d(\overline{9}, \overline{3}) = (\overline{9}, \overline{6}).$$

Next,

$$d(a, b)\alpha(c, d) = (\overline{6}, \overline{8})(\overline{10}, \overline{0}) = (\overline{5}, \overline{0}),$$

and

$$\beta(a, b)d(c, d) = (\overline{0}, \overline{4})(\overline{3}, \overline{9}) = (\overline{0}, \overline{2}).$$

Thus,

$$d(a, b)\alpha(c, d) + \beta(a, b)d(c, d) = (\overline{5}, \overline{2}).$$

Since

$$d((a, b)(c, d)) \neq d(a, b)\alpha(c, d) + \beta(a, b)d(c, d),$$

it follows that d is not an (α, β)-derivation on $R \times R$.

4. Conclusion

This paper investigates several properties of (α, β)-derivations and their extensions to matrix-related structures. It is shown that if $d : R \to R$ is an (α, β)-derivation on a ring R, then it can be extended entrywise to certain subrings of the matrix ring $M_n(R)$, including scalar matrices, diagonal matrices, and triangular matrix rings. The results demonstrate that the structure of these matrices is preserved under the induced (α', β')-derivation, meaning that the image of a matrix remains within the same class of matrices.

Furthermore, several properties of (α, β)-derivations on the direct product ring $R \times R$ are examined. In particular, it is observed that the sum of two (α, β)-derivations does not necessarily yield another (α, β)-derivation, as illustrated by a counterexample. These results provide additional insight into the structural behavior of (α, β)-derivations on matrix rings and product rings.

These findings contribute to a better understanding of the behavior of (α, β)-derivations in matrix-related algebraic structures and may serve as a basis for further investigations on generalized derivations in more complex ring constructions.

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