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Dynamic Analysis of Modified Leslie-Gower the Predator-Prey Model with Fear Effect, Additional Food, and Prey Refuge

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ABSTRACT. This study discusses a Leslie-Gower prey-predator model incorporating three behavioral mechanisms: the fear effect on prey, additional food for predators, and prey refuge, with predation rate of predators using the Holling II response function. The objective of this research is to analyze the system dynamics through stability analysis of equilibrium points and the bifurcation associated with the predator attack rate parameter. The analytical results indicate that the model admits five equilibrium points, namely the extinction of both populations (E_1), prey extinction (E_2), predator extinction (E_3), and two coexistence equilibrium points (E_4 and E_5), where the existence of the interior equilibrium points depends on the values of the system parameters. The prey extinction equilibrium point and the coexistence equilibrium points are asymptotically stable under certain conditions. Computational validation was performed through numerical simulations. Furthermore, variations in the predator attack rate parameter reveal the presence of bistability as well as transcritical and saddle-node bifurcations that lead to changes in the stability of the equilibrium points.



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1. Introduction

Rice is a food crop that produces rice, which is the primary food source for the Indonesian population. Therefore, the availability of rice has an important role in preserving national food security [1, 2]. One of the main problems in rice cultivation is pest and disease attacks, including the rice field rat (*Rattus argentiventer*), which can reduce crop yields and cause losses for farmers [3]. Controlling rat pests with chemical pesticides can negatively impact the environment and the balance of the agricultural ecosystem [4, 5]. Therefore, the use of natural predators as biological control becomes one of the environmentally friendly and sustainable control alternatives. Predators such as barn owls (*Tyto alba*) have been proven effective in suppressing rice field rat populations through predator-prey interactions [6–8], often called predation interactions. The effects of predation on prey can be divided into two categories: direct effects and indirect effects, where the fear of predation can affect the physiological condition of the prey, causing long-term losses [9]. The fear effect experienced by rice field rats due to the sound of barn owls causes behavioral changes, such as reduced rat activity and seeking refuge to avoid predation, thereby limiting predator-prey interactions [10]. The decrease in prey encourages predators to utilize additional food sources to maintain population stability [11].

The problem of pest growth can be explained mathematically through a two-species interaction model by considering the predation process and prey responses to the existence of predators. One model that is commonly used to describe predator-prey dynamics is the

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Leslie–Gower model [12]. In this model, the predator population is assumed to grow logistically with carrying capacity depending on prey availability, so the persistence of the predator is strongly influenced by prey density. The predation mechanism is commonly represented using a Holling-type function [13]. In particular, the Holling Type II functional response indicates that the predator's consumption rate rises with increasing prey density but gradually approaches a saturation level due to the limitation of handling time [14].

Some researchers have developed the Leslie–Gower model by adding various biological assumptions. C. Cortes Garcia [15] analyzed a discontinuous Leslie–Gower model by considering alternative food for predators and harvesting using Holling Type II functional response. In another research, C. Cortes Garcia incorporated prey refuge behavior using linear functional response, where the results showed that the existence of prey refuge can maintain system stability [16]. With the presence of prey refuge, some prey can avoid predation, thereby helping to stabilize populations and support predator–prey coexistence [17, 18]. The fear effect also influences the dynamics of the predator-prey system [19, 20]. The effects of fear and prey refuge have been studied by Xie and Zhang [21] on the Lotka–Volterra predator–prey model using a Holling Type III functional response.

Savitri, et. al [11] studied the modified Leslie–Gower model with the effects of fear and additional food for predators using the Holling Type II functional response, assuming that predators also require additional food to survive. Inspired by the model [22] which discusses the modified Leslie–Gower model by incorporating prey refuge and constant additional food for the predator using a Holling Type I functional response. Even though several studies have analyzed the Leslie-Gower model with various biological assumptions, research that simultaneously includes predator supplementary food, prey refuge, and fear effects in predator–prey systems is still limited. These three factors have the potential to significantly influence population dynamics. Therefore, this article studies a Leslie–Gower predator–prey model with additional food for predators and prey refuge, where the fear effect influences only the reproduction rate of the prey population. The predation process follows a Holling Type II functional response, as predators require time to search for, hunt, and consume prey. This response function is biologically realistic in describing predator–prey interactions in this system, since the predation rate cannot increase indefinitely even when prey are abundant. The system dynamics are analyzed through the stability of equilibrium points and bifurcation analysis using the predator attack rate parameter.

2. Methods

Inspired by the model in [22], we construct a Leslie–Gower predator–prey system with a Holling Type II functional response that considers fear effects on prey, prey refuge, and additional food for predators. The research methodology was carried out as follows [23].

2.1. Model Construction and Assumptions

The dynamics of the predator–prey interaction are modeled using a nonlinear system of ordinary differential equations. In constructing this model, the assumptions used are: the prey population grows following a logistic growth model; prey fear predators, which reduces their reproduction rate; and while part of the prey population can refuge from predation. The predation process uses a Holling Type II functional response given by $\frac{ax}{1+bx}$, where x denotes the prey population, a is the predator attack rate, and b represents the handling time parameter. Predators obtain additional food available in the environment as an alternative

energy source. The growth rate of predators follows a modified Leslie–Gower model, where the predator carrying capacity depends on both prey availability and the contribution of additional food.

2.2. Equilibrium Point and Stability Analysis

The equilibrium points of the system are obtained when the right-hand sides of the equations are equal to zero. These equilibria represent the extinction state, the prey-free or predator-free state, and the coexistence state. To determine the stability of each equilibrium point, the system is first linearized using the Jacobian matrix. The local stability is determined based on the eigenvalues or roots of the characteristic equation of the Jacobian matrix. If the eigenvalues are difficult to determine explicitly, then the Routh-Hurwitz criterion is used

2.3. Numerical Simulation

Numerical simulations were performed using MATLAB to check the accuracy of the local stability analysis calculations for each system solution. The results of the numerical simulation are presented in the form of a phase portrait. Furthermore, to describe the system's dynamic behavior due to variations in the predator attack rate parameter, a bifurcation diagram analysis is performed using MatCont.

3. Results and Discussion

This section begins by explaining the formulation of mathematical models, identification of equilibrium points, local stability analysis, and numerical simulation. The stability of the equilibrium point is analyzed based on the eigenvalue obtained from the Jacobian matrix at each equilibrium point. Numerical simulations and bifurcation analysis are conducted to examine changes in the system's dynamic behavior caused by variations in the predator attack rate parameter.

3.1. Mathematics Model

Based on the above assumptions, a prey–predator interaction model is constructed by incorporating prey refuge and additional food for the predator according to research [22]. Then, it is modified by changing the Holling Type I functional response to a Holling Type II functional response and incorporating the fear effect on the prey, referring to the research [11]. Thus, the new model is obtained as follows:

$$\begin{aligned} F(x, y) &= \frac{dx}{dt} = \frac{rx}{1 + fy} \left(1 - \frac{x}{K}\right) - \frac{q\alpha(1-s)xy}{m + (1-s)x}, \\ G(x, y) &= \frac{dy}{dt} = \beta y \left(1 - \frac{y}{q\rho(1-s)x + (1-q)nA}\right). \end{aligned} \quad (1)$$

In Model (1), $x(t)$ and $y(t)$ denote the prey and predator population densities, respectively, with initial conditions $x(0) > 0$ and $y(0) > 0$. All parameters are positive and defined as follows: r is the intrinsic growth rate of the prey, β is the intrinsic growth rate of the predator, K is the carrying capacity of prey in the environment, f is the fear rate of the prey toward the predator, α is the predator attack rate, m is the half-saturation constant, and ρ is the conversion rate of prey consumed by the predator into predator biomass.

A portion of the prey population is protected from predation, represented by the parameter $s \in [0, 1)$, so that only the proportion $(1-s)x$ is vulnerable to predation. In addition, the parameters q , n , and A represents the additional food available to the predator, where q

is the preference rate of the predator due to the predator has two sources of food. If $q = 0$, the predator consumes only the additional food, and its carrying capacity becomes nA . If $q = 1$, the predator consumes only the prey x , resulting in a carrying capacity of $\rho(1-s)x(t)$. If $q \in (0, 1)$, the predator consumes both food sources according to the proportions defined by q , giving a carrying capacity of $q(1-s)\rho x(t) + (1-q)nA$.

3.2. Equilibrium Points

The equilibrium points of model (1) are obtained when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. Thus, the equilibrium point are given as follows:

1. $E_1 = (0, 0)$ shows the extinction of all populations.
2. $E_2 = (0, \hat{y})$ indicates extinction of the prey, with $\hat{y} = nA(1-q)$, so that the predator population attains its carrying capacity.
3. $E_3 = (K, 0)$ indicates extinction of the predator, while the prey population attains its carrying capacity.
4. $E_4 = (x^*, y^*)$ shows the coexistence of prey and predator population. The point E_4 is also called an interior equilibrium point with

$$y^* = q\rho(1-s)x^* + (1-q)nA$$

and x^* is the roots of quadratic equation

$$A_1(x^*)^2 + A_2(x^*) + A_3 = 0, \quad (2)$$

where

$$\begin{aligned} A_1 &= (1-s)[K\alpha f q^3 \rho^2 (1-s)^2 + r], \\ A_2 &= (1-s)^2 K\alpha q^2 \rho [2Afn(1-q) + 1] - Kr(1-s) + mr, \\ A_3 &= (1-s)AK\alpha nq(1-q)[Afn(1-q) + 1] - Kmr. \end{aligned}$$

The roots obtained from eq. (2) are as follows:

$$x_{1,2}^* = \frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2A_1}.$$

Only equilibrium points with positive real values have biological meaning, so the interior equilibrium points can be classified as follows:

- (a) If $A_3 < 0$ and $A_2^2 - 4A_1A_3 > 0$, then eq. (2) has exactly one positive root x_1^* , so there is one interior equilibrium point, namely E_4 . The coefficient A_3 is negative if $(1-s)AK\alpha nq(1-q)[Afn(1-q) + 1] < Kmr$.
- (b) If $A_3 > 0$, $A_2 < 0$, and $A_2^2 - 4A_1A_3 > 0$, then eq. (2) has two positive roots, so there are two interior equilibrium points, namely E_4 and E_5 . The coefficient A_3 is positive if $(1-s)AK\alpha nq(1-q)[Afn(1-q) + 1] > Kmr$ and the coefficient A_2 is negative if $(1-s)^2 K\alpha q^2 \rho [2Afn(1-q) + 1] < Kr(1-s) - mr$.
- (c) If $A_3 > 0$, $A_2 < 0$, and $A_2^2 - 4A_1A_3 = 0$, then eq. (2) has one positive root, so there is one interior equilibrium point, namely E_4 with $x^* = -\frac{A_2}{2A_1}$.

3.3. Local Stability

The local stability of each equilibrium point is determined based on the eigenvalues (or roots) of the Jacobian matrix's characteristic equation. The Jacobian matrix of eq. (1) is

$$J(x, y) = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{pmatrix}, \quad (3)$$

with

$$\begin{aligned} \frac{\partial F}{\partial x} &= \frac{r}{1 + fy} \left(1 - \frac{2x}{K}\right) - \frac{q\alpha(1-s)ym}{(m + (1-s)x)^2}, \\ \frac{\partial F}{\partial y} &= -\frac{rfx}{(1 + fy)^2} \left(1 - \frac{x}{K}\right) - \frac{q\alpha(1-s)x}{m + (1-s)x}, \\ \frac{\partial G}{\partial x} &= \frac{\beta q\rho(1-s)y^2}{(q\rho(1-s)x + (1-q)nA)^2}, \\ \frac{\partial G}{\partial y} &= \beta \left(1 - \frac{2y}{q\rho(1-s)x + (1-q)nA}\right). \end{aligned}$$

Theorem 1. The equilibrium point E_1 is unstable (unstable node).

Proof. The Jacobian matrix of model (1) at equilibrium point $E_1 = (0, 0)$ is as follows

$$J(E_1) = \begin{pmatrix} r & 0 \\ 0 & \beta \end{pmatrix},$$

than eigen values for E_1 is $\lambda_1 = r > 0$ and $\lambda_2 = \beta > 0$. Therefore, equilibrium point $E_1 = (0, 0)$ is unstable (unstable node). \square

Theorem 2. The equilibrium point E_2 is asymptotically stable if

$$\frac{r}{1 + fnA(1-q)} < \frac{q\alpha(1-s)nA(1-q)}{m}.$$

Proof. The Jacobian matrix of model (1) at equilibrium point $E_2 = (0, nA(1-q))$ is as follows

$$J(E_2) = \begin{pmatrix} \frac{r}{1 + fnA(1-q)} - \frac{q\alpha(1-s)nA(1-q)}{m} & 0 \\ \beta q\rho(1-s) & -\beta \end{pmatrix},$$

than eigen values for E_2 is $\lambda_1 = \frac{r}{1 + fnA(1-q)} - \frac{q\alpha(1-s)nA(1-q)}{m}$ and $\lambda_2 = -\beta < 0$. Hence, equilibrium point E_2 is locally asymptotically stable if

$$\frac{r}{1 + fnA(1-q)} < \frac{q\alpha(1-s)nA(1-q)}{m}.$$

\square

Theorem 3. The equilibrium point E_3 is unstable (saddle point).

Proof. The Jacobian matrix of model (1) at equilibrium point $E_3 = (K, 0)$ is as follows

$$J(E_3) = \begin{pmatrix} -r & -\frac{q\alpha(1-s)K}{m + (1-s)K} \\ 0 & \beta \end{pmatrix},$$

than eigen values for E_3 is $\lambda_1 = -r < 0$ and $\lambda_2 = \beta > 0$. Therefore, equilibrium point $E_3 = (K, 0)$ is unstable (saddle point). \square

Theorem 4. The equilibrium point E_4 is asymptotically stable if $Tr(J(E_4)) < 0$ and $det(J(E_4)) > 0$.

Proof. The Jacobian matrix of model (1) at equilibrium point $E_4 = (x^*, y^*)$ is as follows

$$J(E_4) = \begin{pmatrix} J_{11}^4 & J_{12}^4 \\ J_{21}^4 & J_{22}^4 \end{pmatrix},$$

where

$$\begin{aligned} J_{11}^4 &= \frac{r}{1 + fy^*} \left(1 - \frac{2x^*}{K} \right) - \frac{q\alpha(1-s)y^*m}{(m + (1-s)x^*)^2}, \\ J_{12}^4 &= -\frac{rfx^*}{(1 + fy^*)^2} \left(1 - \frac{x^*}{K} \right) - \frac{q\alpha(1-s)x^*}{m + (1-s)x^*}, \\ J_{21}^4 &= \frac{\beta q\rho(1-s)y^{*2}}{(q\rho(1-s)x^* + (1-q)nA)^2}, \\ J_{22}^4 &= \beta \left(1 - \frac{2y^*}{q\rho(1-s)x^* + (1-q)nA} \right). \end{aligned}$$

The eigenvalues of E_4 are determined from $det(J - \lambda I) = 0$, so we get the characteristic equation,

$$\lambda^2 - Tr(J)\lambda + det(J) = 0, \tag{4}$$

with $Tr(J) = J_{11}^4 + J_{22}^4$ and $det(J) = J_{11}^4 J_{22}^4 - J_{12}^4 J_{21}^4$. Based on the Routh-Hurwitz criterion [24], The equilibrium point E_4 is said to be asymptotically stable if and only if $Tr(J(E_4)) < 0$ and $det(J(E_4)) > 0$. \square

3.4. Numerical Simulation

Numerical simulations were performed to check the accuracy of the analysis results and to illustrate changes in the system dynamics around the equilibrium points through phase portraits and time series. The following are the parameters used in the numerical simulation.

For the simulations, the parameter values in Table 1 are used, so that all equilibrium points E_1, E_2, E_3, E_4 , and E_5 of the system (1) exist. Based on the analysis, the existence condition for the interior equilibrium point E_3 is satisfied when $A_3 > 0$, namely $(1-s)AK\alpha nq(1-q)[Afn(1-q) + 1] > Kmr$. With the parameter values listed in Table 1 and $\alpha = 2.2$, we find $0.6146 > 0.5574$. Therefore, there are two interior equilibrium points, namely E_4 and E_5 .

Table 1. Numerical Simulation Parameter Values

Parameter	Description	Value	Source
r	The intrinsic growth rate of the prey	3.22	[11]
K	The carrying capacity of prey in the environment	3	[22]
β	The intrinsic growth rate of the predator	1.5	[22]
f	The rate of prey's fear of predators	0.31	[11]
α	The predator attack rate	2.2	Assumption
A	Additional food for predators with constant density	3	[22]
n	Amount of energy from additional food converted into predator biomass	0.2	[22]
ρ	Predator conversion rate	0.8	[22]
s	Prey Refuge	0.3	Assumption
q	The preference rate of the predator	0.7	[22]
m	Half-saturation constant	0.0577	[11]

For the parameters in **Table 1**, the eigenvalues of each equilibrium point are obtained as follows:

1. The equilibrium point $E_1(0, 0)$ has two positive eigenvalues, ($\lambda_1 = 1.5 > 0, \lambda_2 = 3.22 > 0$), so E_1 is unstable (unstable node).
2. The equilibrium point $E_2(0, nA(1 - q))$ has two negative eigenvalues ($\lambda_1 = -1.5 < 0, \lambda_2 = -0.313 < 0$), so E_2 is asymptotically stable. Thus, this confirms that **Theorem 2** is satisfied for $\alpha = 2.2$.
3. The equilibrium point $E_3(K, 0)$ has one negative and one positive eigenvalue, ($\lambda_1 = -3.2199 < 0, \lambda_2 = 1.5 > 0$), so E_3 is unstable (saddle point).
4. The equilibrium point $E_4(2.1346, 1.0167)$ has a pair of complex eigenvalues with negative real parts, $\lambda_{1,2} = -1.28101 \pm 1.01628i$, so E_4 is asymptotically stable (spiral point). Thus, this confirms that **Theorem 2** is satisfied for $\alpha = 2.2$.
5. The equilibrium point $E_5(0.0111, 0.1844)$ has one negative eigenvalue and one positive eigenvalue, ($\lambda_1 = 0.28618 > 0, \lambda_2 = -1.43642 < 0$), so E_5 is unstable (saddle point).

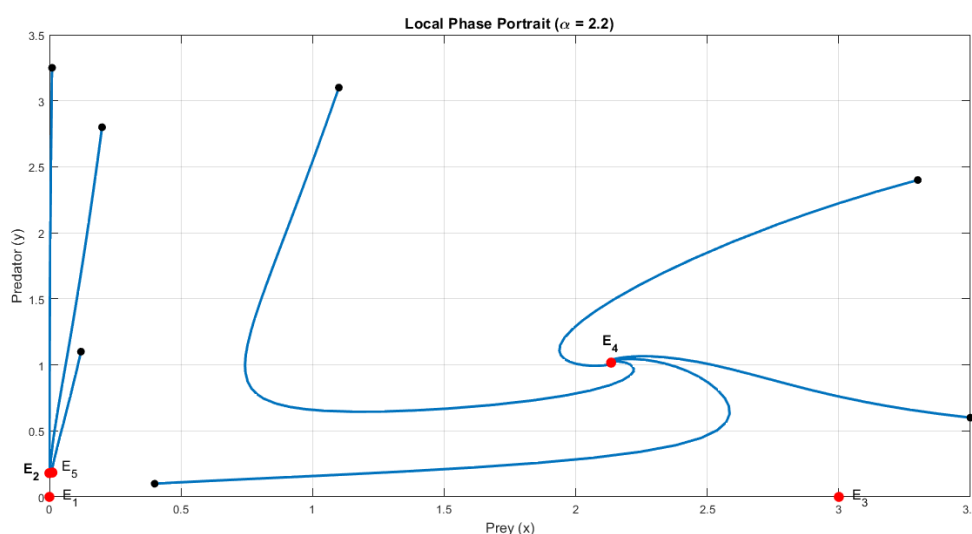


Figure 1. Phase portrait of system (1) for $\alpha = 2.2$

Therefore, the system has double stability because there are two stable equilibrium points, E_2 and E_4 . Biologically, the stability of E_2 indicates a condition where the preda-

tor population survives while the prey population becomes extinct. This can occur when predator attack rate is sufficiently high or when the initial prey population is low. On the other hand, the stability of E_4 represents a coexistence state, where both prey (rice field rats) and predator (barn owls) persist in the long term, indicating a balanced ecosystem.

Numerical simulations are conducted to observe the dynamic behavior of system (1) using the parameter values from Table 1 with variations in the predator attack rate parameter, specifically for $\alpha = 2.2$ and $\alpha = 0.5$. The first simulation was conducted using a variation of the predator attack rate parameter $\alpha = 2.2$.

The phase portrait in Figure 1 shows a double stability system (bistable). The solution of a double stability system is influenced by the initial values of both populations [24]. When initial conditions of $[0.01, 3.25]$, $[0.2, 2.8]$ and $[0.12, 1.1]$ are used, the system converges to the equilibrium point $E_2(0, 0.18)$, so that the equilibrium point E_2 is stable. Meanwhile, with initial conditions of $[0.4, 0.1]$, $[1.1, 3.1]$, $[3.5, 0.6]$, and $[3.3, 2.4]$, the system solution converges to the equilibrium point E_4 , so that the equilibrium point E_4 is also stable. These results are consistent with the analytical calculations and existence conditions.

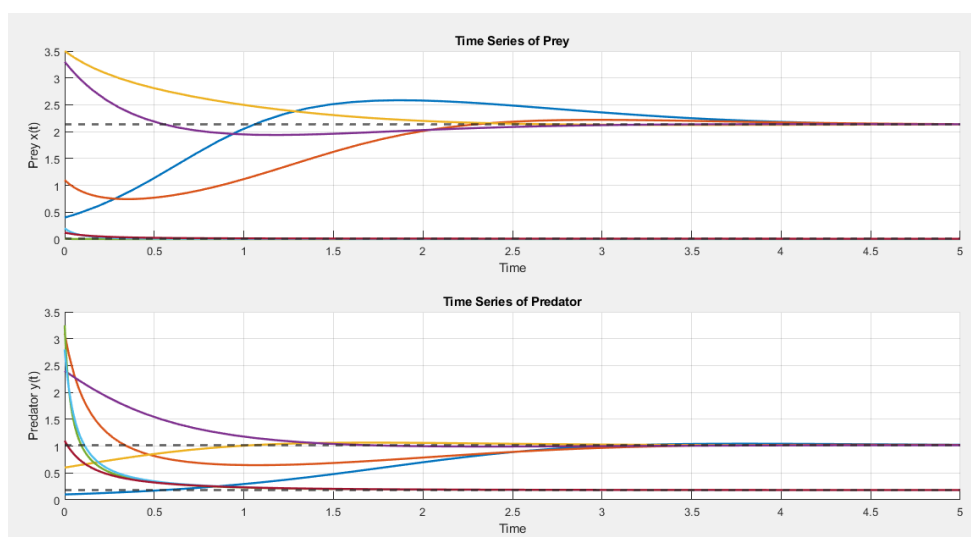


Figure 2. Time Series with $\alpha = 2.2$

The time series graph in Figure 2, shows that for initial conditions where the prey population $x(t)$ is relatively small, the solution converges to the equilibrium point E_2 , where the prey population becomes extinct. Meanwhile, for initial conditions with a relatively large prey population, the solution converges to E_4 , where the prey and predator populations can coexist in the long term.

From a biological perspective, this bistable behavior indicates that the initial population density significantly influences the system's final outcome. A very low prey population can lead to predator dominance and eventually to prey extinction, while sufficient prey availability allows for stable coexistence. This condition is more desirable for maintaining ecological balance, since prey extinction may cause predators to lose their primary food source and reduce their population, potentially triggering a pest outbreak in the future. Therefore, maintaining the prey population at a certain level enables more effective natural control and preserves ecosystem balance.

The second simulation was conducted with a variation in the predator attack rate parameter to $\alpha = 0.5$. The phase portrait in Figure 3 shows that only point E_4 is stable with equilibrium points E_1 , E_2 , E_3 , and E_4 of system (1) exist. Using various different initial values,

all solutions tend to converge to equilibrium point E_4 . Biologically, this indicates that when the predator attack rate is low, the system tends toward stable coexistence, thereby reducing the risk of prey extinction.

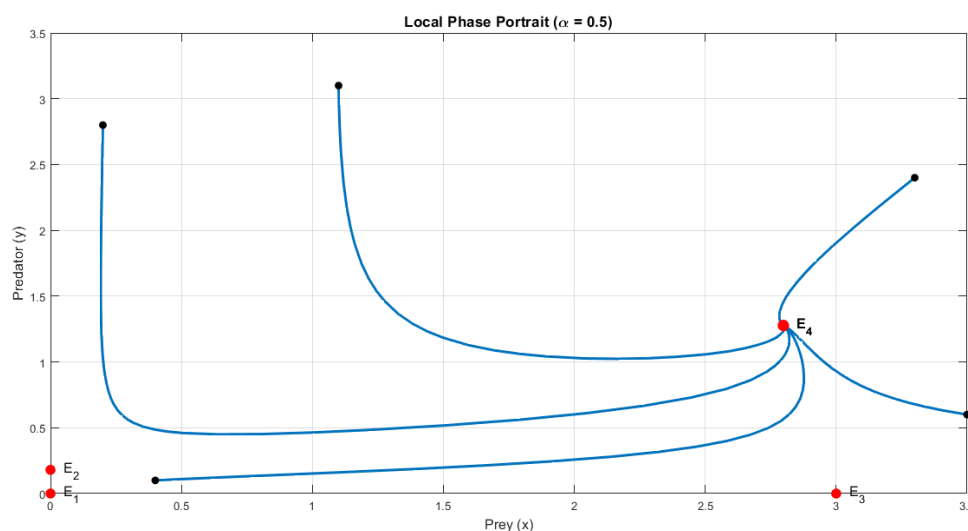


Figure 3. Phase portrait of system (1) for $\alpha = 0.5$

The simulation results in the phase portraits show that a value of $\alpha = 2.2$ strengthens predation pressure in the system. This results in the existence of all five equilibrium points and the occurrence of bistability, as there are two stable equilibrium points. In the bistable case, the system's solutions depend on the initial conditions. Meanwhile, when predation pressure decreases to $\alpha = 0.5$, only four equilibrium points exist, namely E_1 , E_2 , E_3 , and E_4 , with the equilibrium point E_4 being stable. This indicates that the predator attack rate plays a crucial role in shaping system dynamics. High predation rates can increase the risk of prey extinction under certain conditions, while moderate predation promotes stable coexistence. Biologically, this implies that maintaining an optimal level of predation is necessary to control pest populations without causing the extinction of prey, thereby preserving long-term ecological balance.

3.5. Bifurcation

Numerical continuation was performed using MatCont in MATLAB R2019a to investigate how variations in the predator attack rate influence the system's dynamic behavior. Variations in the parameter α can significantly change the stability of some equilibrium points. This is consistent with **Theorem 2**, which shows that the stability of the equilibrium point E_2 depends explicitly on α . Therefore, α is chosen as the bifurcation parameter. The stability changes induced by α are shown in the bifurcation diagram.

The bifurcation diagram in **Figure 4** shows two types of bifurcations, namely *saddle node* bifurcation and *transkritikal* bifurcation. A branch point (BP) is detected at $\alpha = 1.995177$, indicating a *transkritikal* bifurcation, in which the stability is exchanged between the coexistence equilibrium E_5 and the prey extinction point E_2 . The equilibrium point E_2 is initially stable when $\alpha > 1.995177$ and becomes unstable when $\alpha < 1.995177$. On the other hand, the coexistence equilibrium point E_5 is initially unstable for $\alpha > 1.995177$ and becomes stable for $\alpha < 1.995177$. However, when the equilibrium point E_5 is stable, it produces a negative prey density, which is biologically infeasible.

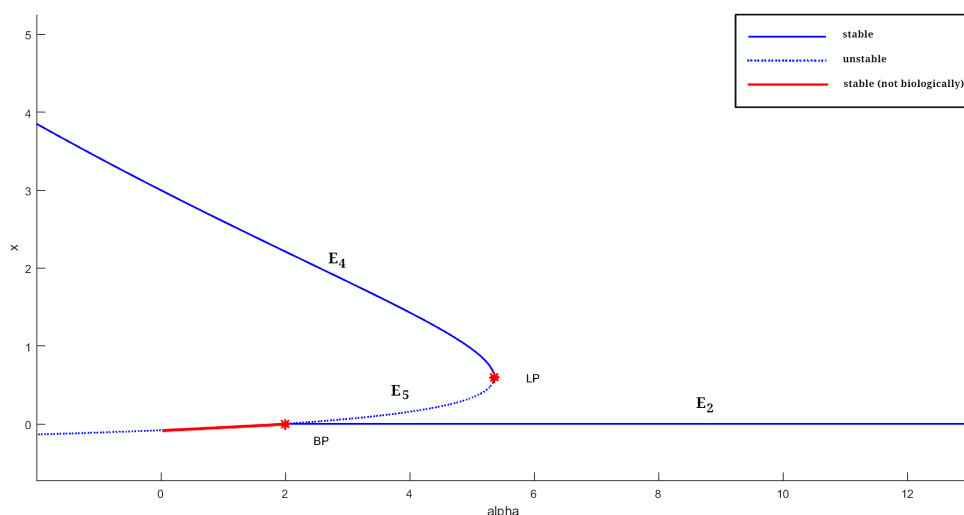


Figure 4. Bifurcation Diagram

Furthermore, in the interval α between BP and LP, which is $1.995177 < \alpha < 5.358781$, bistability phenomenon occurs, where there are two stable equilibrium points, E_2 and E_4 , while E_5 is unstable. In addition, the system has two interior equilibrium points, with (E_4) being stable and (E_5) unstable. The two interior equilibrium points meet at $\alpha = 5.358781$, referred to as the Limit Point (LP), corresponding to a *saddle node* bifurcation. Thus, variations in the predator attack rate parameter play a significant role in shaping the long-term dynamics of the predator–prey system.

4. Conclusion

Based on the analysis results, the studied predator–prey model have five equilibrium points: total population extinction (E_1), prey extinction (E_2), predator extinction (E_3), and two coexistence equilibrium points (E_4 and E_5). The existence of interior equilibrium points depends on the system parameter values, particularly the predator attack rate on prey. Stability analysis shows that the extinction points for all populations and the predator extinction point are always unstable. The stability of the other equilibrium is influenced by the chosen parameter values. For appropriate parameter values, the system exhibits bistability (E_2 and E_4 are stable). Numerical simulation results show consistency with the analysis results for the stability of each equilibrium solution. Using numerical continuation with respect to the predator attack rate parameter identifies *Transkritikal* and *saddle-node* bifurcation that lead to changes in stability and bistability at certain intervals. The results of this study indicate that the predator attack rate parameter plays an important role in influencing the dynamic behavior of the system. For further research, this model can be developed by considering more complex response functions, such as Holling type III and Holling type IV functional response, by incorporating additional variables, such as the effects of time delays.

Author Contributions. Hanifa Febriana Putri: Conceptualization, methodology, software, formal analysis, resources, visualization, writing–original draft preparation and editing. Dian Savitri: Final Conceptualization, validation, writing–review and supervision.

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Data availability. The parameter values used in the simulation were obtained from relevant literature to represent the system dynamics.

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