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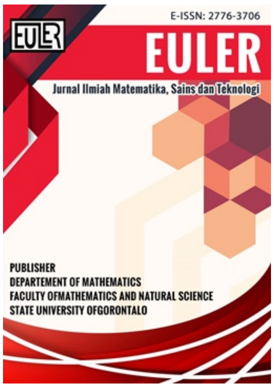

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Research Article

Recidivism-Driven Persistence in Crime Dynamics: A Comparative S–C–P–R Compartmental Model for Kupang City, Indonesia

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ABSTRACT. *Crime remains a significant social problem in Kupang City, which recorded 2,173 criminal cases in the first half of 2024, necessitating a quantitative approach to understand its dynamics. This study aims to construct and analyze mathematical models of crime dynamics in Kupang City by considering the role of recidivism. Two compartmental models (S-C-P-R) are developed: a model with recidivism and a model without recidivism, each formulated as a system of ordinary differential equations. The analysis includes the determination of equilibrium points, the basic reproduction number ($\mathcal{R}_0$) using the Next Generation Matrix method, stability analysis via the Jacobian matrix and Routh–Hurwitz criterion, sensitivity analysis, and numerical simulation using a Nonstandard Finite Difference (NSFD) scheme. Parameter estimation is performed through exponential regression based on crime data from Kupang City for the period 2013–2024. The results show that the model without recidivism yields $\mathcal{R}_0 = 2.3525$, indicating that criminal activity will persist and grow over time. The crime-free equilibrium point is locally asymptotically stable when $\mathcal{R}_0 < 1$, and sensitivity analysis identifies the transmission rate β as the most influential parameter affecting $\mathcal{R}_0$. In the model with recidivism, the feedback loop from the recovered compartment to the criminal compartment causes the crime-free equilibrium point to become unstable, and $\mathcal{R}_0$ is not well-defined due to the absence of a clear generational structure. These findings suggest that recidivism significantly complicates crime control and that strategies focused on reducing the transmission rate β are essential for suppressing criminal activity in Kupang City.*

1. Introduction

Criminal behavior can be viewed as actions that conflict with established legal rules and socially accepted standards, which in turn may jeopardize personal security and disturb the balance of social life [1]. Crime not only has physical impacts but also affects economic and psychological aspects [2]. Globally, the dynamics of crime continue to evolve, influenced by social inequality, urbanization, and technological development [3, 4]. In Indonesia, the number of criminal cases reached 434,768 in 2024 [5], while Kupang City recorded 2,173 cases from January to June 2024 alone [6], making it one of the highest crime areas in East Nusa Tenggara province.

One of the critical factors in crime dynamics is recidivism, which refers to the tendency of offenders to re-engage in criminal activities after release from prison [7, 8]. Limited rehabilitation programs, overcrowding in correctional facilities, social stigma, restricted employment opportunities, and economic hardship contribute to high recidivism rates [9, 10]. Consequently, former offenders may return to criminal behavior, creating a continuous cycle that sustains and even increases crime prevalence within a community [8]. Therefore, incorporating recidivism into mathematical models is essential for quantitatively analyzing its impact

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on crime dynamics and for providing a more realistic representation of criminal behavior in society [11, 12].

Beyond individual reoffending, recidivism also affects crime dynamics through social interaction mechanisms. Former offenders who return to society often maintain connections with previous criminal networks, peer groups, or environments associated with criminal activities. These interactions may facilitate the transmission of criminal knowledge, criminal opportunities, and deviant behavioral norms to susceptible individuals, particularly among unemployed youth and socially vulnerable groups. As a result, recidivists not only contribute directly to the number of active offenders through repeated criminal acts but may also indirectly increase crime incidence by influencing others to engage in criminal behavior. Consequently, high recidivism rates can strengthen the persistence of criminal activities within communities and accelerate the propagation of crime over time.

Although criminal behavior is not a biological disease, its spread exhibits characteristics analogous to the transmission of infectious diseases. Criminal behavior can be influenced by social interactions, peer pressure, imitation, and environmental exposure, whereby individuals may adopt criminal activities after repeated contact with active offenders [13, 14]. Similar to the transmission process of infectious diseases, susceptible members of society may become involved in criminal activities through continuous interaction with criminals. Furthermore, the spread of criminal behavior often occurs within social networks, families, peer groups, and communities, creating a contagion-like effect that facilitates crime propagation [11, 15]. From a mathematical perspective, the analogy between crime transmission and infectious disease transmission arises from the similarity of their underlying population dynamics. In epidemiological models, individuals are typically classified into compartments such as susceptible, infected, and recovered, with transitions occurring through contact and behavioral change. A similar framework can be applied to crime dynamics, where law-abiding individuals may be regarded as susceptible, active offenders as infectious individuals, and rehabilitated or imprisoned individuals as removed or recovered classes. The transition from lawful behavior to criminal involvement is often influenced by exposure to criminal peers, social learning, imitation, and environmental pressures, analogous to infection through contact. Moreover, intervention measures such as rehabilitation programs, law enforcement, imprisonment, and social reintegration can be interpreted as control mechanisms that reduce the transmission of criminal behavior. Therefore, epidemiological compartmental models provide a systematic framework for quantifying crime propagation, identifying key transmission pathways, evaluating threshold conditions for crime persistence, and assessing the effectiveness of crime prevention strategies.

In this context, recidivists can be viewed as a persistent source of criminal influence within the population because their repeated involvement in criminal activities increases the frequency of contact between criminals and susceptible individuals, thereby enhancing the potential transmission of criminal behavior. Therefore, ignoring recidivism may underestimate the long-term persistence and growth of crime in society.

In recent years, the application of epidemiological compartmental models to criminal behavior has gained significant attention, treating crime as a socially contagious phenomenon [13, 14]. Sooknanan and Seemungal [11] provided a comprehensive review confirming the effectiveness of this approach across various crime types. Several notable studies have advanced this field: Kwofie et al. [4] incorporated education programs and derived the reproduction number using the next-generation matrix (NGM) method; Mataru et al. [16] modeled unemployment-driven crime with vocational training as control measures; Ibrahim et al. [17]

developed an age-structured criminal gang model showing backward and Hopf bifurcation, and later applied optimal control theory in a limited-resource setting [12]. More recently, Idisi et al. [18] analyzed crime–corruption interdependence across three countries, Correa-Flórez et al. [19] fitted a compartmental system to Spanish imprisonment data, and Nyaberi and Mushanyu [15] modeled the role of imitation in crime propagation. The field has also been extended to fractional-order models [20–22] and co-dynamics of crime with drug abuse incorporating cross-recidivism [23]. Ayoade et al. [24] further analyzed the influence of family background on crime spread, while Aguadze et al. [25] and Mebratie and Dawed [26] explored the roles of latent criminal stages and media coverage, respectively.

Despite the substantial development of mathematical models for crime dynamics, several important gaps remain in the existing literature. Previous studies have examined various determinants of crime propagation, including education programs, unemployment, family background, media influence, criminal gangs, corruption, drug abuse, and imitation effects [4, 15–18, 23–26]. While some studies incorporated recidivism as one of the model components [23].recidivism has generally been treated as an auxiliary mechanism rather than the primary focus of investigation. To date, few studies have explicitly quantified the impact of recidivism by systematically comparing crime models with and without recidivism under the same modeling framework. As a result, the extent to which recidivism influences crime persistence, transmission intensity, and long-term equilibrium behavior remains insufficiently understood.

Furthermore, most existing studies have focused on theoretical model development, qualitative analysis, or numerical simulations using hypothetical parameter values. Empirical studies based on observed crime data are relatively limited, particularly in developing countries. To the best of our knowledge, empirical studies investigating the influence of recidivism on crime dynamics using crime data from Indonesian cities remain very limited, and evidence from Southeast Asia is still scarce. This limitation reduces the applicability of existing findings to regions with different social, economic, and criminal justice characteristics.

This study contributes to the literature in three important ways. First, it develops and compares two epidemiological compartmental crime models, namely a model with recidivism and a model without recidivism, to explicitly quantify the role of recidivism in crime transmission dynamics. Second, unlike many previous studies that rely on hypothetical parameter values, the proposed models are calibrated using actual crime data from Kupang City during the period 2013–2024, providing an empirical assessment of crime dynamics in an Indonesian urban setting. Third, the study provides quantitative evidence of how recidivism influences the basic reproduction number, equilibrium stability, and long-term crime prevalence through parameter estimation, stability analysis, sensitivity analysis, and numerical simulations, thereby offering deeper insights into the mechanisms through which recidivism sustains crime dynamics. These contributions provide a more comprehensive understanding of crime dynamics and offer evidence-based insights for the design of crime prevention and rehabilitation policies.

Based on the above, this study aims to construct and analyze two compartmental models (S-C-P-R) of crime dynamics in Kupang City: a model with recidivism and a model without recidivism. The study involves identifying equilibrium states and calculating the basic reproduction number ($\mathcal{R}_0$) through the Next Generation Matrix approach [27, 28], stability analysis, sensitivity analysis, and numerical simulations using a Nonstandard Finite Difference (NSFD) scheme. Parameter estimation is performed through exponential regression based on crime data from Kupang City for the period 2013–2024 [6].

2. Model

2.1. Data Sources

This study uses secondary data obtained from the BPS (Badan Pusat Statistik) of Kupang City [6]. The data consist of annual reported criminal cases in Kupang City from 2013 to 2024, which serve as the basis for parameter estimation via exponential regression. Additionally, population data and life expectancy statistics from BPS are used to estimate demographic parameters such as the recruitment rate Λ and the natural death rate μ.

2.2. Model Assumptions and Compartment Description

The population is divided into four compartments:

- $S(t)$: **Susceptible** — individuals who have not engaged in criminal activity but are at risk of becoming criminals through social interaction with active criminals.
- $C(t)$: **Criminal** — individuals who are actively engaged in criminal activity.
- $P(t)$: **Prison** — individuals who have been arrested and incarcerated.
- $R(t)$: **Recovered** — individuals who have been released from prison and have stopped engaging in criminal activity.

The total population at time t is $N(t) = S(t) + C(t) + P(t) + R(t)$.

Two models are formulated based on the following common assumptions:

1. New individuals enter the susceptible compartment at a constant recruitment rate Λ, accounting for births, immigration, and individuals reaching the age of criminal responsibility.
2. Susceptible individuals become criminals through social interaction with active criminals. The transmission follows a *standard incidence* rate $\beta SC/N$, where β is the effective contact rate. The standard incidence formulation is chosen because the probability of criminal influence depends on the *proportion* of criminals in the population rather than the absolute number, which is appropriate for socially transmitted behaviors [4, 11].
3. Criminals are arrested and sent to prison at a rate γ.
4. Imprisoned individuals are released from prison at a rate δ.
5. Each individual in every compartment experiences natural death at a rate μ.
6. The total population is not assumed to be constant; it varies depending on the balance between recruitment and natural death.

The two models differ in a single assumption regarding the recovered compartment:

1. **Model without recidivism:** Individuals released from prison permanently exit the criminal system and remain in the recovered compartment R.
2. **Model with recidivism:** Recovered individuals may return to criminal activity at a rate ω, representing the recidivism phenomenon.

The descriptions of all parameters are summarized in Table 1.

Table 1. Description of model parameters.

Symbol	Description	Unit
Λ	Recruitment rate into the susceptible population	individuals·day^{-1}
μ	Natural death rate	day^{-1}
β	Transmission rate from susceptible to criminal	day^{-1}
γ	Incarceration rate of criminals	day^{-1}
δ	Release rate from prison	day^{-1}
ω	Recidivism rate (model with recidivism only)	day^{-1}

2.3. Model with Recidivism

In the model with recidivism, individuals released from prison may return to criminal behavior. The transfer diagram is shown in Figure 1.

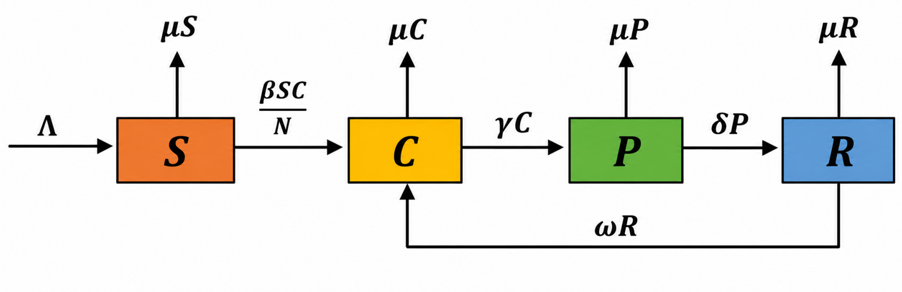

Figure 1. Transfer diagram of the criminal dynamics model with recidivism.

The flow between compartments is governed by the following system of ordinary differential equations:

$$\frac{dS}{dt} = \Lambda - \frac{\beta SC}{N} - \mu S, \tag{1}$$

$$\frac{dC}{dt} = \frac{\beta SC}{N} - \gamma C - \mu C + \omega R, \tag{2}$$

$$\frac{dP}{dt} = \gamma C - \delta P - \mu P, \tag{3}$$

$$\frac{dR}{dt} = \delta P - \mu R - \omega R. \tag{4}$$

In eq. (1), the term Λ represents the inflow of new susceptible individuals, $\beta SC/N$ represents the outflow due to criminal transmission, and μS represents natural death. In eq. (2), the term ωR represents the inflow from the recovered compartment due to recidivism, which is the distinguishing feature of this model. In eq. (3), the term γC represents new incarcerations and $(\delta + \mu)P$ represents outflows due to release and natural death. In eq. (4), the term δP represents individuals released from prison, while $(\mu + \omega)R$ represents outflows due to natural death and recidivism.

2.4. Model without Recidivism

In the model without recidivism, individuals who are released from prison do not return to criminal activity. The transfer diagram is shown in Figure 2. The system is governed by:

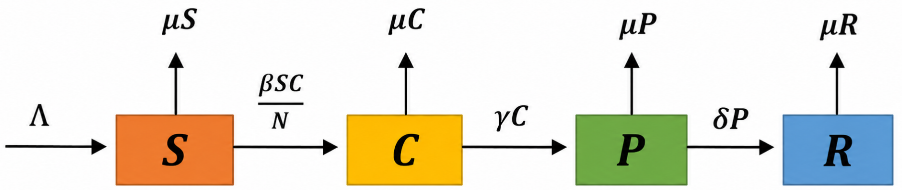

Figure 2. Transfer diagram of the criminal dynamics model without recidivism.

$$\frac{dS}{dt} = \Lambda - \frac{\beta SC}{N} - \mu S, \tag{5}$$

$$\frac{dC}{dt} = \frac{\beta SC}{N} - \gamma C - \mu C, \tag{6}$$

$$\frac{dP}{dt} = \gamma C - \delta P - \mu P, \tag{7}$$

$$\frac{dR}{dt} = \delta P - \mu R. \tag{8}$$

This model is identical to the model with recidivism except that the recidivism term ωR in eq. (2) is absent, and the outflow from the recovered compartment in eq. (4) only includes natural death μR. The model without recidivism thus represents an optimistic scenario where the correctional system is fully effective in rehabilitating offenders.

2.5. *Analytical Methods*

2.5.1. Equilibrium Analysis

Equilibrium states are determined by assigning all time derivatives to zero. Two equilibrium conditions are considered: (i) the *crime-free state* (E_0), characterized by $C = P = R = 0$, which indicates the absence of criminal activity and (ii) the *endemic state* (E^*), where $C > 0$, reflecting the persistence of crime within the population [29].

2.5.2. Basic Reproduction Number

The basic reproduction number, denoted by $\mathcal{R}_0$ is evaluated using the Next Generation Matrix (NGM) approach [27, 28]. In this method, the system at the crime-free equilibrium is separated into two matrices: matrix F, which accounts for the generation of new criminal cases, and matrix V, which represents the transfer of individuals across compartments. The value of ($\mathcal{R}_0$) is expressed as:

$$\mathcal{R}_0 = \rho(FV^{-1}),$$

where $\rho(\cdot)$ denotes the spectral radius, corresponding to the dominant eigenvalue of the matrix FV^{-1}.

2.5.3. Stability Analysis

The asses local stability, the Jacobian matrix is evaluated at each equilibrium point, and its eigenvalues are analyzed. An equilibrium is regarded as locally asymptotically stable if all eigenvalues have negative real components. Furthermore, stability conditions may also be verified using the Routh-Hurwitz criterion applied to the characteristic polynomial, eliminating the need for explicit eigenvalue computation [29].

2.5.4. Sensitivity Analysis

The sensitivity of $\mathcal{R}_0$ with respect to a parameter p is quantified using a normalized sensitivity index [9, 30] defined by :

$$C_p^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial p} \times \frac{p}{\mathcal{R}_0}.$$

A positive value of this index that increasing the parameter will raise the value of $\mathcal{R}_0$, whereas a negative value indicates a decreasing effect. Parameters with the highest absolute sensitivity values are identified as the most significant contributors to the dynamics of criminal activity.

2.5.5. Numerical Simulation

Numerical simulations are performed using the Nonstandard Finite Difference (NSFD) scheme. Unlike standard methods such as Euler or Runge–Kutta, the NSFD scheme is designed to preserve essential qualitative properties of the continuous system, including positivity and boundedness of solutions [29]. The discretization uses the nonstandard denominator function:

$$\phi(\Delta t) = \frac{1 - e^{-\mu \Delta t}}{\mu},$$

which replaces the standard step size Δt in the finite difference approximation. This ensures unconditional stability of the numerical scheme regardless of the step size chosen.

3. Results and Discussion

3.1. *Equilibrium Points of The Model*

The equilibrium points of the model represent the steady-state condition where the system no longer changes over time. They are obtained by setting

$$\frac{dS}{dt} = \frac{dC}{dt} = \frac{dP}{dt} = \frac{dR}{dt} = 0.$$

This condition indicates that all compartments remain constant, meaning the inflow and outflow in each class are balanced. Determining the equilibrium points helps to understand whether criminal activity will disappear or persist in the population. In this study, two types of equilibrium points are considered, namely the crime-free equilibrium and the endemic equilibrium.

3.1.1. Crime-Free Equilibrium Point

The crime-free equilibrium point, both with and without recidivism, occurs when there are no criminals in the population, that is:

$$C^* = 0, \quad P^* = 0, \quad R^* = 0.$$

By substituting into the first equation, we obtain:

$$0 = \Lambda - \mu S.$$

Thus:

$$S^* = \frac{\Lambda}{\mu}.$$

Thus, the crime-free equilibrium point for both the model with and without recidivism is:

$$E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0 \right).$$

3.1.2. Endemic Equilibrium Point of Criminality

The endemic equilibrium point is obtained by substituting the condition $\frac{dS}{dt} = \frac{dC}{dt} = \frac{dP}{dt} = \frac{dR}{dt} = 0$, with the condition $C^* > 0$. This condition indicates that criminal activity persists and remains in the population.

Based on the results of the model calculations, the endemic equilibrium point is obtained as:

$$E^* = (S^*, C^*, P^*, R^*).$$

Model with Recidivism

In the model with recidivism, the endemic equilibrium point is influenced by the presence of the recidivism parameter, which allows individuals who have stopped engaging in criminal activity to return to being criminals. This makes the structure of the equilibrium point more complex. The endemic equilibrium point of the model with recidivism is as follows:

$$\begin{aligned}
S^* &= \frac{\mu N\left(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\mu^2+\mu\omega\right)}{\beta\left(\delta\mu+\delta\omega+\mu^2+\mu\omega\right)},\\
C^* &= \frac{-N\mu^2\Big(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\gamma\omega+\mu^2+\mu\omega)+\Lambda\beta(\delta\mu+\delta\omega+\mu^2+\mu\omega)}{\beta\mu\left(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\gamma\omega+\mu^2+\mu\omega\right)},\\
P^* &= \frac{\gamma(-N\mu^2\Big(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\gamma\omega+\mu^2+\mu\omega)+\Lambda\beta(\delta\mu+\delta\omega+\mu^2+\mu\omega))}{\delta+\mu(\beta\mu\left(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\gamma\omega+\mu^2+\mu\omega\right))},\\
R^* &= \frac{\delta\gamma(-N\mu^2\Big(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\gamma\omega+\mu^2+\mu\omega)+\Lambda\beta(\delta\mu+\delta\omega+\mu^2+\mu\omega))}{(\mu+\omega)(\delta+\mu)(\beta\mu\left(\delta\gamma+\delta\mu+\delta\omega+\gamma\mu+\gamma\omega+\mu^2+\mu\omega\right))}.
\end{aligned}$$

Model without Recidivism

In the model without recidivism, there is no flow of individuals from the recovered compartment back to the criminal compartment. The resulting endemic equilibrium point has a simpler form compared to the model with recidivism. The endemic equilibrium point of the model without recidivism is as follows:

$$\begin{aligned}
S^{**} &= \frac{N(\gamma+\mu)}{\beta},\\
C^{**} &= \frac{-N\gamma\mu-N\mu^2+\Lambda\beta}{\beta(\gamma+\mu)},\\
P^{**} &= \frac{\gamma\left(-N\gamma\mu-N\mu^2+\Lambda\beta\right)}{\beta\left(\delta\gamma+\delta\mu+\gamma\mu+\mu^2\right)},\\
R^{**} &= \frac{\delta\gamma\left(-N\gamma\mu-N\mu^2+\Lambda\beta\right)}{\beta\mu\left(\delta\gamma+\delta\mu+\gamma\mu+\mu^2\right)}.
\end{aligned}$$

3.2. Basic Reproduction Number of the Model with Recidivism

In the model with recidivism, individuals who have left the criminal compartment can return to being criminals through the recidivism rate ω. This increases the potential spread of criminal activity.

Using the *Next Generation Matrix* method, it is obtained that the infected compartment remains the criminal compartment C; however, the effective rate at which individuals leave this compartment is influenced by the presence of recidivism. The resulting matrix is:

$$F=\begin{pmatrix}\beta & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\qquad V=\begin{pmatrix}-(\gamma+\mu) & 0 & \omega\\ \gamma & -(\delta+\mu) & 0\\ 0 & \delta & -(\mu+\omega)\end{pmatrix}.$$

Then,

$$V^{-1} = -\frac{(\delta+\mu)(\mu+\omega)}{(\gamma+\mu)(\delta+\mu)(\mu+\omega)-\omega\gamma\delta}\begin{pmatrix}(\delta+\mu)(\mu+\omega) & -\omega\delta & -\omega(\delta+\mu)\\ \gamma(\mu+\omega) & (\gamma+\mu)(\mu+\omega) & -\omega\gamma\\ \gamma\delta & \delta(\gamma+\mu) & (\gamma+\mu)(\delta+\mu)\end{pmatrix}.$$

Thus,

$$R_0 = \rho(FV^{-1}) = \frac{\beta(\mu+\omega)}{(\gamma+\mu)(\mu+\omega)-\omega\gamma\delta}.$$

Thus, the basic reproduction number for the criminal dynamics model with recidivism is given by:

$$R_0 = \frac{\beta(\mu+\omega)}{(\gamma+\mu)(\mu+\omega)-\omega\gamma\delta}.$$

3.3. *Basic Reproduction Number of the Model without Recidivism*

The basic reproduction number R_0 for the model without recidivism is determined using the *Next Generation Matrix* method . The infected compartment in this model is the criminal compartment C. The transmission and transition matrices are given by:

$$F = \begin{pmatrix}\beta & 0\\ 0 & 0\end{pmatrix}, \qquad V = \begin{pmatrix}-(\gamma+\mu) & 0\\ \gamma & -(\delta+\mu)\end{pmatrix}.$$

Then,

$$V^{-1} = \frac{1}{(\gamma+\mu)(\delta+\mu)}\begin{pmatrix}\delta+\mu & 0\\ \gamma & \gamma+\mu\end{pmatrix}.$$

Thus, we obtain:

$$R_0 = \rho(FV^{-1}) = \frac{\beta}{\gamma+\mu}.$$

Thus, the basic reproduction number for the criminal dynamics model without recidivism is given by:

$$R_0 = \frac{\beta}{\gamma+\mu}.$$

3.4. *Stability of the Crime-Free Equilibrium Point*

3.4.1. Model with Recidivism

The stability of the crime-free equilibrium point is analyzed using the Jacobian matrix evaluated at E_0. The characteristic equation yields the following eigenvalues:

$$\lambda_1 = -\mu, \quad \lambda_2 = \beta-\gamma-\mu.$$

Since $\lambda_2 > 0$ when $\beta > \gamma+\mu$, the system has at least one positive eigenvalue. Therefore, the crime-free equilibrium point is unstable. This condition is equivalent to:

$$R_0 > 1.$$

3.4.2. Model without Recidivism

For the model without recidivism, the eigenvalues of the Jacobian matrix at E_0 are:

$$\lambda_1 = -(\delta+\mu), \quad \lambda_2 = -\mu.$$

Both eigenvalues are negative, indicating that the crime-free equilibrium point is locally asymptotically stable.

Using the Routh-Hurwitz criterion, the stability condition is satisfied when:

$$\lambda^2 + (\gamma + 2\mu - \beta)\lambda + (\gamma + \mu - \beta)\mu > 0.$$

This is equivalent to:

$$R_0 < 1.$$

The results indicate that the presence of recidivism affects the stability of the system. In the model with recidivism, the crime-free equilibrium becomes unstable when $R_0 > 1$, meaning that criminal activity persists in the population. In contrast, the model without recidivism tends to be stable when $R_0 < 1$, indicating that criminal activity can be eliminated over time.

3.5. Estimation Construction

To estimate the transmission parameter of the crime dynamics model, the annual crime data were approximated using an exponential regression approach. Let $X(t)$ represent the number of reported criminal cases at time t. Assuming that the crime cases exhibit an exponential trend during the observation period, the data can be expressed as

$$X(t) = X_0 e^{\lambda t},$$

where X_0 denotes the initial number of criminal cases and λ is the exponential growth rate. In this study, λ is not an eigenvalue associated with the stability analysis of the system, but rather a parameter obtained from the regression process that describes the growth trend of criminal cases. The estimated growth rate is then utilized in the construction and estimation of the transmission parameter of the proposed crime dynamics model.

Model with Recidivism

Substituting the exponential form into the system yields:

$$\begin{aligned}
(\lambda + \beta + \gamma + \mu)C_0 &= \omega R_0, \\
(\lambda + \delta + \mu)P_0 &= \gamma C_0, \\
(\lambda + \mu + \omega)R_0 &= \delta P_0.
\end{aligned}$$

From these equations, the relationships between compartments are obtained as:

$$\frac{R_0}{C_0} = \frac{\lambda + \beta + \gamma + \mu}{\omega}.$$

Model without Recidivism

Using the same approach, we obtain:

$$\begin{aligned}
(\lambda + \gamma + \mu)C_0 &= \beta C_0, \\
(\lambda + \delta + \mu)P_0 &= \gamma C_0.
\end{aligned}$$

The estimation construction shows that the growth rate λ is influenced by the transition rates between compartments. In the model with recidivism, the presence of the parameter ω introduces feedback from the recovered compartment to the criminal compartment, which increases the persistence of criminal activity. In contrast, the model without recidivism does not include this feedback mechanism, resulting in a simpler dynamic structure.

3.5.1. Construction of the Estimation of R_0 and Parameter β

Model with Recidivism

Based on the estimation construction, the basic reproduction number is obtained as:

$$R_0 = \frac{\omega\delta\gamma - (\lambda + \gamma + \mu)}{(\gamma + \mu)(\mu + \omega) - \omega\gamma\delta} - \beta(\lambda(1 + (\delta + \mu))).$$

Furthermore, the estimated value of parameter β is given by:

$$\beta = \frac{\omega\delta\gamma}{(\lambda + \delta + \mu)(\lambda + \mu + \omega)} - (\lambda + \gamma + \mu).$$

From the calculation results, the value of β is $\beta = -8.91387208$.

Model without Recidivism

For the model without recidivism, the basic reproduction number is obtained as:

$$R_0 = \frac{\lambda}{\gamma + \mu} + 1.$$

The estimated value of parameter β is $\beta = \lambda + \gamma + \mu$. Thus, the numerical value of β is $\beta = 0.336385714$.

The results show a significant difference between the two models. In the model with recidivism, the estimated value of β is negative, which indicates that the model may not be biologically or socially feasible under the given parameter assumptions. In contrast, the model without recidivism produces a positive value of β, indicating a more realistic representation of criminal dynamics. This suggests that the inclusion of recidivism introduces additional complexity that strongly affects parameter estimation and system behavior. It should be noted that the crime transmission parameter β was not estimated directly through a parameter fitting procedure. Instead, the value of β was obtained through the estimation construction derived from the exponential growth assumption, as presented in Equation (31), where λ represents the growth rate of crime cases observed from the data. Consequently, the estimated value of β depends on the value of λ as well as the analytical relationships among the model parameters. Therefore, the negative value of β obtained in this study is a mathematical consequence of the estimation construction rather than an assumption that the crime transmission rate itself is negative. From a social perspective, β is still interpreted as a transmission parameter that is theoretically expected to be non-negative. Hence, the negative estimate should be viewed as an indication of limitations in the estimation framework employed and the complexity introduced by the recidivism mechanism in the model, rather than as evidence of a negative crime transmission process.

3.5.2. Curve Fitting of Cumulative Criminal Data

The estimation of the growth parameter was performed using exponential regression based on annual crime data in Kupang City from 2013–2024. The results show that the take-off rate is:

$$\lambda = 1.93199.$$

This value represents the growth rate of criminal cases over time. The parameter λ was estimated using the least-squares method by minimizing the sum of squared errors (SSE) between the observed data and the fitted values. The objective function is defined as

$$\text{SSE} = \sum_{i=1}^{n} \left(X_i - \hat{X}_i \right)^2,$$

where X_i and $\hat{X}_i$ represent the observed and fitted crime cases, respectively.

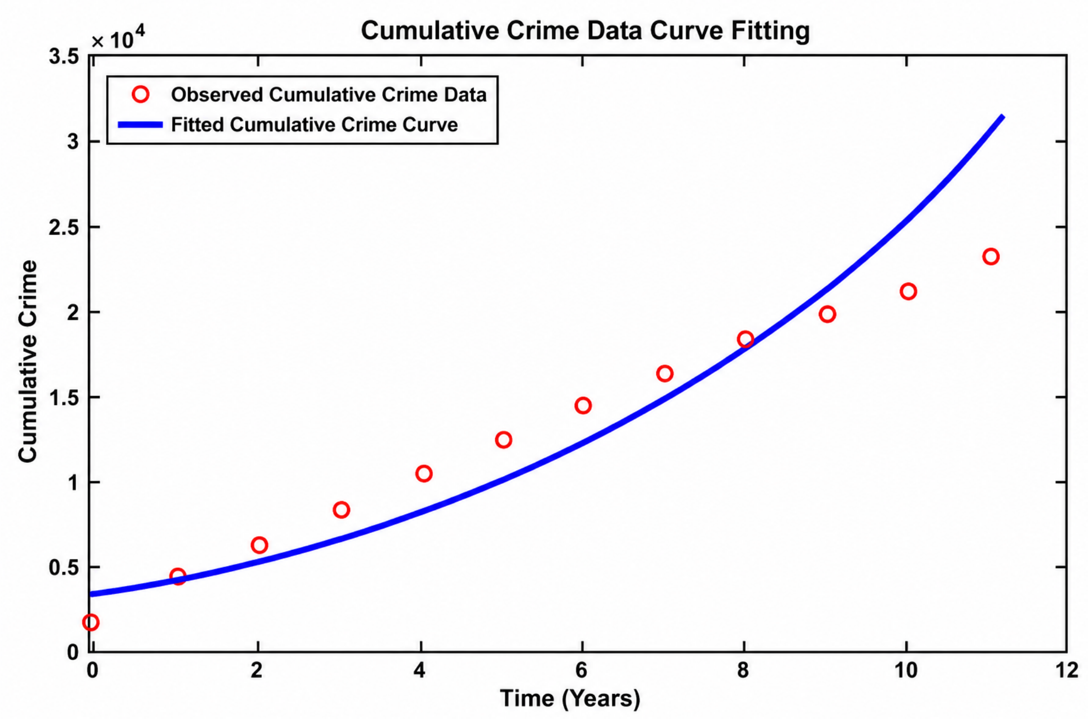

Figure 3. Curve fitting of cumulative criminal data.

The goodness-of-fit of the exponential regression model was evaluated using the coefficient of determination (R^2), which measures the proportion of variation in the observed data explained by the fitted model. The obtained value of $R^2 = 0.7826$ indicates that approximately 78.26% of the variation in the observed crime data can be explained by the exponential regression model. This result suggests that the exponential growth assumption provides a reasonably good representation of the observed crime trend in Kupang City and is therefore suitable for estimating the growth parameter λ.

The estimated value of λ was subsequently used in the parameter construction process to determine the crime transmission parameter β. Therefore, the estimation procedure consisted of two stages: first, estimating the exponential growth rate from crime data through exponential regression, and second, using the obtained growth rate in the analytical parameter construction derived from the mathematical model. Similar approaches have been widely used in compartmental modeling studies when direct estimation of transmission parameters from available data is not feasible [27, 28, 30].

3.5.3. Estimation of the Basic Reproduction Number R_0

Model with Recidivism

In the model with recidivism, the basic reproduction number R_0 is not well-defined. This occurs because individuals in the recovered compartment (R) can return to the criminal compartment (C), creating a feedback loop in the system. As a result, the transmission process does not follow a clear generational structure, which is a key requirement in defining the basic reproduction number. Therefore, R_0 cannot be properly determined for this model.

Model without Recidivism

The estimated value of the basic reproduction number is:

$$R_0 = 2.3525.$$

Since $R_0 > 1$, this indicates that each criminal individual can generate more than two new criminal individuals. Consequently, the number of criminal cases in Kupang City is expected to increase over time. The results highlight a fundamental difference between the two models. The absence of recidivism allows for a clear interpretation of R_0, showing that criminal activity will grow when $R_0 > 1$. In contrast, the presence of recidivism introduces a feedback mechanism that complicates the system dynamics and prevents the proper definition of R_0. This suggests that recidivism plays a significant role in sustaining criminal behavior within the population.

3.6. Sensitivity Analysis

Sensitivity analysis is conducted to assess how variations in model parameters influence the basic reproduction number R_0. This approach enables the determination of parameters that play a dominant role in shaping the dynamics of the system.

Model with Recidivism

In the model with recidivism, sensitivity indices are formally derived for each parameter. However, since the model does not possess a stable equilibrium point and the basic reproduction number R_0 is not well-defined, the sensitivity analysis cannot be meaningfully interpreted. The presence of recidivism introduces a feedback loop from the recovered compartment to the criminal compartment, which leads to an unstable system. As a result, small changes in parameter values do not reflect a consistent or controlled system response. Therefore, numerical sensitivity evaluation is not performed for this model.

Model without Recidivism

For the model without recidivism, sensitivity analysis can be carried out since the system is stable and R_0 is well-defined. The normalized sensitivity indices of R_0 with respect to each parameter are given by:

$$C_\beta^{R_0} = 1, \quad C_\gamma^{R_0} = -\frac{\gamma}{\gamma + \mu}, \quad C_\mu^{R_0} = -\frac{\mu}{\gamma + \mu}.$$

Based on the parameter values, the sensitivity indices are:

$$C_\beta^{R_0} = 1, \quad C_\gamma^{R_0} = -0.2667, \quad C_\mu^{R_0} = -0.999.$$

The results indicate that parameter β is the most influential parameter affecting the value of R_0. A positive sensitivity index of $C_\beta^{R_0} = 1$ means that R_0 changes proportionally with β. Specifically, a 5% increase (or decrease) in β will result in a 5% increase (or decrease) in R_0. On the other hand, parameters γ and μ have negative sensitivity indices, indicating that increasing these parameters will reduce the value of R_0. This suggests that increasing recovery and natural removal rates can help suppress criminal activity.

3.7. Nonstandard Finite Difference Scheme

The nonstandard finite difference (NSFD) scheme is constructed using the general form:

$$\frac{dX}{dt} = \frac{X_{n+1} + X_n}{\phi(t)} = X_n(1 - X_n).$$

The function $\phi(t)$ is a nonstandard time discretization function defined as:

$$\phi(t) = \frac{1 - e^{-\mu(t)}}{\mu},$$

where $\mu > 0$ is a constant parameter. This scheme is applied to each compartment S, C, P, and R for both the model with recidivism and the model without recidivism.

The Nonstandard Finite Difference (NSFD) scheme was employed in this study because it preserves important qualitative properties of the original continuous system. In compartmental crime models, all state variables represent population groups and therefore must remain non-negative for all time [16, 31, 32]. Standard numerical methods, such as the classical Runge–Kutta method, may generate negative solutions or fail to preserve the dynamical properties of the system when relatively large time steps are used. In contrast, the NSFD approach is specifically designed to maintain positivity, boundedness, and stability characteristics of the underlying model. Consequently, NSFD provides a more reliable numerical framework for simulating crime dynamics and ensuring that the numerical solutions remain socially meaningful throughout the simulation period [32].

3.8. Parameter Values and Initial Conditions

The parameter values used in this study are presented in Table 2. All parameters are expressed in units of per day (day^{-1}).

Table 2. Parameter values of the criminal dynamics model.

Symbol	Value	Unit
Λ	$\frac{474.801}{71.83\times365}$	day^{-1}
β (with recidivism)	-8.91387208	day^{-1}
β (without recidivism)	0.336385714	day^{-1}
μ	$\frac{1}{71.83\times365}$	day^{-1}
γ	$\frac{1}{7}$	day^{-1}
δ	$\frac{1}{15\times365}$	day^{-1}
ω	$\frac{1}{3.5\times365}$	day^{-1}

The parameter values are used in all calculations, particularly in the numerical simulations of the criminal dynamics model. The initial conditions used in the simulations are given as:

$$S(0) = 471{,}883, \quad C(0) = 2{,}318, \quad P(0) = 500, \quad R(0) = 100.$$

These initial values are chosen to represent approximate real conditions in Kupang City, although they are not exact measured data.

3.9. Numerical Simulation

Numerical simulations were conducted to illustrate the dynamic behavior of the proposed crime models and to verify the analytical results obtained from the equilibrium and stability analyses. The simulations were implemented using the Nonstandard Finite Difference (NSFD) scheme described in Section 3.7 because this method preserves the positivity, boundedness, and stability properties of the compartmental system. The parameter values and initial conditions used in the simulations are presented in Table 2 and Section 3.8.

Two simulation scenarios were considered. The first scenario corresponds to the crime dynamics model with recidivism, in which individuals released from prison are allowed to return to criminal activity through the recidivism mechanism. This scenario was selected to investigate the influence of repeated offending on the long-term dynamics of crime. The second scenario represents the crime dynamics model without recidivism, assuming that individuals who leave criminal activity do not return to criminal behavior. This comparison enables the impact of recidivism on crime persistence and system stability to be clearly demonstrated.

The simulation results are presented in Figure 4 and Figure 5. The behavior of each compartment is interpreted by relating the numerical trajectories to the analytical findings, including the existence and stability of equilibrium points and the characteristics of the basic reproduction number (R_0).

Model with Recidivism

The susceptible population (S) gradually decreases over time because susceptible individuals continuously move into the criminal compartment through the crime transmission process. Although new individuals enter the susceptible compartment through the recruitment rate, the transition into criminal behavior dominates during the simulation period, causing the susceptible population to decline before eventually approaching an equilibrium level. This behavior indicates that the crime transmission process plays a significant role in determining the size of the susceptible population. Furthermore, it is consistent with the sensitivity analysis, which identifies the crime transmission rate as one of the parameters that strongly influences the system dynamics.

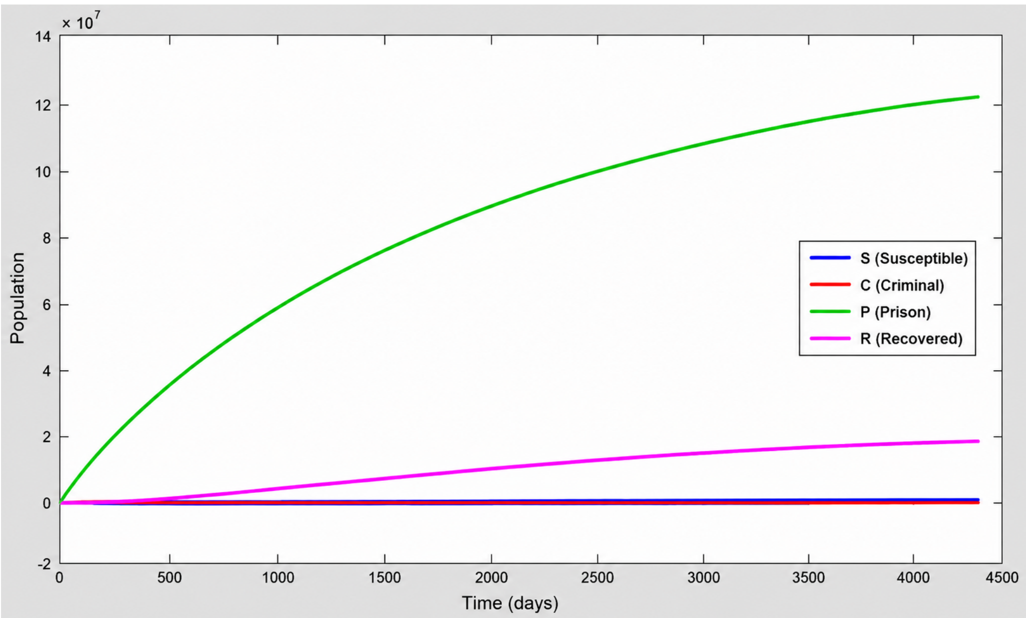

Figure 4. Criminal dynamics model with recidivism.

The criminal population (C) gradually decreases as offenders are transferred to the imprisoned compartment. However, the criminal population persists because individuals released from prison may return to criminal activity through the recidivism mechanism. This behavior is consistent with the equilibrium and stability analyses, indicating that the crime-free equilibrium is unstable when recidivism is incorporated into the model. Furthermore, the simulation supports the analytical finding that the persistence of crime is primarily driven by recidivism, making the basic reproduction number unsuitable as a meaningful threshold parameter in the recidivism model.

The imprisoned population (P) increases as more criminal individuals are transferred into prison. After a certain period, the imprisonment rate becomes balanced with the prisoner release rate, causing the imprisoned population to approach a relatively stable level. Although the imprisoned compartment reaches a steady state, the presence of recidivism prevents the overall system from eliminating criminal activity.

The recovered population (R) continuously increases as more prisoners complete their sentences and are released from prison. However, unlike the model without recidivism, individuals in this compartment may return to criminal activity through the recidivism mechanism. Consequently, the recovered compartment not only represents individuals who have completed their imprisonment but also serves as a source of new criminal individuals. This behavior reinforces the analytical findings, demonstrating that recidivism creates a feedback mechanism that sustains criminal activity over the long term.

Model without Recidivism

The susceptible population (S) initially decreases because susceptible individuals transition into the criminal compartment. As the criminal population gradually declines, fewer susceptible individuals become involved in criminal activity. Together with the continuous recruitment of new individuals, the susceptible population eventually increases and approaches its equilibrium level. This result indicates that reducing crime transmission contributes to the stabilization of the system. Furthermore, this behavior is consistent with the sensitivity analysis, which identifies the crime transmission rate as the parameter with the greatest influence on the system dynamics.

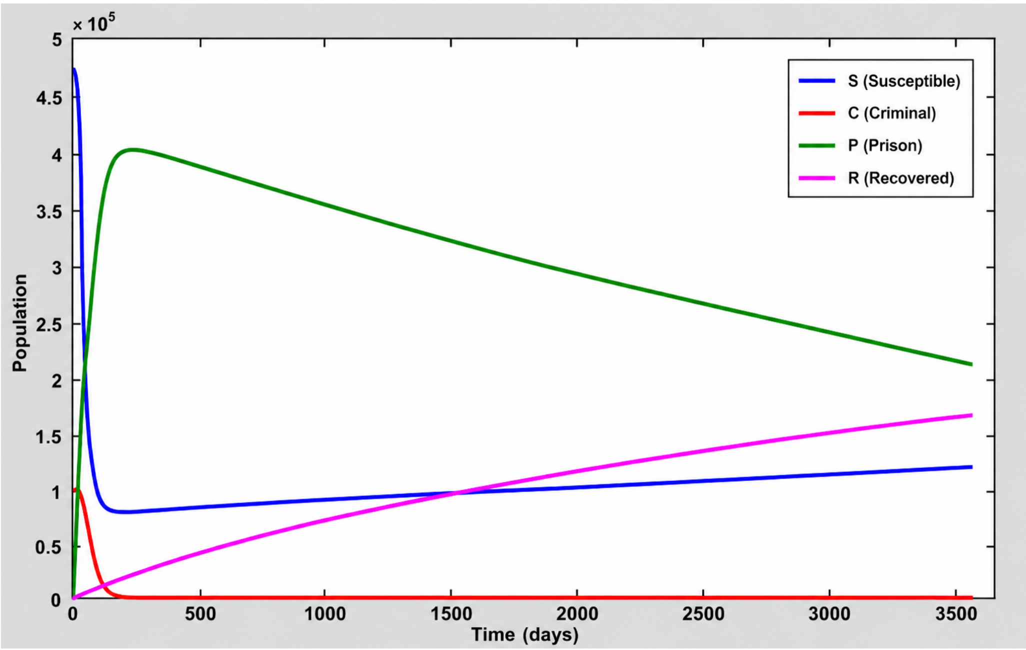

Figure 5. Criminal dynamics model without recidivism.

The criminal population (C) initially increases due to the transition of susceptible individuals into criminal activity. As the simulation progresses, the criminal population gradually decreases because offenders are transferred to the imprisoned compartment and do not return to criminal behavior after completing their prison sentences. Consequently, the criminal population approaches its equilibrium level over time. This behavior is consistent with the analytical findings, indicating that eliminating recidivism results in a more stable long-term system compared with the model that incorporates recidivism. In addition, the sensitivity analysis shows that changes in the crime transmission rate have the greatest impact on the criminal population dynamics. Therefore, reducing the crime transmission rate is one of the most effective strategies for suppressing criminal activity.

The imprisoned population (P) increases during the early stage of the simulation as more criminal individuals are incarcerated. As the criminal population decreases, fewer offenders enter prison, while prisoners who complete their sentences are transferred to the recovered compartment. Consequently, the imprisoned population gradually declines and eventually reaches a relatively stable level. This behavior reflects the balance between the imprisonment and prisoner release processes.

The recovered population (R) continuously increases because individuals who complete their prison sentences remain in the recovered compartment and do not return to criminal activity. In the absence of recidivism, the recovered compartment functions as a terminal compartment and no longer contributes to the generation of new criminal individuals. This behavior supports the analytical findings that eliminating recidivism leads to a more stable long-term system.

3.10. Discussion

The comparative analysis of the two models reveals several important insights.

The model without recidivism yields $\mathcal{R}_0 = 2.3525$, confirming that crime in Kupang City is in an endemic state under current conditions. This value is comparable to reproduction numbers reported in other crime modeling studies; for instance, Kwofie et al. [4] obtained $\mathcal{R}_0$ values greater than unity for crime dynamics in the state of Illinois, USA, and Mataru et al. [16] found similar endemic conditions in developing-country settings. The consistent finding of $\mathcal{R}_0 > 1$ across different contexts reinforces the notion that crime, like infectious diseases, exhibits self-sustaining transmission dynamics when social conditions are favorable [7, 11, 27, 33].

The model with recidivism captures a more realistic aspect of criminal dynamics by allowing recovered individuals to return to criminal behavior [2, 8]. However, the inclusion of recidivism as a feedback mechanism in the model introduces several analytical challenges. While the basic reproduction number $\mathcal{R}_0$ can be formally derived through the Next Generation Matrix (NGM) method, the negative estimated transmission parameter prevents a meaningful epidemiological interpretation of $\mathcal{R}_0$ as a threshold quantity. In addition, the crime-free equilibrium is found to be unstable, indicating that recidivism continuously reintroduces criminal individuals into the system and may contribute to the persistence of criminal activities within the population. These findings highlight the significant role of recidivism in shaping crime dynamics and emphasize the need for effective rehabilitation strategies to reduce repeated criminal behavior. [12, 18, 31]. These findings suggest that the standard epidemiological framework may need to be extended [24, 34] when modeling recidivism, for example by incorporating stochastic elements [19, 35] or fractional-order derivatives that account for memory effects in criminal behavior [20, 22].

The sensitivity analysis identifies the transmission rate β as the most critical parameter affecting $\mathcal{R}_0$ [4, 16, 26]. This implies that prevention strategies should prioritize reducing the social transmission of criminal behavior [9, 36] – for instance, through community-based intervention programs, educational initiatives, and economic empowerment – rather than relying solely on increasing incarceration rates. This recommendation is consistent with the findings of Mataru et al. [16], who demonstrated that vocational training and employment strategies are more effective than law enforcement alone in combating unemployment-related crime. Additionally, the qualitative analysis of the recidivism model suggests that investment in post-release rehabilitation programs (reducing ω) is essential for long-term crime control.

Several limitations should be acknowledged. First, the exponential regression approach for parameter estimation, while straightforward, may not capture the full complexity of the transmission dynamics [18, 23], particularly for the recidivism model. More sophisticated techniques such as Markov Chain Monte Carlo (MCMC) methods or least-squares fitting of the full ODE system could improve parameter estimates [31]. Second, the initial conditions used in the simulations are approximate values, as exact compartmental data for Kupang City are not available. Third, the model assumes homogeneous mixing within the population, which may not reflect [15, 17] the spatial and social heterogeneity of crime in Kupang City [37]. Future studies could address these limitations by incorporating age-structured populations [17, 18], spatial heterogeneity, and time-varying parameters to better capture the complex dynamics of crime.

4. Conclusion

The criminal dynamics model with recidivism results in individuals who have stopped engaging in criminal behavior returning to criminal activity. This creates a feedback loop in the system, which leads to the instability of the crime-free equilibrium point. In this case, the basic reproduction number R_0 is not well-defined, and consequently, the sensitivity analysis parameters cannot be properly determined. In contrast, the criminal dynamics model without recidivism assumes that individuals who have exited criminal behavior do not re-enter the criminal system. As a result, the system is stable at the crime-free equilibrium point, $R_0 > 1$, and the parameters influencing the system can be identified through sensitivity analysis. The most influential parameter affecting the value of R_0 in the criminal dynamics model without recidivism is the transmission rate β, which has a positive relationship with R_0. This implies that every 5% increase in the value of β leads to a 5% increase in R_0, and conversely, every 5% decrease in β results in a 5% decrease in R_0.

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Data availability. The dataset utilized in this research is publicly accessible and can be obtained from the official website of the BPS (Badan Pusat Statistik) Kupang City (https://kupangkota.bps.go.id/id).

References

[1] Z. F. Taufiq, "Covid-19 dan angka kriminalitas: Penerapan teori-teori kriminologi," *JISIP (Jurnal Ilmu Sosial dan Pendidikan)*, vol. 4, no. 4, 2020, doi: https://doi.org/10.36312/jisip.v4i4.1467.

[2] R. Fitriah and H. Yusuf, "Penerapan teori-teori kriminologi dalam sistem peradilan pidana indonesia: Sebuah analisis," *Jurnal Intelek dan Cendikiawan Nusantara*, vol. 1, no. 2, pp. 1498–1507, 2024, doi: https://doi.org/10.51878/cendikiawan.v1i2.2576.

[3] United Nations Office on Drugs and Crime, "World crime trends and emerging issues 2024," UNODC, Vienna, Tech. Rep., 2024, [online] Available at: https://www.unodc.org.

[4] K. Kwofie, M. Dogbatse, and S. E. Moore, "Curtailing crime dynamics: A mathematical approach," *Frontiers in Applied Mathematics and Statistics*, vol. 8, p. 1086745, 2023, doi: https://doi.org/10.3389/fams.2022.1086745.

[5] PUSIKNAS Polri, "Data kasus kriminal di indonesia periode 2024," Pusat Informasi Kriminal Nasional, Kepolisian Republik Indonesia, Tech. Rep., 2024, [online] Available at: https://pusiknas.polri.go.id.

[6] Badan Pusat Statistik Kota Kupang, "Statistik kriminal kota kupang," BPS Kota Kupang, Tech. Rep., 2024, [online] Available at: https://kupangkota.bps.go.id.

[7] A. Priambodo, "Peran kriminalitas sebagai pemoderasi antara kemiskinan dan pengangguran terhadap laju pertumbuhan ekonomi," *Al-KALAM: Jurnal Komunikasi, Bisnis dan Manajemen*, vol. 11, no. 1, pp. 132–153, 2024, doi: https://doi.org/10.31602/al-kalam.v11i1.15377.

[8] M. D. Hersyanda, I. S. Lubis, N. Ikhwan, D. Septriani, and M. Haqqi, "Efektivitas sanksi pidana terhadap pengulangan kejahatan (residivisme) di indonesia," *JIMMI: Jurnal Ilmiah Mahasiswa Multidisiplin*, vol. 1, no. 3, pp. 253–256, 2024, doi: https://doi.org/10.61722/jimmi.v1i3.2024.

[9] M. A. Indraputra, A. Triska, and N. Anggriani, "Model penyebaran kriminalitas dengan pengaruh kebijakan kriminal sebagai pencegahan pada situasi krisis ekonomi," *Jurnal Lebesgue*, vol. 4, no. 2, pp. 949–961, 2023, doi: https://doi.org/10.46306/lb.v4i2.362.

[10] S. Adri, S. Karimi, and I. Indrawari, "Pengaruh faktor sosial ekonomi terhadap perilaku kriminalitas (tinjauan literatur)," *Jurnal Ilmiah Administrasi Publik*, vol. 5, no. 2, pp. 181–186, 2019, doi: https://doi.org/10.21776/ub.jiap.2019.005.02.8.

[11] J. Sooknanan and T. A. R. Seemungal, "Criminals and their models – a review of epidemiological models describing criminal behaviour," *Applied Mathematics and Computation*, vol. 458, p. 128225, 2023, doi: https://doi.org/10.1016/j.amc.2023.128225.

[12] O. M. Ibrahim, D. Okuonghae, and M. N. O. Ikhile, "Optimal control model for criminal gang population in a limited-resource setting," *International Journal of Dynamics and Control*, vol. 11, no. 2, pp. 835–850, 2023, doi: https://doi.org/10.1007/s40435-022-00992-8.

[13] D. McMillon, C. P. Simon, and J. Morenoff, "Modeling the underlying dynamics of the spread of crime," *PLOS ONE*, vol. 9, no. 4, p. e88923, 2014, doi: https://doi.org/10.1371/journal.pone.0088923.

[14] J. C. Nuño, M. A. Herrero, and M. Primicerio, "A triangle model of criminality," *Physica A: Statistical Mechanics and its Applications*, vol. 387, no. 12, pp. 2926–2936, 2008, doi: https://doi.org/10.1016/j.physa.2008.01.076.

[15] H. O. Nyaberi and J. Mushanyu, "Mathematical modeling of the role of imitation in crime dynamics," *arXiv preprint*, 2024, doi: https://doi.org/10.48550/arXiv.2410.09989.

[16] B. Mataru, O. J. Abonyo, and D. Malonza, "Mathematical model for crimes in developing countries with some control strategies," *Journal of Applied Mathematics*, vol. 2023, p. 8699882, 2023, doi: https://doi.org/10.1155/2023/8699882.

[17] O. M. Ibrahim, D. Okuonghae, and M. N. O. Ikhile, "Mathematical modelling of the population dynamics of age-structured criminal gangs with correctional intervention measures," *Applied Mathematical Modelling*, vol. 107, pp. 39–71, 2022, doi: https://doi.org/10.1016/j.apm.2022.02.005.

[18] O. I. Idisi, K. B. Akande, L. A. Suleiman, and O. O. Samson, "Mathematical modeling of crime and corruption dynamics: A cross-national analysis of their interdependencies," *Journal of Computational Social Science*, 2025, doi: https://doi.org/10.1007/s42001-025-00458-9.

[19] D. Correa-Flórez, J.-C. Cortés, E. López-Navarro, and R.-J. Villanueva, "A dynamical mathematical model for crime evolution based on a compartmental system with interactions," *International Journal of Computer Mathematics*, vol. 102, no. 1, pp. 44–59, 2024, doi: https://doi.org/10.1080/00207160.2024.2302840.

[20] K. Bansal, T. Mathur, and S. Agarwal, "Fractional-order crime propagation model with non-linear transmission rate," *Chaos, Solitons & Fractals*, vol. 169, p. 113321, 2023, doi: https://doi.org/10.1016/j.chaos.2023.113321.

[21] K. Bansal and T. Mathur, "Fractional-order crime propagation model: A comparison between logistic and exponential growth," *Ricerche di Matematica*, vol. 74, pp. 1563–1581, 2025, doi: https://doi.org/10.1007/s11587-024-00910-4.

[22] W. Adel *et al.*, "Analysis of a fractional-order mathematical model on crime dynamics incorporating police force and rehabilitation," *Journal of Mathematics*, vol. 2026, p. 1372780, 2026, doi: https://doi.org/10.1155/jom/1372780.

[23] E. Omoregbe *et al.*, "Mathematical modeling and analysis of the co-dynamics of crime and drug abuse," *Scientific Reports*, vol. 14, p. 75034, 2024, doi: https://doi.org/10.1038/s41598-024-75034-8.

[24] A. A. Ayoade, O. M. Ogunmiloro, and S. Thota, "Mathematical modeling and analysis of the influence of family background on the spread of crime," *Quality & Quantity*, vol. 58, no. 6, pp. 5899–5920, 2024, doi: https://doi.org/10.1007/s11135-024-01925-1.

[25] M. Aguadze, A. Vivas, S. Pant, and K. Dagtoros, "Spread of crime dynamics: A mathematical approach," *Turkish Journal of Mathematics and Computer Science*, vol. 16, no. 2, pp. 481–489, 2024, doi: https://doi.org/10.47000/tjmcs.1366566.

[26] S. Mebratie and M. Dawed, "Mathematical model analysis of crime dynamics incorporating media coverage, police force and moral/religious activity," *Journal of Mathematical and Computational Science*, vol. 11, no. 1, 2021, doi: https://doi.org/10.28919/jmcs/5062.

[27] P. van den Driessche and J. Watmough, "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission," *Mathematical Biosciences*, vol. 180, pp. 29–48, 2002, doi: https://doi.org/10.1016/S0025-5564(02)00108-6.

[28] O. Diekmann, J. A. P. Heesterbeek, and M. G. Roberts, "The construction of next-generation matrices for compartmental epidemic models," *Journal of The Royal Society Interface*, vol. 7, no. 47, pp. 873–885, 2010, doi: https://doi.org/10.1098/rsif.2009.0386.

[29] M. Z. Ndii, *Pemodelan Matematika*. Kupang: Penerbit NEM, 2022.

[30] M. Z. Ndii, L. K. Beay, N. Anggriani, K. N. Nukul, and B. S. Djahi, "Estimating the time reproduction number in Kupang City Indonesia, 2016–2020, and assessing the effects of vaccination and different *Wolbachia* strains on dengue transmission dynamics," *Mathematics*, vol. 10, no. 12, p. 2075, 2022, doi: https://doi.org/10.3390/math10122075.

[31] R. De la Cruz, O. Padilla, M. A. Valle, and G. A. Ruz, "Modeling recidivism through bayesian regression models and deep neural networks," *Mathematics*, vol. 9, no. 6, p. 639, 2021, doi: https://doi.org/10.3390/math9060639.

[32] R. E. Mickens, *Dynamic Consistency: A Fundamental Principle for Constructing Nonstandard Finite Difference Schemes*. World Scientific, 2005.

[33] J. Calatayud, M. Jornet, and J. Mateu, "A dynamical mathematical model for crime evolution based on a compartmental system with interactions," *International Journal of Computer Mathematics*, vol. 102, no. 1, pp. 44–59, 2024, doi: https://doi.org/10.1080/00207160.2024.2302840.

[34] J. Chen, "Exploring the legal accuracy of mathematical models in crime sentencing prediction," *Applied Mathematics and Nonlinear Sciences*, vol. 9, no. 1, 2024, doi: https://doi.org/10.2478/amns-2024-2479.

[35] J. Calatayud, M. Jornet, and J. Mateu, "Spatio-temporal stochastic differential equations for crime incidence modeling," *Stochastic Environmental Research and Risk Assessment*, vol. 37, pp. 1839–1854, 2023, doi: https://doi.org/10.1007/s00477-022-02369-x.

[36] O. Adeleke *et al.*, "A mathematical model of criminal gang rivalry: Understanding the dynamics and implications," *Results in Control and Optimization*, vol. 14, p. 100398, 2024, doi: https://doi.org/10.1016/j.rico.2024.100398.

[37] Y. Zhang *et al.*, "Ada-gcnlstm: An adaptive urban crime spatiotemporal prediction model," *Journal of Safety Science and Resilience*, vol. 6, no. 2, pp. 226–236, 2025, doi: https://doi.org/10.1016/j.jnlssr.2024.11.003.