RESEARCH ARTICLE • OPEN ACCESS

Total Absolute Difference Edge Irregularity Strength of Several Ladder-type Graphs

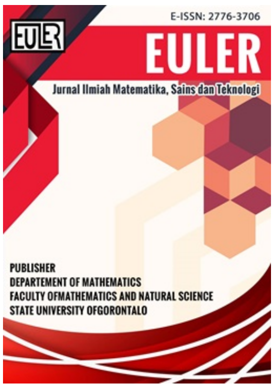
Faisal Susanto and Kristiana Wijaya

Volume 14, Issue 2, Pages 457–466, August 2026

Received 2 May 2026, Revised 15 July 2026, Accepted 25 July 2026, Published 1 August 2026

To Cite this Article : F. Susanto and K. Wijaya,"Total Absolute Difference Edge Irregularity Strength of Several Ladder-type Graphs", *Euler J. Ilm. Mat. Sains dan Teknol.*, vol. 14, no. 2, pp. 457–466, 2026, *https://doi.org/10.37905/euler.v14i2.39334*

JOURNAL INFO • EULER : JURNAL ILMIAH MATEMATIKA, SAINS DAN TEKNOLOGI



Homepage	:	http://ejurnal.ung.ac.id/index.php/euler/index
Journal Abbreviation	:	Euler J. Ilm. Mat. Sains dan Teknol.
Frequency	:	Three times a year
Publication Language	:	English (preferable), Indonesia
DOI	:	https://doi.org/10.37905/euler
Online ISSN	:	2776-3706
License	:	Creative Commons Attribution-NonCommercial 4.0 International License
Publisher	:	Department of Mathematics, Universitas Negeri Gorontalo
Country	:	Indonesia
OAI Address	:	http://ejurnal.ung.ac.id/index.php/euler/oai
Google Scholar ID	:	QF_r-_gAAAAJ
Email	:	euler@ung.ac.id

Euler : Jurnal Ilmiah Matematika, Sains dan Teknologi, Volume 14, Issue 2, pp. 457–466, August 2026 https://doi.org/10.37905/euler.v14i2.39334

Research Article

Total Absolute Difference Edge Irregularity Strength of Several Ladder-type Graphs

Faisal Susanto[1,*], **Kristiana Wijaya**[2]

[1]*Mathematics Department, School of Computer Science, Bina Nusantara University, Jakarta 11480, Indonesia*
[2]*Mathematics Department, Faculty of Mathematics and Natural Sciences, Universitas Jember, Jember 68121, Indonesia*

ARTICLE HISTORY
Received 2 May 2026
Revised 15 July 2026
Accepted 25 July 2026
Published 1 August 2026

KEYWORDS
Edge Irregular Total Absolute Difference Labeling
Triangular Ladder
Alternating Triangulated Ladder
Double Triangulated Ladder

ABSTRACT. *This paper studies edge irregular total absolute difference labeling for several classes of ladder related graphs, namely triangular ladders, alternating triangulated ladders, and double triangulated ladders. By deriving coinciding lower and upper bounds, the precise values for the total absolute difference edge irregularity strength of these graphs are successfully established.*

1. Introduction

Throughout this paper, $G = (V, E)$ denotes a finite simple undirected graph. Here, $V(G)$ and $E(G)$ represent the collections of vertices and edges of G, respectively. The degree of a vertex corresponds to the number of vertices adjacent to it. The notation $\Delta(G)$ represents the largest vertex degree in the entire graph. For integers g and h satisfying $g \leq h$, the notation $[g, h] = \{g, g+1, \ldots, h\}$ is used to represent the set of consecutive numbers from g to h.

Graph labeling represents a crucial subfield within graph theory. It primarily focuses on allocating integer values to graph elements, including vertices, edges, or a combination of the two, under certain prescribed conditions [1]. According to the graph components being assigned labels, labeling schemes are generally categorized into vertex, edge, and total labeling.

One of the well-studied graph labeling techniques is edge irregular total labeling, originally proposed in [2]. In this labeling scheme, positive integers are assigned to both the vertices and edges of a graph in such a way that each edge is uniquely identified by the sum of its own label and the labels assigned to its incident endpoints. There has been a surge of scholarly interest in this area, as evidenced by several recent studies on various graph families, including calendula graphs [3], modified book graphs [4], joint-wheel graphs and path unions of helms [5], the corona product of ladder and null graphs [6], star snake graphs [7], electromagnetic Feynman graphs [8], cycle snake graphs [9], and arithmetic ladder graphs [10].

A variation of edge irregular total labeling, known as edge irregular total absolute difference labeling, was studied in [11]. For a graph G, consider a function $\vartheta : V(G) \cup E(G) \to [1, k]$. The function ϑ is called an edge irregular total absolute difference k-labeling of a graph G whenever distinct edges of G receive distinct weights, where for each edge $xy \in E(G)$ with $x, y \in V(G)$, its weight is determined by $\omega(xy) = |\vartheta(x) + \vartheta(y) - \vartheta(xy)|$. The total

*Corresponding Author.

Email : *faisal.susanto@binus.ac.id* (F. Susanto)
Homepage : http://ejurnal.ung.ac.id/index.php/euler/index / E-ISSN : 2776-3706

absolute difference edge irregularity strength of G, written as tades(G), is the least value of k necessary to successfully construct such a labeling on G.

Over the past few years, the parameter tades has been extensively explored for various graph classes. In [11], this parameter was investigated for paths, star graphs, cycles, sun graphs, and fan graphs. Later, [12] examined the same parameter for T_p-tree graphs. Additional contributions were reported in [13], which addressed double fan graphs, lotus inside the circle graphs, wheel-like graphs, and snake-like graphs. Subsequently, [14] derived corresponding results for hairy cycle graphs, book graphs, and generalized Petersen graphs $P(n, 2)$. The work in [15] focused on transformed trees together with several path related graph families. In addition, [16] examined staircase graphs together with the disjoint union of grids and zigzag graphs.

Although substantial progress has been achieved in the study of tades, ladder-type graph families have not yet been examined in the existing literature. This observation motivates the present study, which focuses on several classes of ladder related graphs, namely triangular ladders, alternating triangulated graphs, and double triangulated ladders. In this study, the exact tades are established for each of these graph families.

2. Methods

This section describes the approach used to derive the precise tades values for triangular ladders, alternating triangulated ladders, and double triangulated ladders. To obtain these values, we derive corresponding lower and upper bounds and show that they coincide. The lower bounds are obtained by applying earlier results established in [11], which are summarized below.

Theorem 1 ([11]). *For an arbitrary graph G having maximum degree $\Delta(G)$, the following lower bound for* tades(G) *is satisfied:*

$$\text{tades}(G) \geq \max\left\{\left\lceil \frac{|E(G)|}{2} \right\rceil, \left\lceil \frac{\Delta(G)+1}{2} \right\rceil\right\}.$$

On the other hand, the upper bounds are derived through constructive arguments by providing explicit edge irregular total absolute difference labelings for triangular ladders, alternating triangulated ladders, and double triangulated ladders. The coincidence of the resulting lower and upper bounds consequently yields precise determinations of tades for these families of graphs.

3. Results and Discussion

In the sequel, attention is directed toward the tades of triangular ladders, alternating triangulated ladders, and double triangulated ladders. Let TL_n, where $n \geq 2$, denote a triangular ladder whose vertex set is defined by $V(TL_n) = \{a_i, b_i : i \in [1, n]\}$ and whose edge set is $E(TL_n) = \{a_i a_{i+1}, b_i b_{i+1}, a_i b_{i+1} : i \in [1, n-1]\} \cup \{a_i b_i : i \in [1, n]\}$. The next theorem establishes the tades of triangular ladders.

Theorem 2. *For $n \geq 2$, it holds that* tades$(TL_n) = 2n - 1$.

Proof. For $n \geq 2$, we know that $|E(TL_n)| = 4n - 3$ and

$$\Delta(TL_n) = \begin{cases} 3 & \text{for } n = 2, \\ 4 & \text{for } n \geq 3. \end{cases}$$

Consequently, applying Theorem 1 yields that for $n \geq 2$,

$$\begin{aligned} \text{tades}(TL_n) &\geq \begin{cases} \max\left\{\left\lceil \frac{5}{2} \right\rceil, \left\lceil \frac{4}{2} \right\rceil\right\} & \text{for } n = 2, \\ \max\left\{\left\lceil \frac{4n-3}{2} \right\rceil, \left\lceil \frac{5}{2} \right\rceil\right\} & \text{for } n \geq 3, \end{cases} \\ &= \left\lceil \frac{4n-3}{2} \right\rceil \\ &= 2n - 1. \end{aligned}$$

To establish the upper bound $\text{tades}(TL_n) \leq 2n - 1$, it suffices to construct an assignment of labels from the set $[1, 2n-1]$ to the vertices and edges of TL_n such that all induced edge weights are mutually distinct. We define a mapping $\vartheta : V(TL_n) \cup E(TL_n) \to [1, 2n-1]$ as follows.

$$\begin{aligned} \vartheta(a_i) &= \begin{cases} 1 & \text{whenever } i = 1, \\ 2i - 2 & \text{whenever } i \in [2, n], \end{cases} \\ \vartheta(b_i) &= \begin{cases} 1 & \text{whenever } i = 1, \\ 2i - 1 & \text{whenever } i \in [2, n], \end{cases} \\ \vartheta(a_i a_{i+1}) &= \begin{cases} 2 & \text{whenever } i = 1, \\ 1 & \text{whenever } i \in [2, n-1], \end{cases} \\ \vartheta(b_i b_{i+1}) &= 1 \quad \text{whenever } i \in [1, n-1], \\ \vartheta(a_i b_{i+1}) &= \begin{cases} 2 & \text{whenever } i = 1, \\ 1 & \text{whenever } i \in [2, n-1]. \end{cases} \\ \vartheta(a_i b_i) &= \begin{cases} 2 & \text{whenever } i = 1, \\ 1 & \text{whenever } i \in [2, n]. \end{cases} \end{aligned}$$

Observe that the largest label assigned under the labeling ϑ is $2n - 1$. We now evaluate the resulting edge weights. Whenever $i \in [1, n-1]$, we get

$$\begin{aligned} \omega(a_i a_{i+1}) &= |\vartheta(a_i) + \vartheta(a_{i+1}) - \vartheta(a_i a_{i+1})| \\ &= \begin{cases} |1 + 2 - 2| & \text{whenever } i = 1, \\ |2i - 2 + 2i - 1| & \text{whenever } i \in [2, n-1], \end{cases} \\ &= 4i - 3, \end{aligned}$$

whenever $i \in [1, n-1]$, we get

$$\begin{aligned} \omega(b_i b_{i+1}) &= |\vartheta(b_i) + \vartheta(b_{i+1}) - \vartheta(b_i b_{i+1})| \\ &= \begin{cases} |1 + 3 - 1| & \text{whenever } i = 1, \\ |2i - 1 + 2i + 1 - 1| & \text{whenever } i \in [2, n-1], \end{cases} \\ &= 4i - 1, \end{aligned}$$

whenever $i \in [1, n-1]$, we get

$$\begin{aligned}\omega(a_i b_{i+1}) &= |\vartheta(a_i) + \vartheta(b_{i+1}) - \vartheta(a_i b_{i+1})| \\ &= \begin{cases} |1+3-2| & \text{whenever } i = 1, \\ |2i-2+2i+1-1| & \text{whenever } i \in [2, n-1], \end{cases} \\ &= 4i - 2.\end{aligned}$$

and whenever $i \in [1, n]$, we get

$$\begin{aligned}\omega(a_i b_i) &= |\vartheta(a_i) + \vartheta(b_i) - \vartheta(a_i b_i)| \\ &= \begin{cases} |1+1-2| & \text{whenever } i = 1, \\ |2i-2+2i-1-1| & \text{whenever } i \in [2, n], \end{cases} \\ &= 4i - 4.\end{aligned}$$

Hence, the values induced on the edges are all different from one another, which confirms that ϑ defines a valid edge irregular total absolute difference $(2n-1)$-labeling of TL_n. Therefore, tades$(TL_n) = 2n - 1$. □

For illustrative purposes, an edge irregular total absolute difference 13-labeling of TL_7 is depicted in Figure 1.

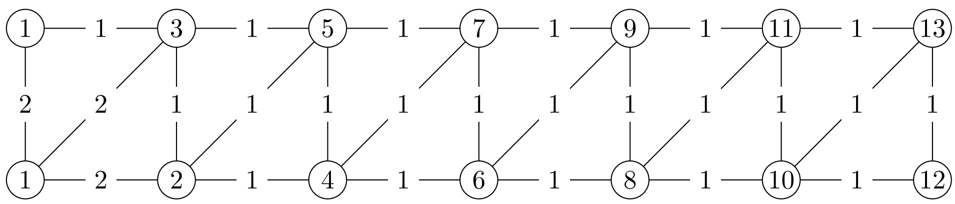


Figure 1. An edge irregular total absolute difference 13-labeling of TL_7.

Let ATL_n, where $n \geq 2$, denote an alternating triangulated ladder whose vertex set is $V(ATL_n) = \{a_i, b_i : i \in [1, n]\}$ and whose edge set is $E(ATL_n) = \{a_i a_{i+1}, b_i b_{i+1} : i \in [1, n-1]\} \cup \{a_i b_{i+1} : i \in [1, n-1] \text{ and } i-1 \text{ is divisible by } 2\} \cup \{a_{i+1} b_i : i \in [1, n-1] \text{ and } i \text{ is divisible by } 2\} \cup \{a_i b_i : i \in [1, n]\}$. The next theorem establishes the tades value of alternating triangulated ladders.

Theorem 3. *For $n \geq 2$, it holds that* tades$(ATL_n) = 2n - 1$.

Proof. Observe that for $n \geq 2$, the graph ATL_n contains exactly $|E(ATL_n)| = 4n - 3$ edges, and its maximum degree is given by

$$\Delta(ATL_n) = \begin{cases} 3 & \text{for } n = 2, \\ 4 & \text{for } n \geq 3. \end{cases}$$

By virtue of Theorem 1, we can establish the following lower bound for all $n \geq 2$:

$$\begin{aligned}\text{tades}(ATL_n) &\geq \begin{cases} \max\left\{\left\lceil \frac{5}{2} \right\rceil, \left\lceil \frac{4}{2} \right\rceil\right\} & \text{for } n = 2, \\ \max\left\{\left\lceil \frac{4n-3}{2} \right\rceil, \left\lceil \frac{5}{2} \right\rceil\right\} & \text{for } n \geq 3, \end{cases} \\ &= \left\lceil \frac{4n-3}{2} \right\rceil = 2n - 1.\end{aligned}$$

To show that $2n-1$ is also an upper bound of tades(ATL_n), it remains to construct a total labeling of ATL_n with labels taken from $[1, 2n-1]$ under which no two edges share the same induced weight. Accordingly, define a function $\vartheta : V(ATL_n) \cup E(ATL_n) \to [1, 2n-1]$ as follows.

$$\vartheta(a_i) = \begin{cases} 1 & \text{whenever } i = 1, \\ 2i-2 & \text{whenever } i \in [2, n], \end{cases}$$
$$\vartheta(b_i) = \begin{cases} 1 & \text{whenever } i = 1, \\ 2i-1 & \text{whenever } i \in [2, n], \end{cases}$$
$$\vartheta(a_i a_{i+1}) = \begin{cases} 2 & \text{whenever } i = 1, \\ 1 & \text{whenever } i \in [2, n-1], \end{cases}$$
$$\vartheta(b_i b_{i+1}) = 1 \quad \text{whenever } i \in [1, n-1],$$
$$\vartheta(a_i b_{i+1}) = \begin{cases} 2 & \text{whenever } i = 1, \\ 1 & \text{whenever } i \in [3, n-1] \text{ is odd}, \end{cases}$$
$$\vartheta(a_{i+1} b_i) = 1 \quad \text{whenever } i \in [2, n-1] \text{ is even},$$
$$\vartheta(a_i b_i) = \begin{cases} 2 & \text{whenever } i = 1, \\ 1 & \text{whenever } i \in [2, n]. \end{cases}$$

It is straightforward to verify that the labeling ϑ assigns values from the prescribed range with the highest assigned value equal to $2n-1$. We next determine the corresponding edge weights. Whenever $i \in [1, n-1]$, we obtain

$$\begin{aligned} \omega(a_i a_{i+1}) &= |\vartheta(a_i) + \vartheta(a_{i+1}) - \vartheta(a_i a_{i+1})| \\ &= \begin{cases} |1+2-2| & \text{whenever } i = 1, \\ |2i-2+2i-1| & \text{whenever } i \in [2, n-1], \end{cases} \\ &= 4i-3, \end{aligned}$$

whenever $i \in [1, n-1]$, we obtain

$$\begin{aligned} \omega(b_i b_{i+1}) &= |\vartheta(b_i) + \vartheta(b_{i+1}) - \vartheta(b_i b_{i+1})| \\ &= \begin{cases} |1+3-1| & \text{whenever } i = 1, \\ |2i-1+2i+1-1| & \text{whenever } i \in [2, n-1], \end{cases} \\ &= 4i-1, \end{aligned}$$

whenever $i \in [1, n-1]$ is odd, we obtain

$$\begin{aligned} \omega(a_i b_{i+1}) &= |\vartheta(a_i) + \vartheta(b_{i+1}) - \vartheta(a_i b_{i+1})| \\ &= \begin{cases} |1+3-2| & \text{whenever } i = 1, \\ |2i-2+2i+1-1| & \text{whenever } i \in [3, n-1] \text{ is odd}, \end{cases} \\ &= 4i-2, \end{aligned}$$

whenever $i \in [2, n-1]$ is even, we obtain

$$\omega(a_{i+1} b_i) = |\vartheta(a_{i+1}) + \vartheta(b_i) - \vartheta(a_{i+1} b_i)|$$

$$= \begin{cases} |4+3-1| & \text{whenever } i = 2, \\ |2i+2i-1-1| & \text{whenever } i \in [4, n-1] \text{ is even}, \end{cases}$$
$$= 4i - 2,$$

and whenever $i \in [1, n]$, we obtain

$$\begin{aligned} \omega(a_i b_i) &= |\vartheta(a_i) + \vartheta(b_i) - \vartheta(a_i b_i)| \\ &= \begin{cases} |1+1-2| & \text{whenever } i = 1, \\ |2i-2+2i-1-1| & \text{whenever } i \in [2, n], \end{cases} \\ &= 4i - 4. \end{aligned}$$

Consequently, the induced weights on all edges are pairwise distinct, showing that the labeling ϑ satisfies the required conditions for ATL_n. This completes the proof. □

Figure 2 illustrates an example of an edge irregular total absolute difference 13-labeling of ATL_7.

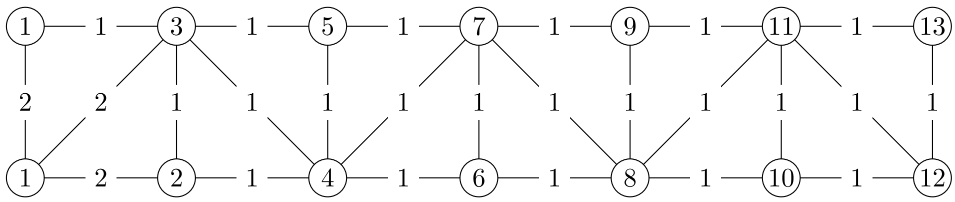


Figure 2. An edge irregular total absolute difference 13-labeling of ATL_7.

Let DTL_n, where $n \geq 2$, denote a double triangulated ladder whose set of vertices is $V(DTL_n) = \{a_i, b_i : i \in [1, n]\}$ and whose set of edges is $E(DTL_n) = \{a_i a_{i+1}, b_i b_{i+1}, a_i b_{i+1}, a_{i+1} b_i : i \in [1, n-1]\} \cup \{a_i b_i : i \in [1, n]\}$. The next theorem establishes the tades of double triangulated ladders.

Theorem 4. *For $n \geq 2$, it holds that* $\text{tades}(DTL_n) = \lceil \frac{5n-4}{2} \rceil$.

Proof. Note that for $n \geq 2$, the graph DTL_n has exactly $|E(DTL_n)| = 5n - 4$ edges. The maximum degree of DTL_n is given by

$$\Delta(DTL_n) = \begin{cases} 3 & \text{for } n = 2, \\ 5 & \text{for } n \geq 3. \end{cases}$$

Therefore, based on Theorem 1, the lower bound for $n \geq 2$ can be deduced as

$$\begin{aligned} \text{tades}(DTL_n) &\geq \begin{cases} \max\left\{\lceil \frac{6}{2} \rceil, \lceil \frac{4}{2} \rceil\right\} & \text{for } n = 2, \\ \max\left\{\lceil \frac{5n-4}{2} \rceil, \lceil \frac{6}{2} \rceil\right\} & \text{for } n \geq 3, \end{cases} \\ &= \left\lceil \frac{5n-4}{2} \right\rceil. \end{aligned}$$

We can verify the upper bound $\text{tades}(DTL_n) \leq \lceil \frac{5n-4}{2} \rceil$ by assigning labels from $[1, \lceil \frac{5n-4}{2} \rceil]$ to all vertices and edges of DTL_n, ensuring that every induced edge weight is distinct. To achieve this, we construct a total labeling $\vartheta : V(DTL_n) \cup E(DTL_n) \to [1, \lceil \frac{5n-4}{2} \rceil]$, defined as follows.

$$\vartheta(a_i) = \begin{cases} 1 & \text{whenever } i = 1, \\ \lceil \frac{5i-5}{2} \rceil & \text{whenever } i \in [2, n], \end{cases}$$

$$\vartheta(b_i) = \begin{cases} 1 & \text{whenever } i = 1 \text{ and } n-1 \text{ is divisible by } 2, \\ \lceil \frac{5i-4}{2} \rceil & \text{whenever } i \in [3, n-2] \text{ is odd and } n-1 \text{ is divisible by } 2, \\ \lceil \frac{5i-2}{2} \rceil & \text{whenever } i \in [2, n-1] \text{ is even and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i \in [1, n-1] \text{ and } n \text{ is divisible by } 2, \\ \lceil \frac{5n-4}{2} \rceil & \text{whenever } i = n, \end{cases}$$

$$\vartheta(a_i a_{i+1}) = \begin{cases} 3 & \text{whenever } i = 1, \\ 2 & \text{whenever } i \in [2, n-1], \end{cases}$$

$$\vartheta(b_i b_{i+1}) = \begin{cases} 1 & \text{whenever } i \in [1, n-1] \text{ and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i = n-1 \text{ and } n \text{ is divisible by } 2, \\ 2 & \text{whenever } i \in [1, n-2] \text{ and } n \text{ is divisible by } 2, \end{cases}$$

$$\vartheta(a_i b_i) = \begin{cases} 2 & \text{whenever } i = 1 \text{ or } i \in [2, n-1] \text{ is even, and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i \in [2, n-1] \text{ and } n \text{ is divisible by } 2, \\ 3 & \text{whenever } i = 1 \text{ and } n \text{ is divisible by } 2, \\ 1 & \text{whenever } i \in [3, n] \text{ is odd and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i = n \text{ and } n \text{ is divisible by } 2, \end{cases}$$

$$\vartheta(a_i b_{i+1}) = \begin{cases} 3 & \text{whenever } i = 1 \text{ and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i = 1 \text{ or } i \in [2, n-2] \text{ is even, and } n \text{ is divisible by } 2, \\ 2 & \text{whenever } i \in [2, n-1] \text{ and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i \in [3, n-3] \text{ is odd and } n \text{ is divisible by } 2, \\ 1 & \text{whenever } i = n-1 \text{ and } n \text{ is divisible by } 2, \end{cases}$$

$$\vartheta(a_{i+1} b_i) = \begin{cases} 1 & \text{whenever } i \in [1, n-1] \text{ and } n-1 \text{ is divisible by } 2, \text{ or} \\ & \text{whenever } i \in [2, n-2] \text{ is even and } n \text{ is divisible by } 2, \\ 2 & \text{whenever } i \in [1, n-1] \text{ and } n \text{ is divisible by } 2 \text{ is odd.} \end{cases}$$

It can be readily verified that the largest label assigned under ϑ is $\lceil \frac{5n-4}{2} \rceil$. We now proceed to determine the induced edge weights. Whenever $i \in [1, n-1]$, we have

$$\begin{aligned} \omega(a_i a_{i+1}) &= |\vartheta(a_i) + \vartheta(a_{i+1}) - \vartheta(a_i a_{i+1})| \\ &= \begin{cases} |1 + 3 - 3| & \text{whenever } i = 1, \\ |\lceil \frac{5i-5}{2} \rceil + \lceil \frac{5i}{2} \rceil - 2| & \text{whenever } i \in [2, n-1], \end{cases} \\ &= 5i - 4, \end{aligned}$$

whenever $i \in [1, n-1]$, we have

$$\omega(b_i b_{i+1}) = |\vartheta(b_i) + \vartheta(b_{i+1}) - \vartheta(b_i b_{i+1})|$$

$$= \begin{cases} |1+4-1| & \text{whenever } i=1 \text{ and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i-4}{2} \rceil + \lceil \frac{5i+3}{2} \rceil - 1| & \text{whenever } i \in [3, n-2] \text{ is odd and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i-2}{2} \rceil + \lceil \frac{5i+3}{2} \rceil - 2| & \text{whenever } i \in [1, n-3] \text{ is odd and } n \text{ is divisible by } 2, \\ |\lceil \frac{5i-2}{2} \rceil + \lceil \frac{5i+1}{2} \rceil - 1| & \text{whenever } i \in [2, n-3] \text{ is even and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i-2}{2} \rceil + \lceil \frac{5i+3}{2} \rceil - 2| & \text{whenever } i \in [2, n-2] \text{ is even and } n \text{ is divisible by } 2, \\ |\lceil \frac{5n-7}{2} \rceil + \lceil \frac{5n-4}{2} \rceil - 1| & \text{whenever } i = n-1, \end{cases}$$
$$= 5i-1,$$

whenever $i \in [1, n-1]$, we have

$$\omega(a_i b_{i+1}) = |\vartheta(a_i) + \vartheta(b_{i+1}) - \vartheta(a_i b_{i+1})|$$
$$= \begin{cases} |1+4-3| & \text{whenever } i=1, \\ |\lceil \frac{5i-5}{2} \rceil + \lceil \frac{5i+3}{2} \rceil - 2| & \text{whenever } i \in [3, n-2] \text{ is odd}, \\ |\lceil \frac{5i-5}{2} \rceil + \lceil \frac{5i+1}{2} \rceil - 2| & \text{whenever } i \in [2, n-3] \text{ is even and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i-5}{2} \rceil + \lceil \frac{5i+3}{2} \rceil - 3| & \text{whenever } i \in [2, n-2] \text{ is even and } n \text{ is divisible by } 2, \\ |\lceil \frac{5n-10}{2} \rceil + \lceil \frac{5n-4}{2} \rceil - 2| & \text{whenever } i = n-1 \text{ and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5n-10}{2} \rceil + \lceil \frac{5n-4}{2} \rceil - 1| & \text{whenever } i = n-1 \text{ and } n \text{ is divisible by } 2, \end{cases}$$
$$= 5i-3,$$

whenever $i \in [1, n-1]$, we have

$$\omega(a_{i+1} b_i) = |\vartheta(a_{i+1}) + \vartheta(b_i) - \vartheta(a_{i+1} b_i)|$$
$$= \begin{cases} |3+1-1| & \text{whenever } i=1 \text{ and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i}{2} \rceil + \lceil \frac{5i-4}{2} \rceil - 1| & \text{whenever } i \in [3, n-2] \text{ is odd and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i}{2} \rceil + \lceil \frac{5i-2}{2} \rceil - 2| & \text{whenever } i \in [1, n-1] \text{ and } n \text{ is divisible by } 2 \text{ is odd}, \\ |\lceil \frac{5i}{2} \rceil + \lceil \frac{5i-2}{2} \rceil - 1| & \text{whenever } i \in [2, n-1] \text{ is even}, \end{cases}$$
$$= 5i-2,$$

and whenever $i \in [1, n]$, we have

$$\omega(a_i b_i) = |\vartheta(a_i) + \vartheta(b_i) - \vartheta(a_i b_i)|$$
$$= \begin{cases} |1+1-2| & \text{whenever } i=1 \text{ and } n-1 \text{ is divisible by } 2, \\ |1+2-3| & \text{whenever } i=1 \text{ and } n \text{ is divisible by } 2, \\ |\lceil \frac{5i-5}{2} \rceil + \lceil \frac{5i-4}{2} \rceil - 1| & \text{whenever } i \in [3, n-2] \text{ is odd and } n-1 \text{ is divisible by } 2, \\ |\lceil \frac{5i-5}{2} \rceil + \lceil \frac{5i-2}{2} \rceil - 2| & \text{whenever } i \in [2, n-1] \text{ is even, or} \\ & \text{whenever } i \in [3, n-1] \text{ is odd and } n \text{ is divisible by } 2, \\ |\lceil \frac{5n-5}{2} \rceil + \lceil \frac{5n-4}{2} \rceil - 1| & \text{whenever } i = n, \end{cases}$$
$$= 5i-5.$$

Since the induced values associated with the edges are all different, ϑ fulfills all the required conditions for DTL_n. Consequently, tades$(DTL_n) = \lceil \frac{5n-4}{2} \rceil$. □

An example of an edge irregular total absolute difference 16-labeling for DTL_7 can be seen in Figure 3.

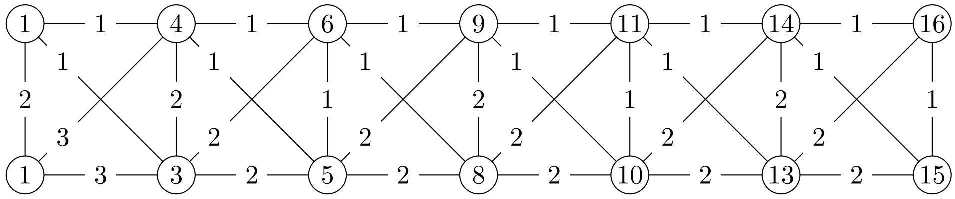


Figure 3. An edge irregular total absolute difference 16-labeling of DTL_7.

4. Conclusion

In this article, the tades of triangular ladders, alternating triangulated ladders, and double triangulated ladders have been established. The obtained results confirm that these graph families achieve the general lower bounds stated in Theorem 1. Furthermore, the constructive methods developed in this work may serve as a useful framework for investigating tades in other classes of structured graphs, thereby providing several possibilities for future investigation.

Author Contributions. **Faisal Susanto**: Conceptualization, methodology, investigation, formal analysis, visualization, writing–original draft preparation. **Kristiana Wijaya**: Investigation, validation, formal analysis, writing–review and editing.

Acknowledgment. We thank the editors and reviewers for their constructive feedback, which resulted in a significant improvement in the quality of this manuscript.

Funding. No external financial support was received for this study.

Conflicts of Interest. The authors have no competing interests to disclose.

Data availability. No external data sources or experimental datasets were used in this work.

References

[1] J. A. Gallian, "A dynamic survey of graph labeling," *Electronic Journal of Combinatorics*, p. #DS6, 2025, doi: 10.37236/27.

[2] M. Bača, S. Jendrol', M. Miller, and J. Ryan, "On irregular total labellings," *Discrete Mathematics*, vol. 307, no. 11-12, pp. 1378–1388, 2007, doi: 10.1016/j.disc.2005.11.075.

[3] M. R. Z. El Deen and G. M. Elmahdy, "Total edge irregularity strength and edge irregular reflexive labeling for calendula graph," *Italian Journal of Pure and Applied Mathematics*, no. 49, pp. 722–753, 2023, Available: https://ijpam.uniud.it.

[4] L. Ratnasari, S. Wahyuni, Y. Susanti, and D. J. Eksi Palupi, "Total edge irregularity strength of modified book graphs," *Southeast Asian Bulletin of Mathematics*, vol. 47, no. 4, pp. 543–562, 2023, Available: http://www.seams-bull-math.ynu.edu.cn/archive.jsp.

[5] A. Sudha and P. Jeyanthi, "Total edge irregularity strength of some graphs," *International Journal of Mathematical Analysis and Research*, vol. 1, no. 2, pp. 16–20, 2025, doi: 10.64137/31082637/IJMAR-V1I2P103.

[6] Nikita, S. Nurhayati, T. M. A. Januaviani, S. Musdalifah, I. Al Fajri, and Andri, "Total edge irregularity strength of graph $L_3 \odot N_m$," *Riemann: Research of Mathematics and Mathematics Education*, vol. 7, no. 2, pp. 104–110, 2025, doi: 10.38114/reimann.v7i2.104.

[7] H. Attiya, N. Ahmed, and F. Salama, "Total edge irregularity strength of star snake graphs," *European Journal of Pure and Applied Mathematics*, vol. 18, no. 2, pp. 5673–5673, 2025, doi: 10.29020/nybg.ejpam.v18i2.5673.

[8] H. Attiya, "Study on electromagnetic Feynman graph and its total edge irregularity strength," *Delta Journal of Science*, vol. 50, no. 1, pp. 120–128, 2025, doi: 10.21608/djs.2025.372919.1210.

[9] S. Pranata, V. H. Krisnawati, and D. Darmajid, "Total edge irregularity strength of cycle snake

graphs," *CAUCHY: Jurnal Matematika Murni dan Aplikasi*, vol. 11, no. 1, pp. 970–986, 2026, doi: 10.18860/cauchy.v11i1.41552.

[10] E. Wulandari and Y. Susanti, "On total edge irregularity strength of arithmetic ladder graphs," *AIMS Mathematics*, vol. 11, no. 4, pp. 10 175–10 190, 2026, doi: 10.3934/math.2026420.

[11] M. Ramalakshmi and K. M. Kathiresan, "Total absolute difference edge irregularity strength of the graph," *Kragujevac Journal of Mathematics*, vol. 46, no. 6, pp. 895–903, 2022, doi: 10.46793/KgJMat2206.895R.

[12] A. Lourdusamy, F. J. Beaula, and F. Patrick, "Total absolute difference edge irregularity strength of T_p-tree graphs," *Proyecciones*, vol. 42, no. 6, pp. 1597–1614, 2023, doi: 10.22199/issn.0717-6279-5411.

[13] A. Lourdusamy and F. Joy Beaula, "Total absolute difference edge irregularity strength of some families of graphs," *TWMS Journal of Applied and Engineering Mathematics*, vol. 13, no. 3, pp. 1005–1012, 2023, Available: https://jaem.isikun.edu.tr/web/images/articles/vol.13.no.3/17.pdf.

[14] K. Shabbir, A. Raza, and L. Guran, "Computation of tades of some families of graphs," *Scientific Inquiry and Review*, vol. 8, no. 4, pp. 80–92, 2024, doi: 10.32350/sir.84.04.

[15] A. Lourdusamy and F. J. Beaula, "On tades of transformed tree and path related graphs," *Jordan Journal of Mathematics and Statistics*, vol. 17, no. 1, pp. 129–143, 2024, doi: 0.47013/17.1.8.

[16] A. Lourdusamy, F. Beaula, and F. Patrick, "On tades of disjoint union of some graphs," *Ars Combinatoria*, vol. 157, pp. 89–94, 2023, doi: 10.61091/ars157-09.