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Research Article

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Dynamics of a predator-prey model incorporating infectious disease and quarantine on prey

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ABSTRACT. *In this article, the dynamics of a predator-prey model incorporating infectious disease and quarantine on prey population is discussed. We first analyze the existence conditions of all positive equilibrium points. Next, we investigate the local stability properties of the proposed model using the linearization method. We also determine the basic reproduction number using the next generation matrix. Finally, some numerical simulations are performed to validate the stability of each equilibrium point.*

1. Introduction

All living things must interact with each other in order to survive. This interaction produces a relationship between every living thing, both as consumers, decomposers, or producers. Ecology is the study of living things and their interactions with the environment as well as interactions with other living things [1]. In ecological systems, the interaction between an organism and its predator is called as predator-prey interaction. Prey is living creature that is preyed upon and a predator as a living creature that preys on. The relationship between prey and predators is very close. This could happen since the predator cannot survive without prey. Vice versa, without predator, an explosion in prey population cannot be avoided and disrupt ecosystem stability [2].

Predator-prey model is a mathematical model that has long been studied because it is directly related to nature and the existence of the creatures in it [3, 4]. Many researchers had study, develop and modify the classical predator-prey model with the aim of conforming to the actual conditions in nature [5].

There are many factors in the ecosystem that affect the population. The spread of disease is one of the noteworthy issues that affect the interactions between one creature and another. The study that focuses on the spread of disease at the population level is called epidemiology [6]. Mathematical modeling in epidemiology provides an understanding of the underlying mechanisms that influence the spread of disease. As in ecological modeling, modifications to epidemiology modelings are usually made between simple models, which omit some detail and design to model general qualitative behavior, and complex models, usually designed for specific situations [7]. If there is an interaction between two populations and there is a spread of disease in one or both of these population, then it refers to eco-epidemiology modeling.

Research on the eco-epidemiology model has been carried out by many previous researchers. Wuhaib and Hasan [8] investigated a predator-prey model by harvesting the infected prey. The infected prey population can recover and becomes susceptible population that can be reinfected. Purnomo, et al [9] also study an eco-epidemiology model by considering predators only kill the infected prey and harvesting susceptible prey. Panigoro, et al. [10] examine an eco-epidemiological model by assuming that prey grows logistically while predator grows exponentially due to disease infection in prey. Furthermore, the eco-epidemiological model developed by Maisaroh, et al. [11] assume that the prey infected with the disease cannot return to being vulnerable prey and then considers harvesting of predators. Ibrahim, et al [12] examine a Gause-type predator-prey model with disease outbreaks in prey by considering no harvesting in both populations.

Predation imbalance against high prey population or low prey population is one of the causes of the extinction of a population. Therefore, this study will discuss the stability analysis of the eco-epidemiological mathematical model with assume that the infected prey population will be given a treatment to reduce the number of infectious populations. This treatment in a quarantine form, so that the infected prey population cannot be eaten by predator. The prey populations that have been successfully treated can be released back into the environment.

We present this article in the following structure. The model formulation is given in Section 2. The existence and stability condition of all equilibrium points are discussed in Section 3. We also confirm the analytical results through some numerical simulations in Section 4. In Section 5, we end of our works with conclusion.

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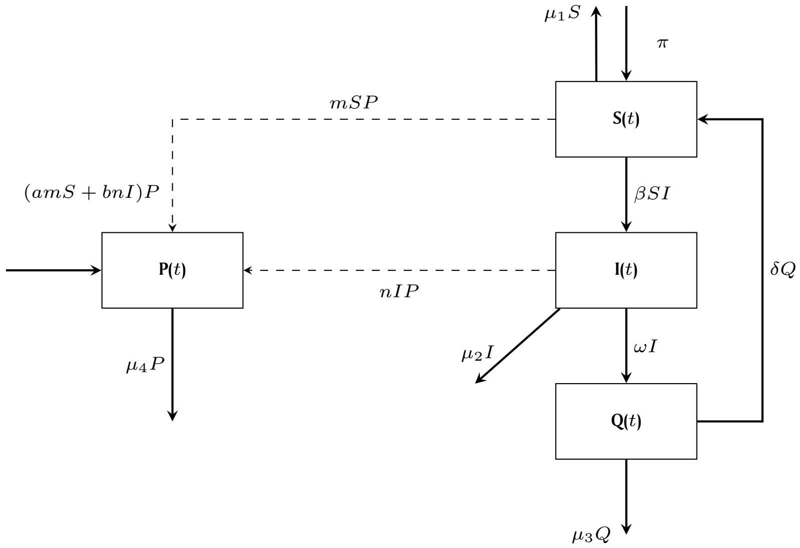

Figure 1. eco-epidemiological model food chain diagram with quarantine in prey population

2. Model Formulation

We first develop the mathematical model for the prey population which formulated based on McKendrick model [13] as follows.

$$\begin{aligned} \frac{dS}{dT} &= \Pi - \beta SI - \mu_1 S, \\ \frac{dI}{dT} &= \beta SI - \mu_2 I, \end{aligned} \tag{1}$$

where $S(t)$ and $I(t)$ are the susceptible and infected prey at time t. The parameters Π, β, μ_1, and μ_2 denote the constant growth rate of prey, the infection rate of prey,the natural death rate of susceptible prey, and the natural death rate of infected prey, respectively. In this case, we assume to protect the exixtence of the population, the infected prey will be quarantined with linear rate ω to receive treatment from human which aims to reduce the death rate due to the infectious disease. Following the linear form, we have the modified model as follows.

$$\begin{aligned} \frac{dS}{dT} &= \Pi - \beta SI + \delta Q - mSP - \mu_1 S, \\ \frac{dI}{dT} &= \beta SI - \omega I - \mu_2 I, \\ \frac{dQ}{dT} &= \omega I - \delta Q - \mu_3 Q, \end{aligned} \tag{2}$$

where δ movement rate of quarantine prey to susceptible prey due to its recovery from disease, and μ_3 is the natural death rate of quarantine prey. Now, by integrating the existence of predator due to the food chain in wild life, the final model of our works is obtained

$$\begin{aligned} \frac{dS}{dt} &= \Pi - \beta SI + \delta Q - mSP - \mu_1 S, \\ \frac{dI}{dt} &= \beta SI - \omega I - nIP - \mu_2 I, \\ \frac{dQ}{dt} &= \omega I - \delta Q - \mu_3 Q, \\ \frac{dP}{dt} &= amSP + bnIP - \mu_4 P \end{aligned} \tag{3}$$

where m, n, a, b and μ_4 are respectively the predation rate of susceptible prey and the predation rate of infected prey. It is clear that when the prey is quarantined, the predator cannot reach them so that the predation does not exists for the compatment. For more details, see food chain diagram given by Figure 1.

3. Analytical Results

Some definitions are given to investigate the existence of biological equilibrium point of model (3)

Definition 1. [14] Suppose there is a system of differential equations

$$\dot{x} = f(x), x(0) = x_0, x \in R^n. \tag{4}$$

The system of differential equations has an equilibrium point $\tilde{x} \in \mathbb{R}^{\ltimes}$ if it satisfies $f(\tilde{x}) = 0$.

Definition 2. $\tilde{x}$ is the biological equilibrium point of eq. (4) if it satisfies Definition 1 and $\tilde{x} \in \mathbb{R}^{\ltimes}{}_{+}$ with $\mathbb{R}^{\ltimes}{}_{+} := \{x_i : x_i \geq 0, i = 1, 2, \dots, n\}$.

Based on Definition 2, the equilibrium point of model (3) is obtained by solving $\frac{dS}{dt} = \frac{dI}{dt} = \frac{dQ}{dt} = \frac{dP}{dt} = 0$. Four equilibrium points are obtained including their local dynamical behaviors given by the next subsections.

3.1. The Predator-Disease-Free Point and Basic Reproduction Number

The Predator-Disease-Free Point (PDFP) biologically show the extinction of predator along with the disappearance of the disease in the population of prey. The PDFP is given by

$$E_1 = \left(\frac{\Pi}{\mu_1}, 0, 0, 0 \right) \tag{5}$$

which always exists. This condition explains that only susceptible prey populations can survive in the ecosystem, while predator populations, sick prey, and quarantined prey populations are extinct. Its population density is the ratio of the constant growth rate and natural mortality of the susceptible prey population. Now, to determine the basic reproduction number R_0, we follow the next generation matrix as given in [15, 16]. We first investigate the transmission and transition matrix denoted by F and V of the following infection subsystem.

$$\begin{aligned}\frac{dI}{dt} &= \beta SI - \omega I - nIP - \mu_2 I,\\ \frac{dQ}{dt} &= \omega I - \delta Q - \mu_3 Q.\end{aligned}$$

We obtain

$$F = \begin{pmatrix}\beta S & 0\\ 0 & 0\end{pmatrix},$$

and

$$V^{-1} = \begin{pmatrix}\dfrac{1}{\omega + nP + \mu_2} & 0\\ \dfrac{\omega}{(\omega+nP+\mu_2)(\delta+\mu_3)} & \dfrac{1}{\delta+\mu_3}\end{pmatrix}.$$

Therefore, the next generation matrix is acquired as follows

$$K = FV^{-1}\begin{pmatrix}\dfrac{\beta S}{\omega+nP+\mu_2} & 0\\ \dfrac{\beta S\omega}{(\omega+nP+\mu_2)(\delta+\mu_3)} & 0\end{pmatrix}.$$

Computing the dominant eigenvalue, we obtain the following basic reproduction number R_0.

$$\begin{aligned}R_0 &= \frac{\beta S_0}{\omega+nP+\mu_2}\\ R_0^1 &= \frac{\beta\pi}{\mu_1(\omega+\mu_2)}\end{aligned} \tag{6}$$

The following theorem shows the dynamics of PDFP based on its basic reproduction number.

Theorem 1. *The PDFP* $E_1 = \left(\frac{\Pi}{\mu_1}, 0, 0, 0\right)$ *is locally asymptotically stable if* $R_0^1 < \min\{1, \bar{R}\}$ *where* $\bar{R} = \dfrac{\beta\mu_4}{am(\omega+\mu_2)}$.

Proof. Simple computation show that the Jacobian matrix at $E_1 = \left(\frac{\Pi}{\mu_1}, 0, 0, 0\right)$ is

$$J_{E_1} = \begin{pmatrix}-\mu_1 & -\dfrac{\beta\pi}{\mu_1} & \delta & -\dfrac{m\Pi}{\mu_1}\\ 0 & (R_0^1-1)(\omega+\mu_2) & 0 & 0\\ 0 & \omega & -\delta-\mu_3 & 0\\ 0 & 0 & 0 & \dfrac{\mu_4}{\bar{R}}(R_0^1-\bar{R})\end{pmatrix},$$

which gives four eigenvalues: $\lambda_1 = -\mu_1$, $\lambda_2 = (R_0^1 - 1)(\omega+\mu_2)$, $\lambda_3 = -(\delta+\mu_3)$, $\lambda_4 = \frac{\mu_4}{\bar{R}}(R_0^1 - \bar{R})$. Obviously, $\lambda_i < 0$, i = 1,3, and hence the stability of PDFP depends on the sign of λ_j, j = 2,4. By arranging $R_0^1 < \min\{1, \bar{R}\}$, the sign of λ_j = 2,4 will be negative and PDFP is locally asymptotically stable. □

3.2. The Predator-Free Point

The predator-free point (PFP) is presented by the extinction of predators and the existence of susceptible, infected, and quarantine preys. This equilibrium point is given by $E_2 = (\bar{S}, \bar{I}, \bar{Q}, 0)$ where

$$\bar{S} = \frac{\omega+\mu_2}{\beta} = \frac{\mu_4}{am\bar{R}}, \bar{I} = \frac{(\delta+\mu_3)\bar{Q}}{\omega}, \bar{Q} = \frac{\beta\mu_1\bar{S}}{\delta}(\hat{R} - R_0^1)$$

This condition explains that only predator populations are extinct in addition to surviving in the ecosystem, namely populations of healthy prey, sick prey, and quarantined prey. The population density is the ratio of the rate of movement of infected prey to quarantined prey, the natural death rate of infected prey, and the rate of movement of susceptible prey to infected prey. The existence of PFP and its local dynamics of PFP is given by the following theorem.

Theorem 2. $E_2 = (\bar{S}, \bar{I}, \bar{Q}, 0)$ *exists if* $\hat{R} > R_0^1$.

Proof. it is clear that $\bar{S} > 0$, $\bar{I} > 0$, and $\bar{a}$. Moreover, $\bar{Q} > 0$ only if $\hat{R} > R_0^1$. Consequently, according to Definition 2 E_3 is the point of biological equilibrium. □

Theorem 3. *Suppose that* $\mu_4 > \dfrac{\bar{\mu}}{R_0^1}$ *where* $\bar{\mu} = \dfrac{am\Pi}{\mu_1} + \left(\dfrac{bn\Pi\beta(\delta+\mu_3)(\hat{R}-R_0^1)}{\omega\delta}\right)$. *The PFP* $E_2 = (\bar{S}, \bar{I}, \bar{Q}, 0)$ *is locally asymptotically stable if* $R_0^1 > \hat{R}$.

Proof. The Jacobian matrix at E_2 is

$$J_{E_2} = \begin{pmatrix}-(\beta\bar{I}+\mu_1) & -\beta\bar{S} & \delta & -m\bar{S}\\ \beta\bar{I} & 0 & 0 & -n\bar{I}\\ 0 & \omega & -(\delta+\mu_3) & 0\\ 0 & 0 & 0 & \dfrac{\bar{\mu}}{R_0^1} - \mu_4\end{pmatrix},$$

which gives $\lambda_1 = R_0^1 > \dfrac{\bar{\mu}}{\mu_4}$ and polynomial characteristic

$$P(\lambda) = \lambda^3 + \bar{\zeta}_1\lambda^2 + \bar{\zeta}_2\lambda + \bar{\zeta}_3, \tag{7}$$

where $\bar{\zeta}_1 = \beta\bar{I} + \delta + \mu_1 + \mu_3$, $\bar{\zeta}_2 = \beta(\omega+\mu_2+\delta+\mu_3)\bar{I} + \mu_1(\delta+\mu_3)$, and $\bar{\zeta}_3 = (\delta\mu_2 + \mu_2\mu_3 + \mu_3\omega)\beta\bar{I}$. Since $\mu_4 > \dfrac{\bar{\mu}}{R_0^1}$, we have $\lambda_1 < 0$ and hence the local stability of PFP depends on eq. (7). We examine that $\bar{\zeta}_1 > 0$, $\bar{\zeta}_2 > 0$, and $\bar{\zeta}_3 > 0$. Thus, according to Routh-Hurwitz criterion, we have to proof that $\bar{\zeta}_1\bar{\zeta}_2 > \bar{\zeta}_3$, which is true for $R_0^1 > \hat{R}$. □

3.3. The Disease-Free Point

The Disease-Free Point (DFP) is a condition when all populations exist except the populations of infected and quarantine class. This condition explains that two populations survive in the ecosystem, namely the susceptible prey population and the predator population. While the other two populations became extinct, namely the sick prey population and the quarantined prey population. The population density is the ratio of the natural death rate of the predator population and the conversion of the predation rate of susceptible prey to the birth rate of predators and the predation rate of susceptible prey to predators. The equilibrium point is given by

$$E_2 = \left(\frac{\mu_4}{am}, 0, 0, \frac{a\Pi}{\mu_4} - \frac{\mu_1}{m}\right),$$

The existence of the equilibrium point E_2 is given by Theorem 4

> **Theorem 4.** $E_3 = \left(\frac{\mu_4}{am}, 0, 0, \frac{a\Pi}{\mu_4} - \frac{\mu_1}{m}\right)$ *exists if* $a > \frac{\mu_1\mu_4}{m\Pi}$.

Proof. This condition can be confirmed by identify the positivity of E_3. □

> **Theorem 5.** *The DFP* E_2 *is locally asymptotically stable if* $\beta < \beta^*$ *where* $\beta^* = \left(\frac{an^2}{\mu_1} + \frac{n\mu_1}{m} + \omega + \mu_2\right)\frac{am}{\mu_4}$.

Proof. When $E_3 = \left(\frac{\mu_4}{am}, 0, 0, \frac{(\Pi - \bar{\Pi})a}{\mu_4}\right)$, the jacobian matrix becomes:

$$J_{E_2} = \begin{pmatrix} c_{11} & c_{12} & \delta & -\frac{\mu_4}{a} \\ 0 & c_{21} & 0 & 0 \\ 0 & \omega & -(\delta+\mu_3) & 0 \\ c_{41} & c_{42} & 0 & 0 \end{pmatrix},$$

where

$$\begin{aligned} c_{11} &= -\left(\frac{(\Pi - \bar{\Pi})am}{\mu_4} + \mu_1\right), \\ c_{12} &= -\frac{\beta\mu_4}{am}, \\ c_{21} &= \frac{(an\mu_1 + \beta\mu_4)\bar{\Pi}}{\mu_1\mu_4} - \left(\frac{an\mu_1 R_0^2 + \beta\mu_4}{\mu_1\mu_4 R_0^2}\right)\Pi, \\ c_{41} &= \frac{(am\Pi - \mu_1\mu_4)a}{\mu_4} \\ c_{42} &= \frac{(am\Pi - \mu_1\mu_4)bn}{m\mu_4}. \end{aligned}$$

This jacobian matrix has eigenvalues $\lambda_1 = -(\delta + \mu_3), \lambda_2 = \frac{(an\mu_1 + \beta\mu_4)\bar{\Pi}}{\mu_1\mu_4} - \left(\frac{an\mu_1 R_0^2 + \beta\mu_4}{\mu_1\mu_4 R_0^2}\right)\Pi$ and quadratic equation

$$P(\lambda) = \lambda^2 + \bar{\zeta}_1\lambda + \bar{\zeta}_2, \tag{8}$$

where $\zeta_1 = \mu_1 + \frac{am\Pi - \bar{\Pi}}{\mu_4}$ and $\zeta_2 = am\Pi - \mu_1\mu_4$. Since $\beta < \beta^*$ we have $\lambda_2 < 0$ and hence the local stability of PDFP depends Equation (8). Obeying Routh-Hurwitz criterion, PDFP is locally asymptotically stable if $\zeta_1 > 0$ dan $\zeta_2 > 0$. It is easy to compute that those criterions are satisfied by eq. (8). □

3.4. The Co-exixtence Point

The point of existence of all populations $E_4 = \left(\frac{\mu_4 - bn\bar{I}}{am}, \bar{I}, \frac{\omega\bar{I}}{\delta + \mu_3}, \beta\frac{(\mu_4 - bn\bar{I}) - am(\omega - \mu_2)}{amn}\right)$ where $\bar{I}$ is a quadratic equation, so we get:

$$d_1\bar{I}^2 + d_2\bar{I} + d_3 = 0, \tag{9}$$

with

$$\begin{aligned} d_1 &= \left(1 - \frac{b}{a}\right)bn\beta \\ d_2 &= \frac{\delta\omega am}{\delta + \mu_3} + \frac{2\mu_4\beta b}{a} + \mu_1 bn - \mu_4^2\beta - b(\omega + \mu_2) \\ d_3 &= amn + \frac{\mu_4 m(\omega + \mu_2)}{n} - \frac{\mu_4^2\beta}{an} - \mu_1\mu_4 \end{aligned}$$

The existence of the equilibrium point E_4 is given by Theorem 6

> **Theorem 6.** *Suppose* $d_1d_2 < 0, d_1d_3 > 0$, *and* $\bar{I} < \min\left\{\frac{\mu_4}{bn}, \frac{\mu_4 - am(\omega - \mu_2)}{bn}\right\}$ *the equilibrium point* E_4 *exists if one of the following conditions is satisfied.*
> 1. *If* $d_2^2 < 4d_1d_3$ *then there is no equilibrium point* E_4 *in the interior*
> 2. *If* $d_2^2 = 4d_1d_3$ *then there is one equilibrium point* E_4 *in the interior*
> 3. *If* $d_2^2 > 4d_1d_3$ *then there are two equilibrium points* E_4 *in the interior*

Proof. When $d_2^2 < 4d_1d_3$, the roots of eq. (9) are a pair of complex conjugate numbers. When $d_2^2 = 4d_1d_3$, only $\bar{I} = -\frac{d_2}{2d_1}$ is the root of eq. (9). Since $d_1d_2 < 0$, this root is positive. When $d_2^2 > 4d_1d_3$, we have a pair of real numbers of eq. (9) which clearly positive numbers when $d_1d_3 > 0$ and $d_1d_2 < 0$. □

> **Theorem 7.** *Equilibrium point* E_4 *is locally asymptotically stable if* $m < \frac{n\left(\mu_1 an + \beta\bar{I}an + \beta(\mu_4 - bn\bar{I})\right)}{an(\omega - \mu_2)}$.

Proof. For E_4, the Jacobian matrix is

$$J_{E_4} = \begin{pmatrix} d_{11} & d_{12} & \delta & \frac{-\mu_4 + bn\bar{I}}{a} \\ \beta\bar{I} & 0 & 0 & -n\bar{I} \\ 0 & \omega & -\delta - \mu_3 & 0 \\ d_{41} & d_{42} & 0 & 0 \end{pmatrix},$$

where

$$\begin{aligned} d_{11} &= -\beta\bar{I} - \frac{\beta(\mu_4 - bn\bar{I}) + am(\omega - \mu_2)}{an} - \mu_1, \\ d_{12} &= -\beta\left(\frac{\mu_4 - bn\bar{I}}{am}\right), \\ d_{41} &= \frac{\beta(\mu_4 - bn\bar{I}) - am(\omega - \mu_2)}{n}, \end{aligned}$$

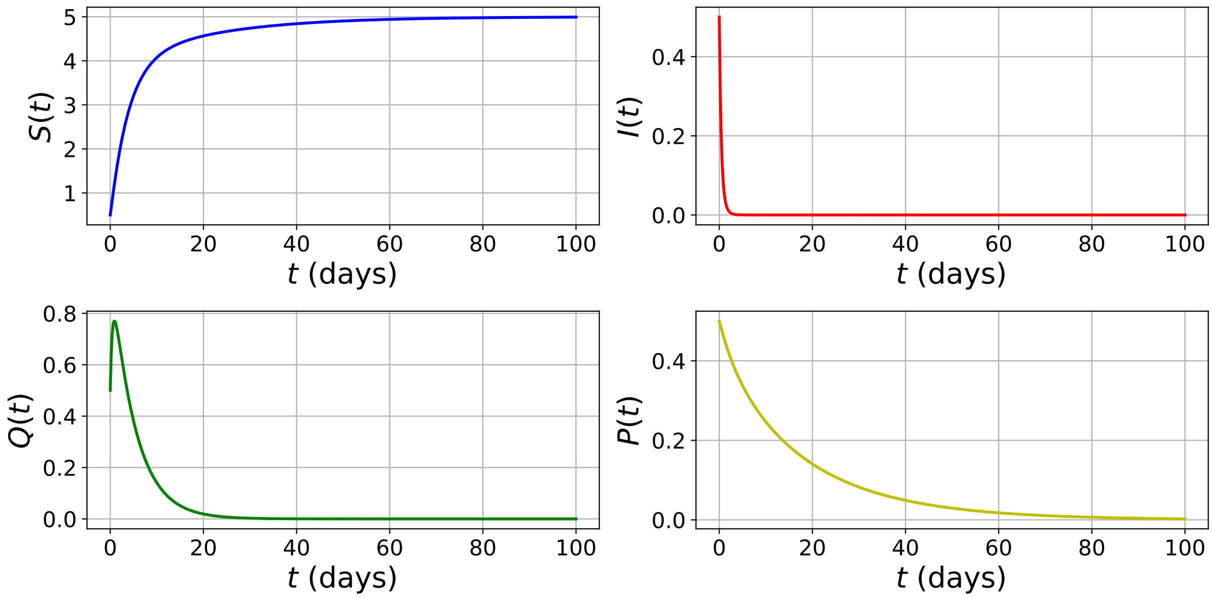

Figure 2. equilibrium point simulation E_1

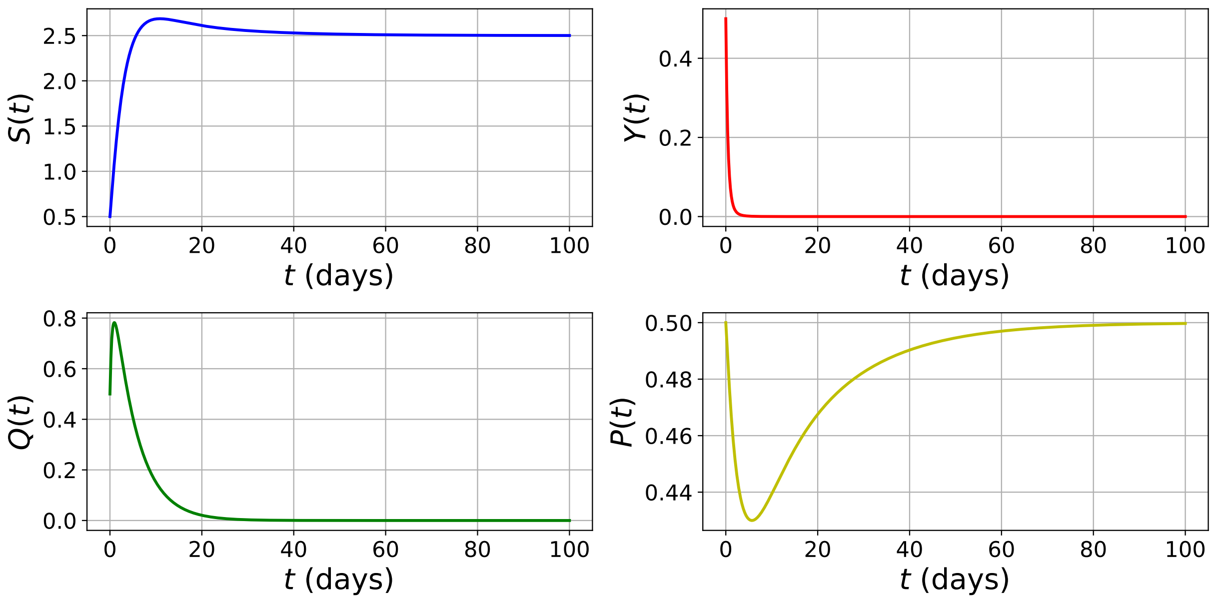

Figure 3. equilibrium point simulation E_2

$$d_{42} = b\left(\frac{\beta(\mu_4 - bn\bar{I}) - am(\omega - \mu_2)}{am}\right),$$

It is easy to confirm that all real parts of eigenvalues are negative when the necessary condition $m < \dfrac{n\left(\mu_1 an + \beta\bar{I}an + \beta(\mu_4 - bn\bar{I})\right)}{an(\omega - \mu_2)}$ is satisfied. □

4. Numerical Simulation

In this section, a numerical simulation will be carried out to see the dynamics of the Equation around the equilibrium points $E_1, E_2, E_3, danE_4$ which have been analyzed previously. This numerical simulation was carried out using the Runge-Kutta method of order 4 with the help of Python 3.8 software

4.1. Dynamics around the equilibrium point E_1

The parameter values used are $\Pi = 1, \beta = 0.4, \delta = 0.1, \omega = 0.2, \mu_1 = 0.2, \mu_2 = 0.4, \mu_3 = 0.1, \mu_4 = 0.1, a = 0.1, b = 0.1, m = 0.1, n = 0.1$, then get the eigenvalues -0.2, -0.2, -0.4, -0.05 because all of them are negative then the value is stable. Furthermore, the initial values for each population are presented in Table 1. From the values given to the parameters and initial values, the simulation shown in Figure 2 which shows that $E_1 = (5, 0, 0, 0)$ is stable or locally asymptotically stable and

Table 1. Initial value of equilibrium point E_1

z0	0.5	0.5	0.5	0.5
z10	0.1	0.5	0.1	0.5
z20	0.5	0.35	0.5	0.45
z30	0.1	0.35	0.1	0.35

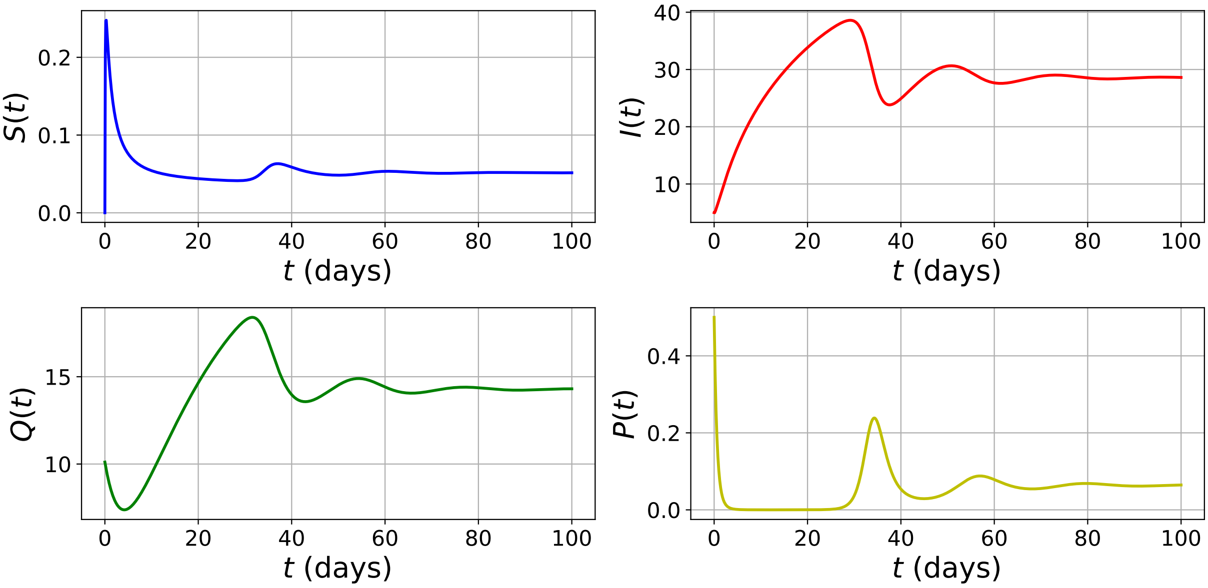

Figure 4. equilibrium point simulation E_4

has only one equilibrium point. Where it is seen that for each predetermined initial value, the solution goes to the equilibrium point E_1. This condition explains Healthy prey populations will be maintained and sick prey populations, quarantined prey populations, and predator populations will become extinct.

4.2. *Dynamics around the equilibrium point E_2*

The parameter values used are $\Pi = 1, \beta = 0.8, \delta = 0.1, \omega = 2, \mu_1 = 0.2, \mu_2 = 0.4, \mu_3 = 0.1, \mu_4 = 0.1, a = 0.1, b = 0.1, m = 0.4, n = 0.4$, then get the eigenvalues -0.341, -0.058, -0.2, -0.6 because all of them are negative then the value is stable. Furthermore, the initial values for each population are presented in Table 2. From the values given to the parameters and initial

Table 2. Initial value of equilibrium point E_2

z0	0.5	0.5	0.5	0.5
z10	0.1	0.5	0.1	0.5
z20	0.5	0.35	0.5	0.45
z30	0.1	0.35	0.1	0.35

values, the simulation shown in the Figure 3. Figure 3 shows that $E_2 = (2.5, 0, 0, 0.5)$ has three equilibrium points, namely E_1 and E_3 unstable (saddle) , because there is a positive value λ while E_2 is stable or locally asymptotically stable because every predefined initial value, the solution goes to E_2. This condition explains that when the population of sick prey and quarantined sick prey becomes extinct, the ecosystem will only be inhabited by healthy prey and predator populations.

4.3. *Dynamics around the equilibrium point E_3*

The parameter values used are $\Pi = 1, \beta = 0.8, \delta = 0.1, \omega = 2, \mu_1 = 0.2, \mu_2 = 0.4, \mu_3 = 0.1, \mu_4 = 0.1, a = 0.1, b = 0.1, m = 0.1, n = 0.1$, then get the eigenvalues -0.258, -0.714, -0.110, -0.067 because all of them are negative then the value is stable. Furthermore, the initial values for each population are presented in Table 3. From the values given to the parameters and initial values, the simulation shown in the following figure is

Table 3. Initial value of equilibrium point E_3

z0	0.5	0.5	0.5	0.5
z10	0.1	0.5	0.1	0.5
z20	0.5	0.35	0.5	0.45
z30	0.1	0.35	0.1	0.35

obtained.

Figure 5 shows that $E_3 = (3, 0.285, 2.857, 0)$ has two equilibrium points,E_1 is unstable (saddle) because there is a positive value of λ while E_3 is stable or locally asymptotically stable because each initial value has been determined, the solution goes to the equilibrium point E_3.This condition explains that the ecosystem will only be inhabited by healthy prey populations, sick prey, and sick prey which are quarantined when the predator population becomes extinct.

4.4. *Dynamics around the equilibrium point E_4*

The parameter values used are $\Pi = 3, \beta = 3, \delta = 0.1, \omega = 0.1, \mu_1 = 0.2, \mu_2 = 0.01, \mu_3 = 0.1, \mu_4 = 2, a = 0.1, b = 0.1, m = 0.04, n = 0.7$, then get the eigenvalues -85.745, -0.066, -0.276, -0.221 because all of them are negative then the value is stable. Furthermore, the initial values for the population z0 are 0.00008, 5, 10.11, 0.5. From the values given to the parameters and initial values, the simulation shown in Figure 4 which shows that $E_4 = (0.051, 28.564, 14.282, 0.063)$ has three equilibrium points with E_1 and E_3 unstable (saddle) , because there is a positive value of λ. Equilibrium point $E_4$is stable or locally asymptotically stable because for each predetermined initial value, the solution goes to the equilibrium point E_4. This condition explains that the populations of healthy prey, sick prey, quarantined sick prey and predators will not become extinct , because the four populations will depend on each other so that no population will become extinct and no population will grow excessively.

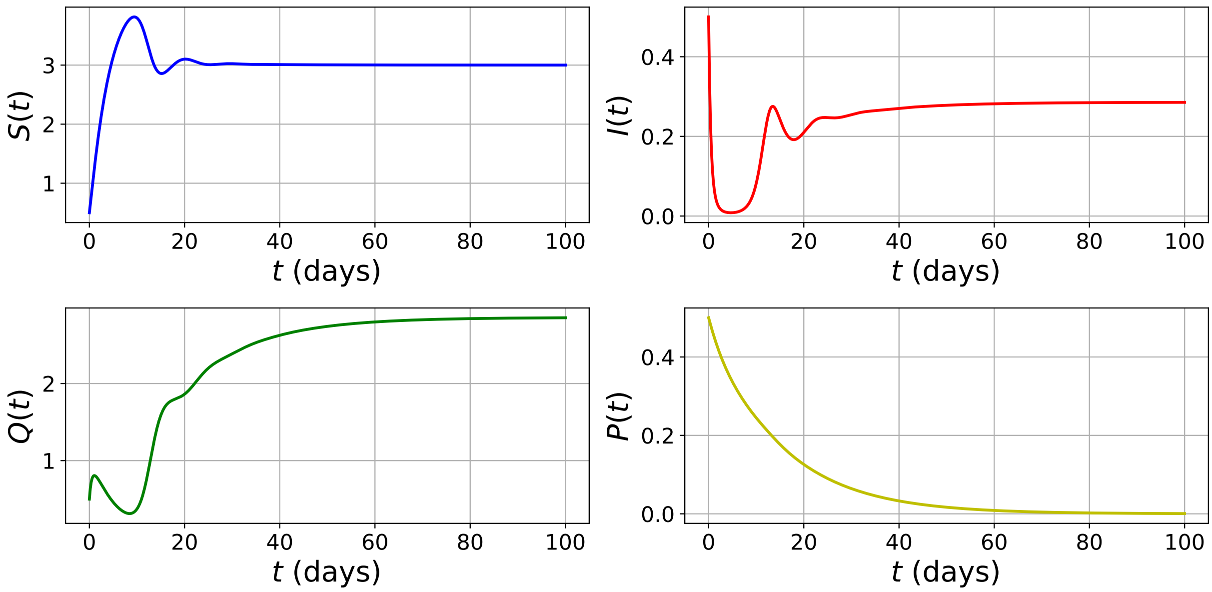

Figure 5. equilibrium point simulation E_3

5. Conclusion

This study discusses the dynamics of the eco-epidemiological model with quarantine in prey populations. This model was modified by adding a quarantined prey population. Therefore, to prevent an imbalance in an ecosystem, quarantine is given to a sick prey population, then the prey that has been successfully treated can be released back into the wild. the only population that is unlikely to become extinct while the other equilibrium points have the conditions for existence. Numerical simulations were carried out to confirm the analysis results. It can be interpreted that this model guarantees that each population will survive with the number of their respective populations of both prey and predators going to their equilibrium values.

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