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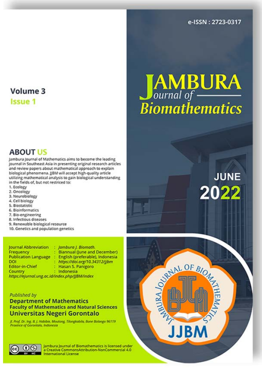

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Mathematical model of Dynamics of Covid-19 Transmission in Response to Reinfection

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ABSTRACT. *SARS-CoV-2 brings on the pandemic known as Coronavirus Disease 2019 (Covid-19). The Covid-19 spread model developed by Bugalia et al.(2020) has been modified in this study by incorporating reinfection and covid-19-related death during medical isolation. There are two equilibrium points in this model: the point of equilibrium without disease and the point of equilibrium with the disease. In addition, the equilibrium point's stability and the basic reproduction number ($\mathcal{R}_0$) will be discussed. A sensitivity analysis based on Covid-19 data from India was carried out to identify sensitive parameters. Lockdown's effectiveness is one of the sensitivity analysis parameters that have the greatest impact on changes in $\mathcal{R}_0$.*

1. Introduction

The rapidly spreading pandemic known as Coronavirus Disease 2019 or Covid-19 is caused by SARS-CoV-2 [1]. In December 2019, Covid-19 was first detected in Wuhan, China [2, 3]. Covid-19 is a disease that can only be passed from one person to another through direct contact, surfaces, or respiratory droplets inhaled by an infected person, regardless of whether they exhibit clinical symptoms [4]. It should be noted that individuals may reinfect Covid-19 even after being declared recovered from Covid-19. Reinfection is when the body is again infected with the same virus or pathogen or a different pathogen [5]. Most reinfected persons are asymptomatic or have mild to moderate respiratory symptoms [6]. In a public hospital in Wuhan, China, around 2.4% of recovered people tested positive for Covid-19 and showed no symptoms of infection (asymptomatic) [7]. Therefore, Covid-19 is an epidemic that must stop its spread immediately. There are two approaches to Covid-19 prevention: medical treatment and not receiving any medical treatment. Vaccination is the only medical method to prevent Covid-19 infection, but vaccines are still ineffective. This demonstrates that despite the availability of numerous vaccines, Covid-19 has yet to be contained. As a result, non-medical strategies like lockdowns, social distancing, wearing masks in public places, contact tracing, isolation, and quarantine must be implemented [8, 9]. To prevent the disease, infected individuals can be separated from the general population through isolation, and quarantine [10]. In most cases, people who appear healthy but may be infected are placed in quarantine; individuals infected with Covid-19 are placed in isolation [11]. Several nations have implemented a lockdown policy to control the Covid-19 outbreak. Wuhan's local government initiated this lockdown implementation strategy on January 23, 2020, and other cities in the Hubei province quickly adopted it [12]. It has been reported that the number of daily cases of Covid-19 in China has decreased significantly since the lockdown policy was implemented. [13]. The death rate was lower in Germany, New Zealand, Canada, Norway, and other early-pandemic countries [14]. According to reports, a lockdown policy can downsize the number of confirmed cases and deaths in the United States by more than 60% [15]. Instead of using lockdowns to combat the spread of Covid-19, the Indonesian government implements Large-Scale Social Restrictions (PSBB) in stages per a law passed on March 31, 2020. This intervention establishes that the relevant ministries must approve the regional government's PSBB requests [16, 17]. This policy is unique worldwide because it is not the central government that makes decisions, but it is up to the regions to apply for PSBB. Another policy implemented after the PSBB was the "Implementation of Restrictions on Community Activities" (PPKM) which has been in effect since early 2021. These two policies show that the transmission of Covid-19 in Indonesia can be handled. Based on the research that Bugalia et al. [10] have done, this research will change the model of how Covid-19 spreads. It will assume that people who have been cured can get Covid-19 again. The modified model will then undergo an equilibrium point analysis and numerical simulation to examine the behavior and dynamics of the spread of Covid-19 in the presence of various strategies, such as a lockdown.

2. Methods

In this study, a modification of the Covid-19 distribution model will be carried out, which refers to the main model studied by Bugalia et al. [10], i.e., SQ_1AQ_2IMR model, which the population is divided into seven compartments: Susceptible individuals (S), compartment of quarantined susceptible individuals (Q_1), compartment of asymptomatic individuals (A), compartment of self-quarantined individuals who are categorized as

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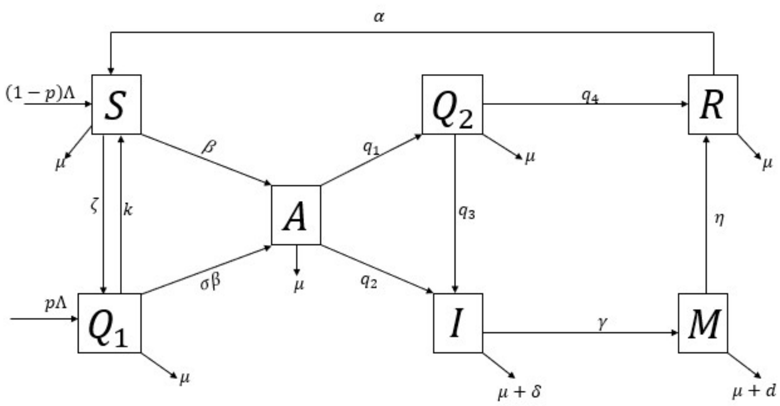


Figure 1. Schematic Diagram of the Model System

asymptomatic individuals (Q_2), compartment of symptomatic individuals (I), compartment of medically isolated individuals (M), and compartment of recovered individuals (R). The assumptions that apply in this modified model are as follows.

1. Due to the lockdown policy, a certain proportion of newly recruited individuals were quarantined. Also, some susceptible individuals were being quarantined (those in compartment Q_1). Where $1 - \sigma$ describes lockdown effectiveness [10].
2. After contact with infected individuals, individuals can get the virus again because they don't get complete immunity or their immunity wears out after a particular time. at a rate of α [18–20].
3. Infected individuals in medical care could die at a rate d [5, 21].

Schematically, the pattern of the modification model for the Covid-19 transmission is depicted in the compartment diagram in Figure 1. The model is given in the following system of equations.

$$\begin{aligned}
\frac{dS}{dt} &= (1-p)\Lambda - \beta S(A+I) - \mu S - kS + \zeta Q_1 + \alpha R,\\
\frac{dQ_1}{dt} &= p\Lambda - \sigma\beta Q_1(A+I) + kS - (\mu+\zeta)Q_1,\\
\frac{dA}{dt} &= \beta S(A+I) + \sigma\beta Q_1(A+I) - (q_1+q_2+\mu)A,\\
\frac{dQ_2}{dt} &= q_1 A - (q_3+q_4+\mu)Q_2,\\
\frac{dI}{dt} &= q_3 Q_2 + q_2 A - (\delta+\gamma+\mu)I,\\
\frac{dM}{dt} &= \gamma I - (\eta+d+\mu)M,\\
\frac{dR}{dt} &= q_4 Q_2 + \eta M - (\alpha+\mu)R,
\end{aligned} \tag{1}$$

with nonnegative initial conditions $S(0) \geqslant 0$, $Q_1(0) \geqslant 0$, $A(0) \geqslant 0$, $Q_2(0) \geqslant 0$, $I(0) \geqslant 0$, $M(0) \geqslant 0$, $R(0) \geqslant 0$, and $N(0) > 0$. It is assumed that all the parameters are nonnegative. Table 1 displays the biological interpretations of the system's parameters (1).

3. Results and Discussion

3.1. Non-Negativity and Boundedness of the Solution for the Model (1)

Since the model (1) represents the different human populations, all the variables are non-negative for all $t \geq 0$.

> **Theorem 1.** *Solutions of the model (1) with positive initial data will remain positive for all time $t \geq 0$ and the biologically feasible region $\Omega = (S, Q_1, A, Q_2, I, M, R) \in \mathbb{R}^7_+ : 0 < S + Q_1 + A + Q_2 + I + M + R \leq \frac{\Lambda}{\mu}$ is positively invariant fo the system (1).*

Proof. Consider total population $N(t) = S(t) + Q_1(t) + A(t) + Q_2(t) + I(t) + M(t) + R(t)$ are positive. Adding all the equations of the model (1), the total population satisfies the following equations.

$$\begin{gathered}
\tfrac{dN}{dt} = \Lambda - \mu N - dM - \delta I \Rightarrow\\
\Lambda - (\mu + d + \delta)N \leq \tfrac{dN}{dt} \leq \Lambda - \mu N
\end{gathered}$$

Using the initial conditions and integrating the aforementioned inequality, we now obtain

$$\begin{gathered}
\tfrac{\Lambda}{\mu+d+\delta} + (N(0) - \tfrac{\Lambda}{\mu+d+\delta})e^{-(\mu+d+\delta)} \leq N(t) \leq\\
\tfrac{\Lambda}{\mu} + (N(0) - \tfrac{\Lambda}{\mu})e^{-(\mu)}
\end{gathered}$$

considering $t \to \infty$ or $N(t)$ approaches $\frac{\Lambda}{\mu}$ asymptotically

$$\tfrac{\Lambda}{\mu+d+\delta} \leq N(t) \leq \tfrac{\Lambda}{\mu}$$

Thus, the region Ω is positively-invariant so that no solution path moves beyond the boundary of Ω. □

3.2. Equilibrium Point

The videlicet disease-free equilibrium point (DFE) and the endemic equilibrium point (EEP) are the two equilibriums that are reached by the system of equation eq. (1). The following is how the disease-free equilibrium point is found.

$$\begin{aligned}
E_1&(S, Q_1, A, Q_2, I, M, R)\\
&= (S^0, Q_1^0, A^0, Q_2^0, I^0, M^0, R^0)\\
&= \frac{\Lambda(\zeta + \mu(1-p)}{\mu(k+\zeta+\mu)}, \frac{\Lambda(k+p+\mu}{\mu(k+\zeta+\mu)}, 0, 0, 0, 0, 0),
\end{aligned}$$

Table 1. Parameters with Biological Interpretations

Parameter	Biological interpretation	Unit
Λ	The new recruitment of susceptible individuals	$individual \times times^{-1}$
p	Proportion of individuals those are in quarantine due to lockdown	
β	The transmission rate of infected individuals	$(individual \times times)^{-1}$
σ	Scaling factor that describes the percentage of lockdown effectiveness, where $1-\sigma$ describes lockdown effectiveness	
μ	The natural death rate	$times^{-1}$
k	The rate of transmission by which susceptible individuals move into quarantine class	$times^{-1}$
ζ	The rate of transmission of quarantine (Q_1) individuals to the susceptible compartment (S)	$times^{-1}$
q_1	Rate of asymptomatic individuals to self- quarantine	$times^{-1}$
q_2	The rate of asymptomatic individuals showing symptoms	$times^{-1}$
q_3	The rate by which the infected individuals come from self- quarantine class	$times^{-1}$
q_4	Self-quarantine individual recovery rate	$times^{-1}$
δ	Rate of individual death due to Covid-19 without medical isolation	$times^{-1}$
d	Rate of individual death due to Covid-19 when the individual is in medical isolation	$times^{-1}$
γ	Medically isolated rates of symptomatically infected individuals	$times^{-1}$
η	The rate of infection recovery following medical isolation	$times^{-1}$
α	Reinfection rate of recovered individuals	$times^{-1}$

while the endemic equilibrium point is the point obtained when a disease in a population has not completely disappeared as follows.

$$E_2(S,Q_1,A,Q_2,I,M,R)=(S^*,Q_1^*,A^*,Q_2^*,I^*,M^*,R^*)$$

where

$$\begin{aligned}
S^* &= \frac{R^*\alpha+Q_1^*\zeta+\Lambda(1-p)}{k+(A^*+I^*)\beta+\mu},\\
Q_1^* &= \frac{kS^*+p\Lambda}{\zeta+\mu+(A^*+I^*)\beta\sigma},\\
A^* &= \frac{\beta I^*(S^*+\sigma Q_1^*)}{q_1+q_2+\mu-\beta(S^*+\sigma Q_1^*)},\\
Q_2^* &= \frac{A^*q_1}{q_3+q_4+\mu},\\
I^* &= \frac{A^*q_2+Q_2^*q_3}{\gamma+\delta+\mu},\\
M^* &= \frac{I^*\gamma}{d+\eta+\mu},\\
R^* &= \frac{Q_2^*q_4+M^*\eta}{\alpha+\mu}.
\end{aligned}$$

3.3. Basic Reproduction number

A basic reproduction number is a number that indicates the number of susceptible individuals who can suffer from a disease caused by one infected individual. The next-generation matrix can be used to determine the basic reproduction number.

$$\mathcal{R}_0=\rho(FV^{-1}),$$

where ρ is the dominant eigen value [22].

$$F=\begin{bmatrix}\beta(S^0+\sigma Q_1^0) & 0 & \beta(S^0+\sigma Q_1^0) & 0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\end{bmatrix},$$

$$V=\begin{bmatrix}q_1+q_2+\mu & 0 & 0 & 0\\ -q_1 & q_3+q_4+\mu & 0 & 0\\ -q_2 & -q_3 & \delta+\gamma+\mu & 0\\ 0 & 0 & -\gamma & \eta+d+\mu\end{bmatrix}.$$

The fundamental reproduction number, or ($\mathcal{R}_0$), is then derived as follows.

$$\mathcal{R}_0=\frac{\beta\Lambda(q_1q_3+(q_3+q_4+\mu)(q_2+\gamma+\delta+\mu))(\zeta+\mu(1-p)+(p\mu+k)\sigma)}{\mu(q_1+q_2+\mu)(q_3+q_4+\mu)(\gamma+\delta+\mu)(k+\zeta+\mu)}$$

4. Stability Analysis

DFE Stability Analysis

Theorem 2. *If $\mathcal{R}_0$ is less than 1, DFE (E_1) is locally asymptotically stable, while $\mathcal{R}_0$ is greater than 1.*

Proof. The following formula can be used to demonstrate that the eigenvalues of the corresponding Jacobi matrix (J) only have a non-positive real part, demonstrating the local stability of DFE.

$$|J-\lambda I|=0$$

where λ is eigenvalue of J [23].

$$\begin{vmatrix}
j_{11}-\lambda I & j_{12} & j_{13} & 0 & j_{15} & 0 & j_{17}\\
j_{21} & j_{22}-\lambda I & j_{23} & 0 & j_{25} & 0 & 0\\
0 & 0 & j_{33}-\lambda I & 0 & j_{35} & 0 & 0\\
0 & 0 & j_{43} & j_{44}-\lambda I & 0 & 0 & 0\\
0 & 0 & j_{53} & j_{54} & j_{55}-\lambda I & 0 & 0\\
0 & 0 & 0 & 0 & j_{65} & j_{66}-\lambda I & 0\\
0 & 0 & 0 & j_{74} & 0 & j_{76} & j_{77}-\lambda I
\end{vmatrix}=0$$

where

$$\begin{aligned}
j_{11} &= -k-\mu, & j_{12} &= \zeta,\\
j_{13} &= -\beta S^0, & j_{15} &= -\beta S^0,\\
j_{17} &= \alpha, & j_{21} &= k,\\
j_{22} &= -(\zeta+\mu), & j_{23} &= -\beta\sigma Q_1^0,\\
j_{25} &= -\beta\sigma Q_1^0, & j_{33} &= -q_1-q_2-\mu+\beta(S^0+\sigma Q_1^0),\\
j_{35} &= \beta(S^0+\sigma Q_1^0), & j_{43} &= q_1,\\
j_{44} &= -q_3-q_4-\mu, & j_{53} &= q_2,\\
j_{54} &= q_3, & j_{55} &= -(\gamma+\delta+\mu),
\end{aligned}$$

$$j_{65} = \gamma, \qquad j_{66} = -(d+\eta+\mu),$$
$$j_{74} = q_4, \qquad j_{76} = \eta,$$
$$j_{77} = -(\alpha+\mu),$$

obtained seven eigenvalues, four negative eigenvalues: $\lambda_1 = -k-\zeta-\mu$, $\lambda_2 = -\mu$, $\lambda_3 = -\alpha-\mu$, and $\lambda_4 = -d-\eta-\mu$, while the other three eigenvalues are obtained by solving the following equation.

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0,$$

where

$$\begin{aligned}
C_1 &= q_1+q_2+q_3+q_4+2\mu \\
&\quad + (\gamma+\delta+\mu)\left(1-\mathcal{R}_0\tfrac{(q_1+q_2+\mu)(q_3+q_4+\mu)}{(q_1q_3+(q_3+q_4+\mu)(q_2+\gamma+\delta+\mu))}\right) \\
C_2 &= (q_3+q_4+\mu)(\gamma+\delta+\mu) \\
&\quad \left(1-\mathcal{R}_0\tfrac{(q_1+q_2+\mu)(q_2+q_3+q_4+\gamma+\delta+2\mu)}{(q_1q_3+(q_3+q_4+\mu)(q_2+\gamma+\delta+\mu))}\right) \\
&\quad + (q_1+q_2+\mu)(q_2+q_3+q_4+\gamma+\delta+2\mu) \\
C_3 &= (q_1+q_2+\mu)(q_3+q_4+\mu)(\gamma+\delta+\mu)(1-\mathcal{R}_0).
\end{aligned}$$

The Routh-Hurwitz stability criterion, as in [24], is also required because the analysis for determining the model's stability in this study is extremely complex if only eigenvalues are used. The Routh-Hurwitz stability criterion, C_1, C_2, and C_3 are positeve, and $C_1C_2 > C_3$ when $\mathcal{R}_0 < 1$, were found to be satisfied by E_1. The eigenvalues of the matrix J have real negative parts with the conditions $\mathcal{R}_0 < 1$ because each parameter is positive and meets the Routh-Hurwitz criteria. Therefore, E_1 will be unstable if $\mathcal{R}_0 > 1$ and local asymptotically stable if $\mathcal{R}_0 < 1$. □

EEP Stability Analysis

Theorem 3. *if $\mathcal{R}_0 > 1$, then EEP (E_2) is locally asymptotically stable.*

Proof. Using the theorem in [25], endemic equilibrium point stability analysis was carried out. Choose β as the bifurcation parameter, then setting $\mathcal{R}_0 = 1$ gives

$$\begin{aligned}
\beta &= \beta^* \\
&= \frac{\mu(q_1+q_2+\mu)(q_3+q_4+\mu)(\gamma+\delta+\mu)(k+\zeta+\mu)}{(\Lambda(q_1q_3+(q_3+q_4+\mu)(q_2+\gamma+\delta+\mu))(\zeta+(\zeta+\mu(1-p))+p\mu\sigma)+k\sigma)}.
\end{aligned}$$

Equilibrium point E_1 has one zero eigenvalue at $\mathcal{R}_0 = 1$. The zero eigenvalue has a left eigenvectors $(w_1, w_2, w_3, w_4, w_5, w_6, w_7)$ and right eigenvectors $(v_1, v_2, v_3, v_4, v_5, v_6, v_7)$ as follows.

$$\begin{aligned}
w_1 &= \tfrac{q_1q_4\alpha\mu(\zeta+\mu)(k+\zeta+\mu)}{\mu^2(k+\zeta+\mu)^2(q_3+q_4+\mu)(\alpha+\mu)} + \tfrac{\alpha\gamma\eta\mu(\zeta+\mu)(k+\zeta+\mu)}{\mu^2(k+\zeta+\mu)^2(\alpha+\mu)(d+\eta+\mu)} \\
&\quad - \tfrac{\beta\Lambda((\zeta+\mu)(\zeta+\mu(1-p))+\zeta(k+p\mu)\sigma)(1+w_5)}{\mu^2(k+\zeta+\mu)^2}, \\
w_2 &= \tfrac{k\mu(k+\zeta+\mu)w_1-\beta\sigma\Lambda(k+p\mu)(1+w_5)}{\mu(k+\zeta+\mu)(\zeta+\mu)}, \\
w_3 &= 1, \\
w_4 &= \tfrac{q_1}{q_3+q_4+\mu}, \\
w_5 &= \tfrac{(q_3+q_4+\mu)q_2+q_3q_1}{(\gamma+\delta+mu)(q_3+q_4+\mu)}, \\
w_6 &= \tfrac{\gamma((q_3+q_4+\mu)q_2+q_3q_1)}{(\gamma+\delta+\mu)(q_3+q_4+\mu)(d+\eta+\mu)}, \\
w_7 &= \tfrac{w_3(q_1q_3\gamma\eta+q_2\gamma\eta(q_3+q_4+\mu)+dq_1q_4(\gamma+\delta+\mu)+q_1q_4(\gamma+\delta+\mu)(\eta+\mu))}{(q_3+q_4+\mu)(\alpha+\mu)(\gamma+\delta+\mu)(d+\eta+\mu)}, \\
v_1 &= \tfrac{\alpha+\mu}{\alpha}, \\
v_2 &= \tfrac{(k+\mu)(\alpha+\mu)}{k\alpha}, \\
v_3 &= \tfrac{\beta\Lambda((\zeta+\mu(1-p))v_1+\sigma(k+p\mu)v_2)+\mu(k+\zeta+\mu)((\gamma+\delta+\mu)v_5-\gamma v_6)}{\beta\sigma(\zeta+\mu(1-p)+p\mu\sigma+k\sigma)}, \\
v_4 &= \tfrac{q_3v_5+q_4v_7}{q_3+q_4+\mu}, \\
v_5 &= 1, \\
v_6 &= \tfrac{\eta v_7}{d+\eta+\mu}, \\
v_7 &= 1.
\end{aligned}$$

The second-order partial derivative of the function g, evaluated at E_1, can then be determined by selecting the function $g, x_1 = S, x_2 = Q_1, x_3 = A, x_4 = Q_2, x_5 = I, x_6 = M, x_7 = R$. So that a and b are obtained as follows.

$$\begin{aligned}
a &= 2\beta(w_3+w_5)(w_1(v_3-v_1) \\
&\quad + w_2\sigma(v_3-v_2)), \\
b &= 2(w_3+w_5)((\tfrac{\Lambda(\zeta+\mu(1-p))}{\mu(k+\zeta+\mu)})(v_3-v_1) \\
&\quad + (\tfrac{\Lambda(k+p\mu)\sigma}{\mu(k+\zeta+\mu)})(v_3-v_2)).
\end{aligned}$$

Since the coefficient b is positive and $a > 0$, if

$$\begin{aligned}
\tfrac{(\alpha\mu)}{(\alpha+\mu)} &> \tfrac{\beta\Lambda((\zeta+\mu)(\zeta+\mu(1-p))+\zeta(k+p\mu)\sigma)(d+\eta+\mu)}{\gamma\eta(\zeta+\mu)(k+\zeta+\mu)}, \text{ and} \\
\tfrac{(\alpha\mu)}{(\alpha+\mu)} &> \tfrac{\beta\Lambda((\zeta+\mu)(\zeta+\mu(1-p))+\zeta(k+p\mu)\sigma)(d+\eta+\mu)}{q_1q_4(\zeta+\mu)(k+\zeta+\mu)},
\end{aligned} \tag{2}$$

so the following conclusions are established.

Theorem 4. *The system of equation (1) undergoes backward branching at $\mathcal{R}_0 = 1$ because of inequality (2) being fulfilled.*

It should be noted that for cases where individuals who have recovered do not experience reinfection ($\alpha = 0$), then $w_1 < 0$ and $w_2 < 0$. So, the coefficient a becomes

$$a = 2\beta(w_3+w_5)(w_1(v_3-v_1)+w_2\sigma(v_3-v_2)) < 0.$$

In other words, this study demonstrates that reinfection is the cause of the backward bifurcation of the equations (1) system because if a=0, the system will not undergo a backward bifurcation. A transcritical bifurcation occurs at $\mathcal{R}_0 = 1$ whenever $\alpha = 0$ because $a < 0$ and $b > 0$ at $\beta = \beta^*$. If $\mathcal{R}_0$ is greater than 1, then EEP at $\alpha = 0$ is locally asymptotically stable. □

5. Sensitivity Analysis

This sensitivity analysis aims to ascertain how each parameter affects $\mathcal{R}_0$. It should be noted that this parameter has the greatest impact when the sensitivity index value is higher. The relationship between these parameters and the analyzed variables, specifically the basic reproduction number, is shown by positive and negative signs. The analysis occurs when the condition $\mathcal{R}_0 > 1$ indicates that Covid-19 has spread. A parameter sensitivity analysis should be conducted to determine the steps to stop the spread of Covid-19.

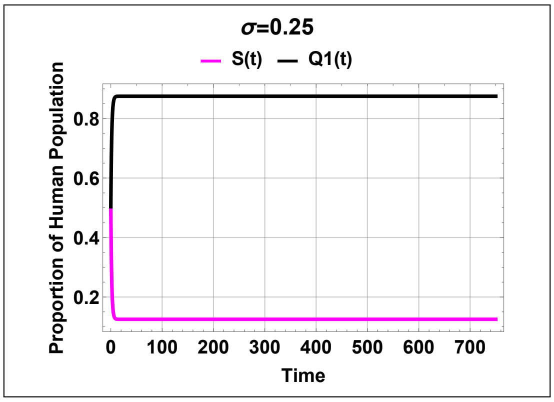
This research is a deterministic analysis of local stability, so the sensitivity analysis is carried out by local analysis. However,

Table 2. Estimated Parameters

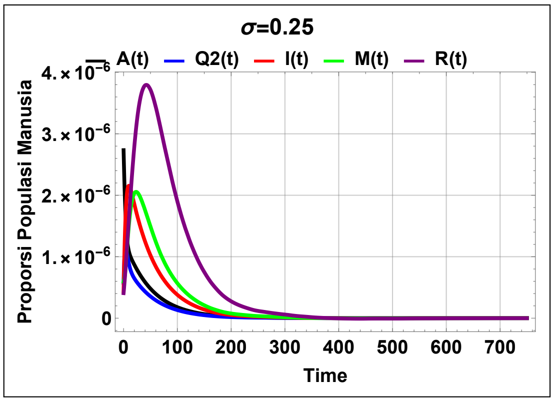
Parameter	Value	Reference	Parameter	Value	Reference
Λ	$1/(365 \times 65)$	Bugalia *et al.* (2020)[10]	q_2	$1/7$	Bugalia *et al.* (2020)[10]
p	0.5	Bugalia *et al.* (2020)[10]	q_3	0.2108	Bugalia *et al.* (2020)[10]
β	0.3004	Bugalia *et al.* (2020)[10]	q_4	0.08	Bugalia *et al.* (2020)[10]
σ	0.5	Bugalia *et al.* (2020)[10]	δ	0.05	Bugalia *et al.* (2020)[10]
μ	$1/(365 \times 65)$	Bugalia *et al.* (2020)[10]	d	0.00145	Rai *et al.* (2020)[21]
k	0.5	Bugalia *et al.* (2020)[10]	γ	0.11	Bugalia *et al.* (2020)[10]
ζ	$1/14$	Bugalia *et al.* (2020)[10]	η	0.0917	Bugalia *et al.* (2020)[10]
q_1	0.2	Bugalia *et al.* (2020)[10]	α	0.05	Rai *et al.* (2020)[21]

Table 3. Sensitivity Index of each Parameter

Parameter	Sensitivity Index Value	Parameter	Sensitivity Index Value
Λ	1	p	0
β	1	σ	0.78
μ	-1.00031	k	-0.0972
ζ	0.0972	q_1	-0.2595
q_2	-0.0976	q_3	0.089
q_4	-0.089	δ	-0.2008
d	0	γ	-0.442
η	0	α	0



(a)



(b)

Figure 2. The relationship of $\sigma = 0.25$ to $\mathcal{R}_0$

the global analysis, including Latin Hypercube Sampling (LHS) and Partial Rank Correlation Coefficient (PRCC), were beyond the scope of this study.

The sensitivity index is calculated using partial derivatives, as shown below:

$$\Upsilon_{\ell}^{R_0} = \frac{\partial \mathcal{R}_0}{\partial \ell} \times \frac{\ell}{\mathcal{R}_0},$$

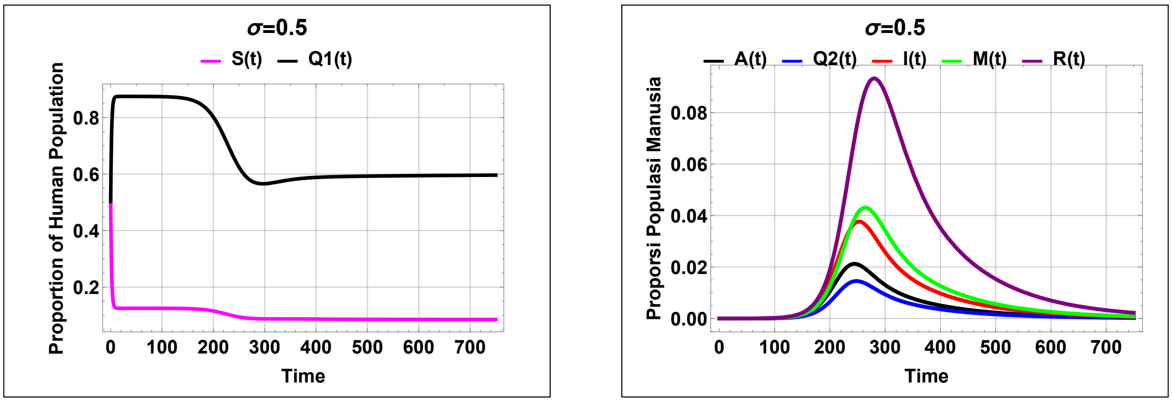
where the changing value of parameter ℓ affects the basic reproduction number $\mathcal{R}_0$ [26]. Table 2 shows the estimated parameter values for Covid-19 cases in India. As an example, the sensitivity index of $\mathcal{R}_0$ towards parameter σ, γ and α is

$$\begin{aligned}
\Upsilon_{\sigma}^{R_0} &= \frac{(\partial \mathcal{R}_0)}{\partial \sigma} \times \frac{\sigma}{\mathcal{R}_0} \\
&= \frac{((k+p\mu)\sigma)}{(\zeta+\mu+p\mu(-1+\sigma)+k\sigma)} = 0.78, \\
\Upsilon_{\gamma}^{R_0} &= \frac{(\partial \mathcal{R}_0)}{\partial \gamma} \times \frac{\gamma}{\mathcal{R}_0} \\
&= -\frac{\gamma(q_1q_3+q_2(q_3+q_4+\mu))}{(\gamma+\delta+\mu)(q_1q_3+(q_3+q_4+\mu)(q_2+\gamma+\delta+\mu))} \\
&= -0.442,
\end{aligned}$$

$$\Upsilon_{\alpha}^{R_0} = \frac{(\partial \mathcal{R}_0)}{\partial \alpha} \times \frac{\alpha}{\mathcal{R}_0} = 0$$

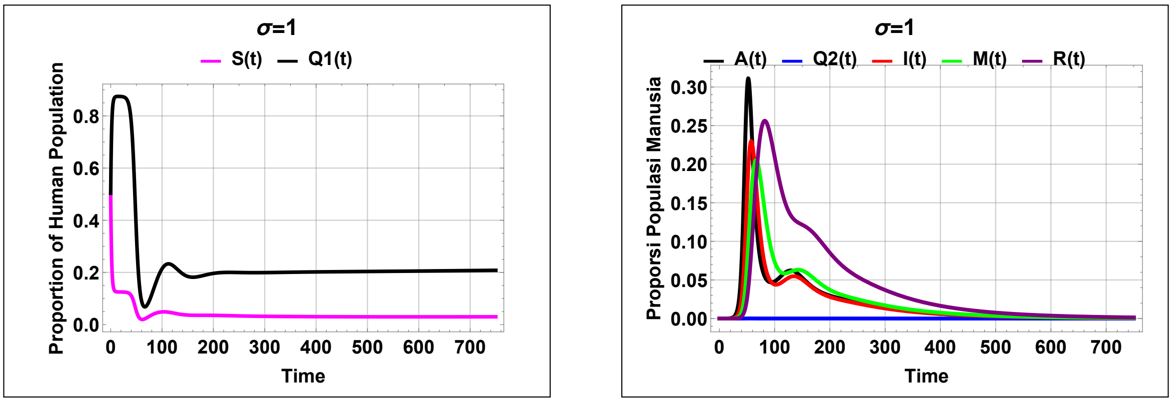
Table 3 is the result of the sensitivity value at $\mathcal{R}_0$. According to Table 3, the parameters Λ, β, σ and μ have a significant impact on $\mathcal{R}_0$. The fact that the sensitivity index displays a positive sign indicates that an increase in the parameter will increase $\mathcal{R}_0$. On the other hand, a negative sign on the sensitivity index indicates that an increase in the parameter will decrease $\mathcal{R}_0$. For instance, the sensitivity index of γ is = -0.442, indicating that increasing the parameter value by 10% will decrease the $\mathcal{R}_0$ by 4.42%, and vice versa. similar explanation for $\Upsilon_{\sigma}^{R_0} = 0.78$ indicates that increasing the parameter value by 10% will increase the $\mathcal{R}_0$ value by 7.8%, and vice versa.

According to the sensitivity analysis, one of the parameters most sensitive to changes in the basic reproduction number is the scaling factor that describes lockdown effectiveness. The Covid-19 transmission rate will decrease as the lockdown's effectiveness increases, lowering the value of the basic reproduction number. As a result, the lockdown can be more effective at stopping the spread of Covid-19.



(a) (b)

Figure 3. The relationship of σ to $\mathcal{R}_0$



(a) (b)

Figure 4. The relationship of σ to $\mathcal{R}_0$

6. Numerical Simulation

Numerical simulations are expected to provide an overview of the influence of the spread of Covid-19. The calculation process is carried out using Mathematica 11.2 with a long observation of up to 750 days. The proportion of the initial population taken is as follows.

$$\begin{aligned} S(0) &= 0.496, & Q_1(0) &= 0.5, \\ A(0) &= 0.0000023, & Q_2(0) &= 0.00000058, \\ I(0) &= 0.00000043, & M(0) &= 0.00000059, \\ R(0) &= 0.00000041 \end{aligned}$$

A simulation involving varying the lockdown's effectiveness will be carried out to determine how the spread of Covid-19 is affected by lockdown effectiveness $(1-\sigma)$.

1. $\sigma = 0.25$
   In this case, it describes the condition when the effectiveness of the lockdown was 75%. In this case $\mathcal{R}_0$=0.843 with the equilibrium point $E_1(0.125, 0.875, 0, 0, 0, 0, 0)$, it is obtained that the behavior at this equilibrium point is asymptotically stable. Figure 2 shows the time series of impact of $\sigma = 0.25$ on the value of $\mathcal{R}_0$. It demonstrates that at t=300, at a value of =0.25, the number of infected individuals will begin to stabilize. This demonstrates that a lockdown with a 75 percent success rate can stop the spread of Covid-19 and prevent reinfection.
2. $\sigma = 0.5$
   In this case, it describes the condition when the effectiveness of the lockdown was 50%. In this case, there are two equilibrium points, namely $E_1(0.125, 0.875, 0, 0, 0, 0, 0)$ which is unstable and

$$E_2(0.091, 0.634, 0.000124, 0.000085, \\ 0.000223, 0.00026, 0.00062),$$

   is asymptotically stable, with $\mathcal{R}_0$=1.379. Figure 3 shows the time series of the impact of $\sigma = 0.5$ on the value of $\mathcal{R}_0$ which demonstrates that if sigma= 0.5, the susceptible individual (S) will be lower, and the infection rate will initially increase before significantly decreasing, but the population will still contain Covid-19-infected individuals. This indicates that a 50% effective lockdown can prevent reinfection; however, to control the spread of Covid-19, the lockdown's effectiveness must be increased.
3. $\sigma = 1$
   This case describes the conditions when the lockdown was

not implemented. In this case, there are two equilibrium points, namely $E_1(0.125, 0.875, 0, 0, 0, 0, 0)$ which is unstable and

$$E_2(0.051, 0.357, 0.000267, 0.000184, \\ 0.00048, 0.00057, 0.00133),$$

is asymptotically stable, with $\mathcal{R}_0$=2.4515. Figure 4 shows the time series of the impact of $\sigma = 1$ on the value of $\mathcal{R}_0$ which describes that despite no lockdown, the infection rate will decrease due to self-quarantine and medical isolation. But at the time point t = 100, infected individuals will increase again, then decrease significantly and start to stabilize at t = 600. This increase is due to reinfection, so it is necessary to implement a lockdown.

7. Conclusion

The SQ_1AQ_2IMRS model is used to modify the Covid-19 transmission model in this study. This model considers the individual's reinfection and death while receiving medical attention. In this modified model, there are two equilibrium points: a disease-free equilibrium point (DFE), which is locally asymptotically stable in conditions $\mathcal{R}_0 < 1$, and a disease-endemic equilibrium point (EEP), which is locally asymptotically stable in conditions $\mathcal{R}_0 > 1$. The sensitivity analysis parameter that significantly impacts changes in $\mathcal{R}_0$ is the rate of disease transmission, the rate at which new people are recruited, the efficiency of lockdowns, and the rate of natural deaths.

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Data availability. Not applicable.

Abbreviations.

DFE	:	Disease free equilibrium point
EEP	:	Endemic equilibrium point
Covid-19	:	Coronavirus disease 2019

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