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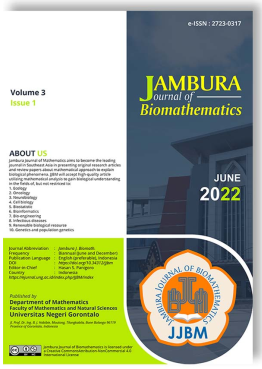


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# The dynamics of prisoner population model in Indonesia with a rehabilitation regulation for drug users to overcome prison overcapacity issue

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**ABSTRACT.** *This paper is to discuss about a mathematical model of the prisoner population with a new regulation regarding punishments to the drug users in an effort to overcome the prison overcapacity issue in Indonesia. This new regulation is launched by the Indonesia government after a fact about overcapacity of prison in Indonesia is revealed through a fire incident in a prison on 8 September 2021 that causes 41 people died and a number of people were injured. Besides, prisons in Indonesia are mostly occupied by the drug user. The model is constructed by using a compartmental model approach. The stability analysis of the equilibrium points is carried out along with its existence conditions. The analytical studies are equipped by calculating the basic reproduction number. Furthermore, this study is also equipped by numerical simulations with some scenarios. The results of this study confirm that the new regulation is able to reduce overcrowded issue in prisons in Indonesia. However, if it compares to recent prisons capacity, this new regulation has not been able to suppress the number of prisoners below to its capacity limit in the short time so that it is needed to consider other solutions as the additional regulation and policy.*



## 1. Introduction

Overcapacity of prisons is a condition where the occupancy is disproportionate to its capacity. Recently, Indonesia has a serious issue regarding the prison overcapacity. The Indonesia government through the Ministry of Law and Human Rights responsible for the policy-making in this issue. A fire incident on September 8, 2021, resulted in 41 intimates died, eight intimate severe injuries, and 31 of them experience minor injuries in at Block C, Class I of Tangerang Prison in Banten [1]. This incident becomes a concern of many people since it causes a number of casualties. The fire incident also revealed that Class I Tangerang Prison was overcrowded. The Head of Public Relations and Protocol at the Directorate General of Corrections, Ministry of Law and Human Rights, stated that overall the number of prisoner in prisons in 2021 reached 2,072, while the capacity is only about 600 [2]. Meanwhile, the Minister of Law and Human Rights, Yasonna H. Laoly, mentioned that Class I Tangerang Prison is overcapacity by 400% [3].

According to the World Prison Brief (WPB) report in [4], the number of prisoners in Indonesia has been steadily increasing over the past decade. The WPB recorded that the number of prisoners in Indonesia reached approximately 275.518 on October 2022, while the national prison capacity is only about 132,007 [5]. The WPB assessed the occupancy rate of prisons in Indonesia to be 208%. This fact brings Indonesia becomes 21st ranked as the most densely populated prisons out of 207 countries worldwide [4]. Again, according to the report of the Directorate General of Corrections (Ditjenpas), the Ministry of Law and Human Rights, the majority of prisoner in Indonesian prisons are drug-related offenders, namely as 145,413 individuals as of August 2021 [4]. Moreover, the latest data states that there are 135,758 drug-related prisoners as of April 2022, consisting of 120,042 as users and 15,176 as dealers, receivers, or producers [6].

Under these circumstances, the Indonesia government believes that it is needed an adjustment to the drugs law by prioritizing rehabilitation approaches over imprisonment for drug user prisoner as an alternative solution to address the problem of prison overcrowding. In its implementation, the Law number 35 year 2009 about narcotics has not provided a clear conception regarding a drugs addict, drugs abuser, and victim of drugs abuse. Treating a drugs addict, drugs abuser, and victim of drugs abuse in the same way as drugs dealer or traffickers results in an injustice in sentencing them. The Minister of Law and Human Rights stated that the treatment of the drugs addict, drugs abuser, and victim of drugs abuse should focus on rehabilitation [7]. The rehabilitation approach instead of imprisonment is a form of restorative justice, which emphasizes the recovery of a victim of drugs abuse involving various parties. The restorative justice also emphasizes that justice should be no longer based on proportional retaliation from victims to offenders but it must focus on supporting the victims recovery. This rehabilitation-oriented policy is in line with efforts to reduce prison overcrowding in Indonesia.

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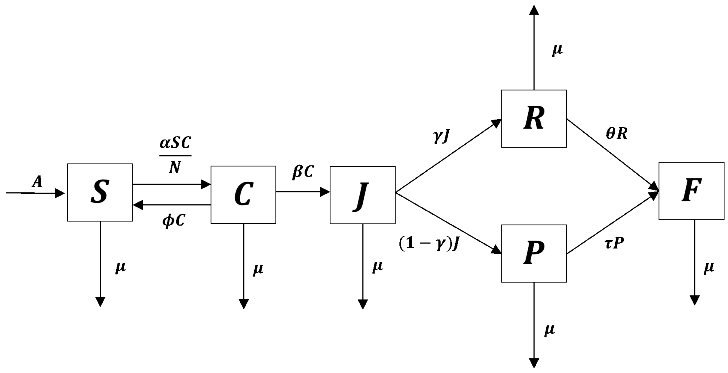



**Figure 1.** Schematic diagram of the model of prisoner population in Indonesia with new regulations against drugs users.

Several studies have discussed the issue of prison overcrowding in Indonesia from legal and social perspectives [8]. Those studies state that prison overcapacity issue results in an ineffectiveness on disciplining the prisoner so it must be addressed immediately. However, it is needed a accompanying study on the analysis of a policy implemented by the government in order to solve those problems. Thus, those problems allows us to conduct a study of the newly implemented rehabilitation regulations from a mathematical perspective. This study uses the mathematical modeling approach [9] which helps to represent a complex system in mathematical form for easier understanding. A such study will give a scientific point of view whether this new rehabilitation regulation is able to overcome the excess of prison capacity issue. If it doesn't then the government may consider other ways.

Generally, the model of prisoner population in Indonesia in this study uses the epidemiology mathematical models approach, especially Susceptible, Infected, Treated, and Recovered (SITR) model. The SITR epidemiological model is a development of the classic SIR model which assumes a situation where infected individuals need a treatment to recover [10, 11]. In the present study, rehabilitation for drugs users instead the imprisonment is considered as a treatment for their recovery. According to the introduction above, this study will focus on constructing of a dynamics prisoner population model in Indonesia with rehabilitation regulations for drug users. Through this model, the effectiveness of government policies in addressing the prison overcrowding issue in Indonesia through a new regulations is revealed.

This paper is presented in several sections as follows. In the next section, we present the constructed mathematical model of the prisoner population in Indonesia. Subsequently, in Section 3, an quantitative analysis of the model is conducted, namely the existence and stability of the equilibrium points and basic reproduction number as well. Numerical simulations are carried out to equip and emphasize the results which is presented in Section 4. In the last section, we briefly provide conclusions and remarks.

## 2. The Mathematical Model

In this study, the model is constructed by dividing the population into six subpopulations, i.e., the susceptible population ($S$), criminal population ($C$), population who is going to the judicial process ($J$), rehabilitation population ($R$), prison population ($P$), and free population ($F$). These six compartments in the model is used to observe the changes of the number of prisoner in Indonesia due to the new rehabilitation regulation for drugs users and its impact. The model works under the following assumptions:

1. The population in the model is assumed to be over 15 years old and the total population is assumed to be constant.
2. The recruitment rate and natural death rate are assumed to be constant.
3. The susceptible population $S$ in the model is a subset of the actual population, representing individuals who are likely to come into close contact with criminals.
4. Criminals are individuals who violate any applicable laws.
5. Individuals involved in criminal activities can return to the susceptible population $S$ if they stop engaging in criminal behavior.
6. Individuals involved in criminal activities, other than drug-related cases, move into the $P$ compartment to undergo detention in the prison, while drug users move into the $R$ compartment as a result of the implementation of the new regulation.
7. Individuals in the $F$ compartment are those who have completed rehabilitation or have been released from prison, assumed not to engage in criminal activities again.
8. The number of criminal cases is assumed to represent one person per case, and the $C$ compartment is used for this purpose.
9. The number of cases in the Supreme Court (MA) is assumed to represent one person for each case, and the $J$ compartment is used for this purpose.

Based on the assumptions above, the process of moving individuals to subpopulations in the model can be described in a schematic diagram presented in Figure 1. The description of variables and parameters is displayed in Table 1 and Table 2.

**Table 1.** Description of variables in the model of prisoner population with the new rehabilitation regulation.

| Variable | Description | Unit |
|---|---|---|
| $S$ | Population of susceptible individuals (Susceptible) | Person |
| $C$ | Population of criminal offenders (Criminal) | Person |
| $J$ | Population of individuals undergoing court (Justice) | Person |
| $R$ | Population of individuals undergoing rehabilitation (Rehabilitation) | Person |
| $P$ | Population of individuals serving sentences in prisons (Prisioner) | Person |
| $F$ | Population of free individuals (Free) | Person |

**Table 2.** Description of parameter in the model of prisoner population with the new rehabilitation regulation.

| Parameter | Description | Dimension |
|---|---|---|
| $A$ | The recruitment rate | $\frac{person}{time}$ |
| $\alpha$ | The crime rate resulting from the interaction between the Susceptible population and criminals | $\frac{1}{time}$ |
| $\phi$ | The rate of transition of a criminal back to the Susceptible population | $\frac{1}{time}$ |
| $\beta$ | The justice rate, i.e., the rate at which the judicial process is carried out to determine punishment | $\frac{1}{time}$ |
| $\gamma$ | The rate of drug users undergoing rehabilitation | $\frac{1}{time}$ |
| $\theta$ | The rate of recovery of drugs users after taking the rehabilitation treatment | $\frac{1}{time}$ |
| $\tau$ | The rate of freedom of the criminals after taking sentences in prisons | $\frac{1}{time}$ |
| $\mu$ | The natural death rate | $\frac{1}{time}$ |

From the schematic diagram above, a mathematical model is constructed in the form of a system of differential equations as follows:

$$\begin{aligned}
\frac{dS}{dt} &= A - \frac{\alpha SC}{N} + \phi C - \mu S,\\
\frac{dC}{dt} &= \frac{\alpha SC}{N} - \phi C - \beta C - \mu C,\\
\frac{dJ}{dt} &= \beta C - \gamma J - (1-\gamma) J - \mu J,\\
\frac{dR}{dt} &= \gamma J - \theta R - \mu R,\\
\frac{dP}{dt} &= (1-\gamma) J - \tau P - \mu P,\\
\frac{dF}{dt} &= \theta R + \tau P - \mu F.
\end{aligned} \tag{1}$$

System (1) is then normalized by dividing each subpopulation by the total population ($N$) for simplification, where $N = S + C + J + R + P + F$. Redefining

$$S^* = \frac{S}{N},\ C^* = \frac{C}{N},\ J^* = \frac{J}{N},\ R^* = \frac{R}{N},\ P^* = \frac{P}{N},\ F^* = \frac{F}{N},\ \text{and } \Lambda = \frac{A}{N}, \tag{2}$$

then system (1) can be transformed into a normalization model by substituting eq. (2) into system (1) and remove the asterisk for simplification of writing. Thus, it is obtained a normalization model as follows

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \alpha SC + \phi C - \mu S,\\
\frac{dC}{dt} &= \alpha SC - \phi C - \beta C - \mu C,\\
\frac{dJ}{dt} &= \beta C - \gamma J - (1-\gamma) J - \mu J,\\
\frac{dR}{dt} &= \gamma J - \theta R - \mu R,\\
\frac{dP}{dt} &= (1-\gamma) J - \tau P - \mu P,\\
\frac{dF}{dt} &= \theta R + \tau P - \mu F.
\end{aligned} \tag{3}$$

## 3. Quantitative Analysis of the Model

### 3.1. Positive Invariant

All variables of system (1) denote the number of subpopulation so that all of them must be non-negative for time $t \geq 0$ when the initial conditions are also non-negative to have biological meaning. From the first equation in system (1), $\frac{dS}{dt} \geq -\mu S$ for all non-negative initial conditions. Therefore,

$$S(t) \geq S_0 e^{-\mu t} \geq 0. \tag{4}$$

It means that $S(t)$ remains non-negative for all times $t > 0$. Analogous results hold for the other variables $C(t)$, $J(t)$, $R(t)$, $P(t)$, and $F(t)$.

The summation of all equations in system (1) yields a differential equation for the total population $N(t)$ as follows

$$\frac{dN}{dt} = \Lambda - \mu N. \tag{5}$$

We can solve eq. (5) to yield $N(t) = \frac{\Lambda}{\mu} + (N(0) - \frac{\Lambda}{\mu})e^{-\mu t}$, where $N(0)$ is the initial total population. As $t \to \infty$, then $N(t) \to \frac{\Lambda}{\mu}$. Hence, the feasible domain of system (1) is

$$\Omega = \left\{(S, C, J, R, P, F) \in \mathbb{R}^6_+ : 0 \leq N \leq \frac{\Lambda}{\mu}\right\}, \tag{6}$$

which is positively invariant. Thus, system (1) is well-posed.

### 3.2. Equilibrium Points

The equilibrium point is obtained when the population growth rate in system (3) does not change or experiences "zero growth rate." Mathematically, the equilibrium point of system (3) can be determined when each equation in system (3) equals zero [12]. Thus, the equilibrium points of system (3) is obtained which consist of two points, which is named as a criminal-free ($E_0$) and criminals equilibrium point ($E_1$).

The criminal-free equilibrium point is

$$E_0 = (S^0, C^0, J^0, R^0, P^0, F^0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0\right),$$

which represents a condition that there is no criminals in the population. Roughly speaking, $E_0 =$ always exist since all variables are non-negative without any condition. Meanwhile, the criminals equilibrium point is $E_1 = (S^*, C^*, J^*, R^*, P^*, F^*)$ where

$$S^* = \frac{\beta+\mu+\phi}{\alpha}, \tag{7}$$

$$C^* = \frac{\Lambda\alpha-\beta\mu-\mu^2-\mu\phi}{\alpha(\beta+\mu)}, \tag{8}$$

$$J^* = \frac{\beta\left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)}{\alpha(\beta+\mu)(\mu+1)}, \tag{9}$$

$$R^* = \frac{\beta\gamma\left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)}{\alpha(\beta+\mu)(\mu+1)(\mu+\theta)}, \tag{10}$$

$$P^* = -\frac{\beta(\gamma-1)\left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)}{\alpha(\beta+\mu)(\mu+1)(\tau+\mu)}, \quad (11)$$

$$F^* = -\frac{\beta(\gamma\mu\tau-\gamma\mu\theta-\mu\tau-\tau\theta)\left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)}{\alpha(\beta+\mu)\mu(\mu+1)(\mu+\theta)(\tau+\mu)}. \quad (12)$$

The criminal equilibrium point represents a situation where there are criminals in the population which results in other subpopulations exist. The existence conditions of $E_1$ can be calculated from each variable in Sections 3.2 to 3.2 to be non-negative. However, we prefer to represent it in term of the basic reproduction number for simplicity which is discussed in the next section.

### 3.3. Basic Reproduction Number

The basic reproduction number $\mathcal{R}_0$ of system (3) can be interpreted as the expectation of the average number of the new criminals due to contact between the susceptible and criminal individuals [11]. Here, the contact implies the influences transmitted by the criminals to susceptible. In system (3), only $C$ compartment affects the crime. Thus, we can easily guess the element of **F** and **V** matrices which correspond to the emergence of a new criminal and all other transitions, respectively, namely

$$\mathbf{F} = \begin{bmatrix} \frac{\alpha\Lambda}{\mu} \end{bmatrix}, \quad \text{and} \quad \mathbf{V} = \begin{bmatrix} \beta+\mu+\phi \end{bmatrix}. \quad (13)$$

Thus, the basic reproduction number of system (3) $\mathcal{R}_0$ is obtained as follows

$$\mathcal{R}_0 = \rho\left(\mathbf{F}\mathbf{V}^{-1}\right) = \frac{\alpha\Lambda}{\mu\left(\beta+\mu+\phi\right)}. \quad (14)$$

Moreover, the criminals equilibrium point $E_1$ can be expressed in term of $\mathcal{R}_0$, namely

$$\begin{aligned} S^* &= \tfrac{\beta+\mu+\phi}{\alpha}, \\ C^* &= \tfrac{\mu(\beta+\mu+\phi)(\mathcal{R}_0-1)}{\alpha(\beta+\mu)}, \\ J^* &= \tfrac{\beta\mu(\beta+\mu+\phi)(\mathcal{R}_0-1)}{\alpha(\beta+\mu)(\mu+1)}, \\ R^* &= \tfrac{\beta\gamma\mu(\beta+\mu+\phi)(\mathcal{R}_0-1)}{\alpha(\beta+\mu)(\mu+1)(\mu+\theta)}, \\ P^* &= \tfrac{\beta\mu(1-\gamma)(\beta+\mu+\phi)(\mathcal{R}_0-1)}{\alpha(\beta+\mu)(\mu+1)(\tau+\mu)}, \\ F^* &= \tfrac{(\mu\tau(1-\gamma)+\gamma\mu\theta+\tau\theta)(\beta+\mu+\phi)(\mathcal{R}_0-1)}{\alpha(\beta+\mu)\mu(\mu+1)(\mu+\theta)(\tau+\mu)}. \end{aligned} \quad (15)$$

Therefore, the existence conditions of $E_1$, i.e., the conditions to yield all variables are non-negative so that they have biologically meaning, become easier to determine. From Equation (15), it is obvious that when $\mathcal{R}_0 > 1$ then all variables are non-negative. Thus, the criminals equilibrium point exist when $\mathcal{R}_0 > 1$.

### 3.4. Local Stability Analysis of the Equilibrium Points

In this section, the local stability analysis of system (3) is carried out by using the eigenvalue approach which is obtained by substituting each equilibrium point into the following Jacobian matrix

$$\mathbf{J} = \begin{bmatrix} -C\alpha-\mu & -S\alpha+\phi & 0 & 0 & 0 & 0 \\ C\alpha & S\alpha-\beta-\mu-\phi & 0 & 0 & 0 & 0 \\ 0 & \beta & -\mu-1 & 0 & 0 & 0 \\ 0 & 0 & \gamma & -\mu-\theta & 0 & 0 \\ 0 & 0 & 1-\gamma & 0 & -\tau-\mu & 0 \\ 0 & 0 & 0 & \theta & \tau & -\mu \end{bmatrix}. \quad (16)$$

### 3.4. Criminals-free Equilibrium Points

> **Theorem 1.** *The criminals-free equilibrium point $E_0$ of system (3) is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable otherwise.*

*Proof.* First, the Jacobian matrix **J** in (16) is evaluated at the criminals-free equilibrium point $E_0$, than we obtain

$$\mathbf{J}(E_0) = \begin{bmatrix} -\mu & -\frac{\Lambda\alpha}{\mu}+\phi & 0 & 0 & 0 & 0 \\ 0 & \frac{\Lambda\alpha}{\mu}-\beta-\mu-\phi & 0 & 0 & 0 & 0 \\ 0 & \beta & -\mu-1 & 0 & 0 & 0 \\ 0 & 0 & \gamma & -\mu-\theta & 0 & 0 \\ 0 & 0 & 1-\gamma & 0 & -\tau-\mu & 0 \\ 0 & 0 & 0 & \theta & \tau & -\mu \end{bmatrix}. \quad (17)$$

The eigenvalues of the matrix $\mathbf{J}(E_0)$ can be obtained by solving the characteristic polynomial $|\mathbf{J}(E_0) - \lambda I| = 0$. Thus, the characteristic polynomial of matrix (17) is yielded as follows

$$(\lambda+\mu+1)(\lambda+\mu)^2(\lambda+\mu+\theta)(\lambda+\mu+\tau) \left(\Lambda\alpha-\beta\mu-\lambda\mu-\mu^2-\mu\phi\right) = 0. \quad (18)$$

The eigenvalues of the matrix (17) are the solutions of eq. (18), which are $\lambda = -\mu$ (with multiplicity is 2), $-\mu-1$, $-\mu-\theta$, $-\mu-\tau$, and

$$\lambda = \frac{\Lambda\alpha-\beta\mu-\mu^2-\mu\phi}{\mu}.$$

It is clear that the first five eigenvalues are real negative values since all parameters are positive, while the last eigenvalue will be negative

$$\begin{aligned} \frac{\Lambda\alpha-\beta\mu-\mu^2-\mu\phi}{\mu} &< 0, \\ \frac{\Lambda\alpha}{\mu(\beta+\mu+\phi)} &< 1, \\ \mathcal{R}_0 &< 1. \end{aligned} \quad (19)$$

Therefore, the criminals-free equilibrium point is locally stable asymptotic when $\mathcal{R}_0 < 1$ and unstable otherwise. □

### 3.4. Criminals Equilibrium Points

> **Theorem 2.** *The criminals equilibrium point $E_1$ of system (3) is locally asymptotically stable if $\mathcal{R}_0 > 1$ and unstable otherwise.*

*Proof.* The local stability of the criminals equilibrium points of system (3) is analyzed in the same way as before, by substituting the criminals equilibrium points into the Jacobian matrix (16), resulting in

$$\mathbf{J}(E_1) = \begin{bmatrix} a_{11} & -\beta-\mu & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -\mu-1 & 0 & 0 & 0 \\ 0 & 0 & \gamma & -\mu-\theta & 0 & 0 \\ 0 & 0 & 1-\gamma & 0 & -\mu-\tau & 0 \\ 0 & 0 & 0 & \theta & \tau & -\mu \end{bmatrix}. \quad (20)$$

where $a_{11} = -\frac{\Lambda\alpha-\beta\mu-\mu^2-\mu\phi}{\mu+\beta} - \mu$ and $a_{21} = \frac{\Lambda\alpha-\beta\mu-\mu^2-\mu\phi}{\mu+\beta}$. Thus, the characteristic polynomial of matrix (20) is given as follows

$$(\lambda+\mu)(\lambda+\mu+1)(\lambda+\mu+\theta)(\lambda+\mu+\tau)\left((\beta+\mu)\lambda^2 + (\Lambda\alpha-\mu\phi)\lambda + \left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)(\beta+\mu)\right) = 0. \quad (21)$$

By solving eq. (21) respect to $\lambda$, the eigenvalues are obtained as listed as the following, $\lambda = -\mu, -\mu-1, -\mu-\theta, -\mu-\tau$ and the roots of polynomial

$$h_1\lambda^2 + h_2\lambda + h_3 = 0, \quad (22)$$

where

$$\begin{aligned} h_1 &= (\beta+\mu), \\ h_2 &= (\Lambda\alpha-\mu\phi), \\ h_3 &= \left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)(\beta+\mu). \end{aligned}$$

Since all parameters are positive than $h_1 > 0$. From the previous results, it is known that the existence condition of $E_1$ is $\mathcal{R}_0 > 1$ which implies that $\alpha\Lambda > \mu(\beta+\mu+\phi)$. It is also hold that $\alpha\Lambda > \mu(\beta+\mu+\phi) > \mu\phi$ so that $\alpha\Lambda > \mu\phi$. Thus, under this circumstances $h_2 > 0$. Meanwhile, the $h_3$ term can be written as follows

$$\begin{aligned} h_3 &= \left(\Lambda\alpha-\beta\mu-\mu^2-\mu\phi\right)(\beta+\mu), \\ &= (\Lambda\alpha-\mu(\beta+\mu+\phi))(\beta+\mu)\frac{\mu(\beta+\mu+\phi)}{\mu(\beta+\mu+\phi)}, \\ &= \mu(\beta+\mu+\phi)\left(\frac{\Lambda\alpha}{\mu(\beta+\mu+\phi)} - \frac{\mu(\beta+\mu+\phi)}{\mu(\beta+\mu+\phi)}\right)(\beta+\mu), \\ &= \mu(\beta+\mu+\phi)(\mathcal{R}_0-1)(\beta+\mu). \end{aligned}$$

Due to $\mathcal{R}_0 > 1$, it can be concluded that $h_3 > 0$. Since all coefficients of eq. (22) are positive, then it has two real negative roots as the consequences [13]. Therefore, when the criminals equilibrium point of system (3) exists in term of biologically meaning, i.e. $\mathcal{R}_0 > 1$, then it is locally asymptotically stable and unstable otherwise. □

Thus, from the local stability analysis of the model of prison population in Indonesian with rehabilitation regulations for drug users, the local stability conditions of its equilibrium points are obtained as shown in Table 3.

Table 3. Local stability condition of the equilibrium points of system (3).

| Equilibrium point | Locally asymptotically stable conditions |
|---|---|
| $E_0$ | $\mathcal{R}_0 < 1$. |
| $E_1$ | $\mathcal{R}_0 > 1$. |

Figure 2 shows bifurcation diagram of $C$ subpopulation as a representation of the equilibrium points of system (3) with respect to $\alpha$ parameter and other parameters value based on Table 4. Figure 2 illustrates that when values of $\alpha$ result in $\mathcal{R}_0 < 1$ then there is only one equilibrium point exist, i.e., $E_0$, and it is locally stable denoted by the red solid curve. As value of $\alpha$ increases so that $\mathcal{R}_0$ is greater than 1, $E_0$ lose its stability (red-dashed curve) and emerge other equilibrium point, i.e., $E_1$ (blue solid curve), and it is locally stable.

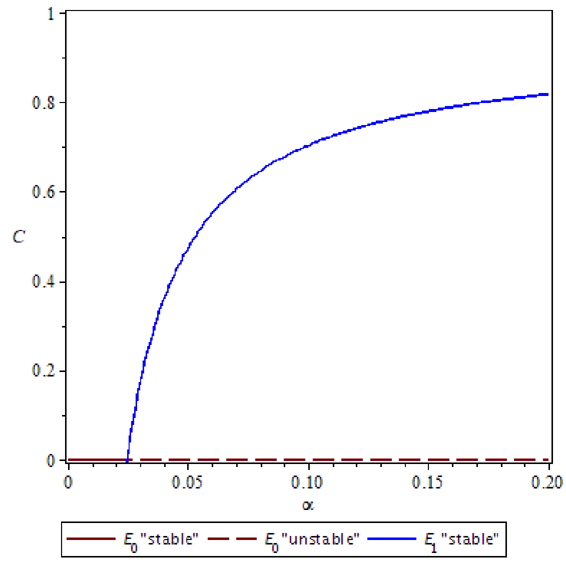



Figure 2. The Bifurcation diagram of of prison population of system (3) with parameters value is in Table 4 and varying values of $\gamma$.

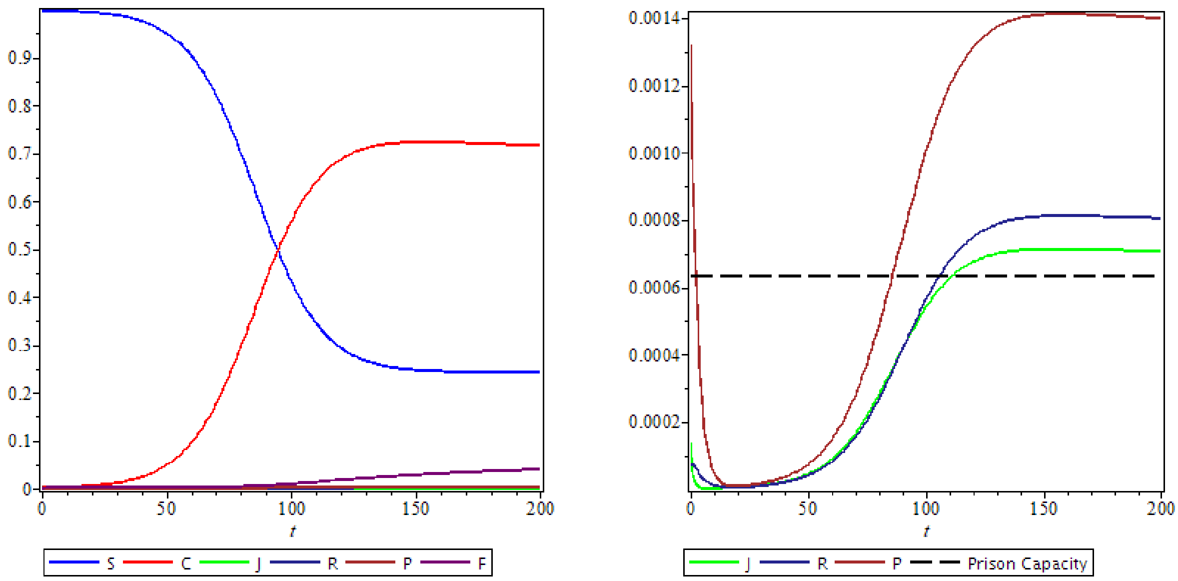
## 4. Numerical Simulations Results

In this section, we perform numerical simulation with some scenarios to get a better understanding. Numerical simulations are conducted to observe the population dynamics of system (3) based on the initial value of variables and parameters value in Table 4. All of the initial value of variables shown in Table 4 are based on related data, whereas some of parameters are assumed. The following is given simulation results with some scenarios which present several circumstances. The first scenario depicts a situation before the implementation of rehabilitation regulations, assuming a value of $\gamma = 0,3$. This parameter value represents the rate at which the subpopulation undergoes to rehabilitation treatment following the court decision. Before the new rehabilitation regulation exist, sometimes judges decide to punish a drugs users with rehabilitation.

However, the second scenario illustrates a situation where the new rehabilitation regulations is implemented so that the values of $\gamma$ is gradually increased to observe the impact of the new regulations. Here, the $\gamma$ values are set as $0,5$, $0,6$ and $0,7$ to represent the variations of implementation rigidity level of the new regulation for drug users. Subsequently, the simulation results of both scenarios is compared, especially between the number of prison subpopulation and number of its capacity to assess the impact of the new regulations in addressing prison overcrowding issue in Indonesia.

### *4.1. Numerical Simulation Before the Implementation of the New Rehabilitation Regulation*

The first scenario in the numerical simulation uses parameters and initial values of variable based on Table 4 with $\gamma = 0.3$. Beforehand, the initial values for each of the subpopulation presented in Table 4 are normalized with total population, i.e., $N = S+C+J+R+P+F$, which is accordance with system (3) in all simulations. Parameters $\gamma = 0.3$ is chosen in order to capture the current situation where prison in Indonesia is overcapacity




(a) (b)

**Figure 3.** (a) The dynamics of system (3) with parameters value is in Table 4 and $\gamma = 0,3$. (b) The dynamics of $J$, $R$ and $P$ of system (3) with parameters value is in Table 4 and $\gamma = 0,3$.

**Table 4.** Parameters value and initial value of variables

| Variables/ parameters | Value | Unit | Reference |
|---|---|---|---|
| $S$ | 208.544.086 | person | [14] |
| $C$ | 247.218 | person | [15] |
| $J$ | 28.284 | person | [16] |
| $R$ | 14.122 | person | [17] |
| $P$ | 275.518 | person | [5] |
| $F$ | 58.054 | person | [18] |
| $\Lambda$ | $N\mu$ | $\frac{person}{year}$ | - |
| $\alpha$ | 0,1 | $\frac{1}{year}$ | Assumed |
| $\beta$ | 0,001 | $\frac{1}{year}$ | Assumed |
| $\phi$ | 0,01 | $\frac{1}{year}$ | Assumed |
| $\gamma$ | 0,4 | $\frac{1}{year}$ | Assumed |
| $\theta$ | 0,25 | $\frac{1}{year}$ | [19, 20] |
| $\tau$ | 0,34 | $\frac{1}{year}$ | Assumed |
| $\mu$ | $\frac{1}{73,83}$ | $\frac{1}{year}$ | [21] |

about 208%. The dynamics of each subpopulation of system (3) under this circumstance are shown in Figure 3.

From Figure 3, it can be observed that the population dynamics of system (3) is asymptotically stable which converge to the criminals equilibrium point for a long time period. The dashed black line in Figure 3(b) denotes the total capacity of all prison in Indonesia which is 132,107 individuals [5]. Figure 3(b) shows that the number of $P$ subpopulation is over the capacity. Under this circumstances, the numerical simulation is conducted to represent the initial state when the rehabilitation process for drug users is low. As the consequences, the criminal individuals which over half of them are the drug users [2, 3] are punished by the prison sentence leading to overcrowded of the $P$ subpopulation.

### *4.2. Numerical Simulation After the Implementation of New Drug Regulation*

Next, numerical simulations are then conducted to illustrate the scenario after the implementation of the new rehabilitation regulations for drug users by varying the parameter values of the rate of drug users undergo rehabilitation treatment, i.e., $\gamma = 0.5, 0.6$, and $0.7$. The variations of $\gamma$ parameter describe the implementation rigidity level of the new rehabilitation regulation. A bigger value of $\gamma$ parameter implies that the new regulation is able to be implemented wider. By varying the value of $\gamma$, the populations dynamics of system (3) are observed to assess the impact of the new regulation in addressing the issue of prison overcapacity in Indonesia, specifically on $P$ subpopulation.

**Table 5.** The variations of $\gamma$ parameter value to describe the implementation rigidity level of the new rehabilitation regulation.

| Parameter | Value | Description |
|---|---|---|
| $\gamma$ | 0,3 | without the rehabilitation regulation |
| $\gamma_1$ | 0,5 | with the rehabilitation regulation |
| $\gamma_2$ | 0,6 | with the rehabilitation regulation |
| $\gamma_3$ | 0,7 | with the rehabilitation regulation |

Figure 4 shows variations of the $P$ subpopulation dynamics of system (3) with respect to the several values of $\gamma$. According to simple logical thinking, the simulation results also confirm that as the value of $\gamma$ increases, the population of prisoner ($P$) decreases. However, from Figure 4 it can be observed that not all values of $\gamma$ is able to suppress the number of $P$ subpopulation below the prison capacity. The number of $P$ subpopulation decreases until below to the prison capacity denoted by dashed black line when

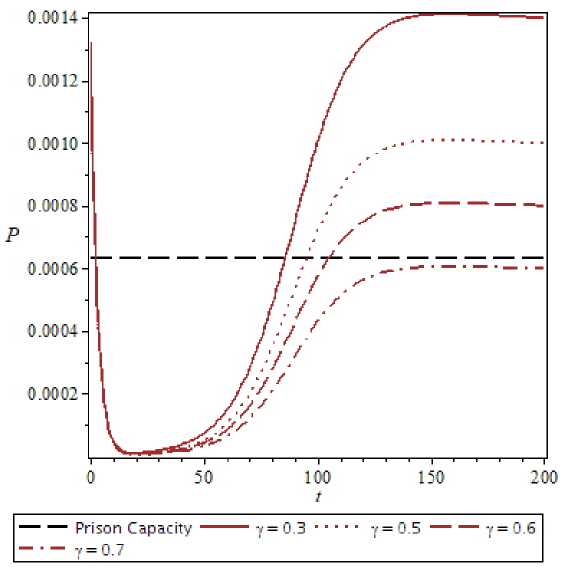



**Figure 4.** The dynamics of prison population of system (3) with parameters value is in Table 4 and varying values of $\gamma$.

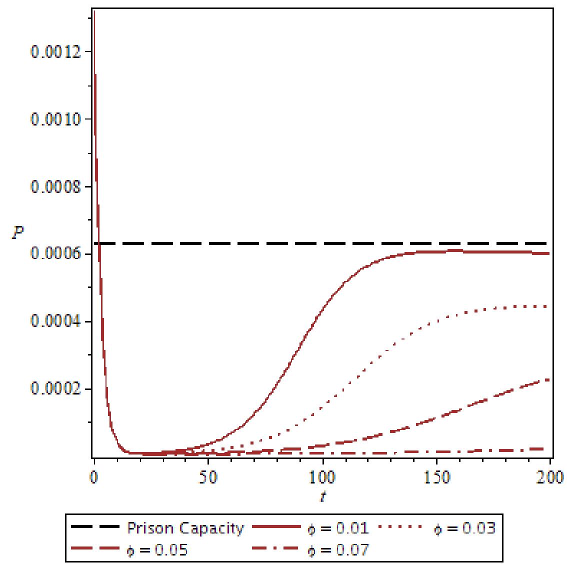



**Figure 5.** The dynamics of prison population of system (3) with parameters value is in Table 4, $\gamma = 0.5$ and varying values of $\phi$.

$\gamma$ value is high enough, i.e., about $0.7$ or higher. In mathematical calculation, it can be interpreted that this new regulation is able to address the issue of prison overcapacity through rehabilitation policy instead of prison sentence if it can lead many cases related to the using of drugs to rehabilitation treatment. However, Figure 4 also indicate that the prison overcapacity issue has not been fully resolved, as the numerical simulations conducted on system (3) show that it would take hundreds of years to achieve the steady state. In real-life scenarios, the results of the new regulation implementation is less efficient since it would require a significantly long time to experience its benefits.

In addition, numerical simulations are also conducted by varying the values of $\phi$ parameter and fixing $\gamma = 0.5$. Figure 5 shows that when the increasing of $\gamma$ is followed by the increasing of $\phi$ parameter value then number of $P$ subpopulation decrease below to the Indonesian prison limit capacity faster. Therefore, under the circumstances in this present study, the new regulation for drug users in addressing prison overcapacity in Indonesia is still insufficient to solve the issue. The government need to consider other policies to overcome the prison overcapacity issue, such as an approach to the criminals include drug user to repent so that it decrease the number of $C$ subpopulation. The decreasing of the criminals number will also decrease the number of prisoner indirectly.

## 5. Conclusion

A mathematical dynamics population model of prisoner in Indonesian with the new regulation for drug users to overcome the issue of prison overcapacity is constructed. The new regulation, i.e., rehabilitation, directs the model so that it consists of six compartments which represent the stages of an individual becoming a criminal, undergoing punishment or rehabilitation until they are released and becomes a free individual. Based on the analytical and numerical studies in this paper, as long as the assumptions and conditions in the model are met, it can be concluded that the new regulation is certainly able to reduce the number of prisoners in Indonesia to below the existing capacity limit, but it takes a long time to experience the results. Therefore, it is needed to add and consider other regulations that can expedite the resolution of the prison overcrowding issue in Indonesia.

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**Data availability.** Several parameter values are cited from some references. See Table 2.

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