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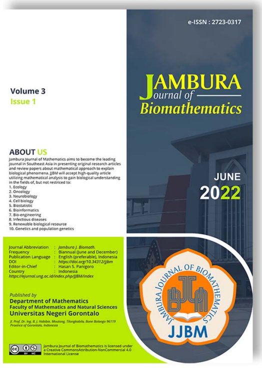

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Research Article

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Sensitivity analysis and optimal control of COVID-19 Model

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ABSTRACT. *Coronavirus infection is a disease that causes death and threatens human life; for prevention, it is necessary to quarantine susceptible, exposed, and infected populations and vaccinate the entire population. This kind of quarantine and vaccination is intended to reduce the spread of coronavirus. Epidemiological models are a strategy used by public health practitioners to prevent and fight diseases. However, to be used in decision making, mathematical models must be carefully parameterized and validated using epidemiological and entomological data. Epidemiological models: susceptible, symptomatic, contagious, and recovering. In this study, sensitivity analysis and optimal control were performed to determine the relative importance of the model parameters and to minimize the number of infected populations and control measures against the spread of the disease. Sensitivity analysis was carried out using a sensitivity index to measure the relative change in the basic reproduction number for each parameter, and this control function was applied to the dynamic modeling of the spread of COVID-19 using the Pontryagin Minimum Principle. We will describe the formulation of a dynamic system for the spread of COVID-19 with optimal control and then use Pontryagin's Minimum Principle to find optimal control solutions. In this article, COVID-19 cases in the USA and India serve as examples of the efficiency of control measures. The results obtained revealed that the parameters that became the basis for reducing the number of infected with COVID-19 for the two countries, the USA and India, are effective transmission rates from S to E, (β), transmission rates from E to I, (α), and transmission rates from S to R, (p_s), which are the main parameters to watch for growth with respect to Basic Reproduction rates (R_0). Finally, three controls were simulated in cases I (in the USA) and II (in India) in the interval $t \in [0, 15]$. For all controls, the effectiveness was close to 50% in India and 100% in the USA to reduce the spread of COVID-19. According to the findings, if these three controls were implemented ideally from the start of the pandemic, the number of sufferers.*

1. Introduction

The COVID-19 outbreak has caused numerous health and economic issues for health officials and governments worldwide. Preventive strategies for other coronaviruses (SARS and MERS) are being applied to reduce the rate of COVID-19 transmission. Other new measures are also being employed, such as travel limitations, stay-at-home orders, and lockdowns [1].

To study the dynamics of COVID-19, mathematical models and optimal controls are required to predict and emphasize disease transmission, recovery, death, and other significant parameters individually for disparate countries, that is, the reported range of high too low for specific regions of the COVID-19 cases. Several modeling approaches have been used in numerous research studies [2–7]. Kamrujjaman et al. (2022) [2] proposed a deterministic mathematical model for the COVID-19 outbreak and validated the model using real data from Italy from 15th Feb 2020 to 14th July 2020. Zaitri et al. (2022) [3] proposed and analyzed a delayed SEIQRP model for COVID-19 with vaccination. Al-Qadi et al. [4] extended the standard susceptible-infected-recovered (SIR) model by incorporating the global dynamics of the COVID-19 pandemic. Sinha et al. [5] found infected cases, infected death rates, and COVID-19 recovery rates, and validated the model using the rough set method. The accuracy for infected cases was 90.19%, COVID-19 infection-fatality was 94%, and recovery was 85.57%, approximately the same as the actual situation reported by the WHO.

The World Health Organization (WHO) has recommended the following precautions to lower the general risk of acute respiratory infections while traveling or migrating from impacted areas: (i) Avoiding close contact with people who have acute respiratory illnesses, (ii) frequent hand washing, (iii) avoiding farms and wild animals, (iv) wearing masks, (v) avoiding crowded areas, (vi) keeping at home except to meet needs, and (vii) self-isolation, even if symptoms are moderate, and so on (NSW (2022) [8].

The rate of COVID-19 dissemination has been slowed by preventive measures that have been employed for previous coronaviruses, including SARS and MERS. Other recent measures are also being deployed, including lockdowns, stay-at-home orders, and travel limitations [9]. The WHO has suggested the following precautions to lessen the risk of acute respiratory infections in general while traveling or relocating from afflicted areas: (i) preventing close contact with individuals who have severe respiratory illnesses, (ii) cleansing hands frequently, (iii) keeping away from farms and wildlife, (iv) donning masks, (v) preventing congested areas, (vi) remaining at home, excluding meeting necessi-

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ties, (vii) isolation despite only mild symptoms, and so forth [10].

Efforts to prevent the spread of COVID have also been carried out by several researchers by determining optimal controls. The use of facemasks, hand sanitizers, and social distancing as control variables, as mentioned in Refs. [11–24]. Care of COVID-19 patients, active screens, and testing as control variables, as mentioned in Refs. [9, 14–18]. Relapse and reinfection in humans who have recovered from COVID-19 as a control variable, as mentioned in Refs. [9, 15]. Vaccination of the exposed population per unit time at t is a control variable, as mentioned in Refs. [18, 20, 25–27]. The recovery rate of asymptomatic infected individuals in treatment per unit time at time t is the control variable, as mentioned in Refs. [18, 19, 21, 22, 24]. The recovery rate of symptomatic infected individuals in treatment per unit time at time t is the control variable, as mentioned in Refs. [18, 21, 27]. Rapid testing of individuals in the exposed stage and identification of asymptomatic and asymptomatic individuals is a control variable, as mentioned in Refs. [22]. The individuals that were not tested yet but identified the patients of COVID- 19, either asymptomatic or asymptomatic, can be treated and should be restricted to their places or hospitals and quarantines as control variables, as mentioned in Refs. [22]. The isolation level of an infectious individual who was not hospitalized at time t was used as a control variable, as mentioned in Refs. [23], and the recovery rate of symptomatic infected individuals in treatment per unit time at time t is the control variable, as mentioned in Refs. [27].

Based on the description of several types of control strategies as previously studied, the motivation for choosing a control is often considered, and the three controls chosen to stop the spread of the epidemic in this article are controlled through public education about health problems, as u_1. The next control involves the use of personal defenses, such as seclusion, increasing immunity, and reducing contact with other people, which is symbolized by u_2 The last control is the care given to COVID-19 patients in a hospital or isolation facility to reduce their suffering from the disorder, which is presented by u_3.

To achieve this goal, we examined the impact of control measures on the spread of COVID-19 from the time of symptoms and infection. To the best of our knowledge, this study assessed the impact of the control measures used. The most severe cases of COVID-19 after China are in the USA and India, so we selected the parameters recorded from the US and India. From the study [26], the model in Wintachi & Prathom (2021) [28] corresponds to cases in the USA and India. This model considers the administration of a vaccine to every human who is susceptible to, symptomatic of, and infected with COVID-19. This is in accordance with the opinion of Khan et al. (2019) that vaccine administration is an effective method for preventing and reducing viral infections. Therefore, for the first time, we decided to use a model [28] as an experiment. In addition, we applied the parameters recorded in [28] and predicted the potential for COVID-19 given controls in both countries when the vaccine was out. In addition, this article will also examine the effect of vaccines on the selected control variables and sensitivity analysis to identify some of the characteristics that contribute to the spread and persistence of this disease in the community.

2. Model Structure

A 4-compartment model called SEIR from [28] where $S(t), E(t), I(t)$, and $R(t)$ are the fractions of the susceptible, exposed, infectious, and recovered populations, respectively, at time t. The trivial solutions $S = 0, E = 0, I = 0$, and $R = 0$ are not of interest. S is the fraction of susceptible cases, E is the fraction of exposed cases, I is the fraction of infectious cases, R is the fraction of recovered cases, β is the effective transmission rate of COVID-19, α is the change rate from E to I, γ is the changing rate from I to R, ν is the vaccination rate of the population, p_s is the potency of vaccination in S, p_e is the potency of vaccination in E, p_i is the potency of vaccination in I, b_0 is the birth rate of the population, d_0 is the death rate of the population without COVID-19, d_1 is the death rate of the exposed population plus d_0, and d_2 is the death rate of the infectious population plus d_0. The model is motivated by the fact that the vaccination rate per day (ν) cannot stop the system's flow instantaneously, because the entire population cannot be immunized at once. Once a person is susceptible, exposed, or contagious, they can receive a vaccination. In the first equation of the SEIR model, the rate of change in susceptibility is dependent on the number of people who have received vaccinations, $\nu p_s S$, and non-vaccinated humans,$(1 - \nu p_s)S$. The system of differential equations related to the equation in [28] as

$$\begin{aligned}
\frac{dS}{dt} &= b_0 - (\nu p_s + d_0)S - \beta * (1 - \nu p_s)SI,\\
\frac{dE}{dt} &= \beta(1 - \nu p_s)SI - (d_1 + \alpha + (1 - \alpha)\nu p_e)E,\\
\frac{dI}{dt} &= \alpha E - (d_2 + \gamma + (1 - \gamma)\nu p_i)I,\\
\frac{dR}{dt} &= \nu p_s S + \nu p_e(1 - \alpha)E + (\gamma + (1 - \gamma)\nu p_i I - d_0 R.
\end{aligned}$$

under the conditions that $0 \le S(0), E(0), I(0), R(0) \le 1$.

3. Analysis of The Model

3.1. Positively Invariant Set

Note that the nonlinear system in [28] has a unique solution set $(S(t), E(t), I(t), R(t))$, according to the basic existence-uniqueness theorem for nonlinear systems. Making sure the densities $S(t), E(t), I(t)$, and $R(t)$ in the model are non-negative at any time $t > 0$, If SEIR is the continuous solution of the model with initial condition, the $(S(t), E(t), I(t), R(t)) \in [0, \infty)^4$ for any positive time $t > 0$.

3.2. Equilibrium Points and Basic Reproduction Number

The model in [28] has two types of equilibrium points, first one is disease free, E_0, and second is endemic equilibrium, E^*. We can write very easily our E_0, by setting the disease classes and derivatives to zero, it would have the form $E_0 = (\frac{b_0}{\nu p_s + d_0}, 0, 0, \frac{b_0}{d_0})$. Meanwhile, endemic equilibrium points were employed to show the potential for disease propagation. Due to endemic circumstances and the spread of the disease, the populations $S \neq 0, E \neq 0, I \neq 0$, and $R \neq 0$. The endemic equilibrium points and basic reproduction number as

$$R_0 = \sqrt{\frac{\alpha\beta(1-\nu p_S)b_0}{[d_1+\alpha+(1-\alpha)\nu p_e][d_2+\gamma+(1-\gamma)\nu p_i][\nu p_s+d_0.]}}.$$

Table 1. Parameter values and initial populations of the USA (Case I) and India (Case II)

Initial Parameter	Case I (USA)	Case II (India)
$S(0)$	0.97286	0.994
$E(0)+I(0)$	0.00905	3.813×10^{-4}
$R(0)$	0.01809	5.569×10^{-3}
β	0.462	0.32
α	$\frac{1}{11.5}$ per day	$\frac{1}{11.5}$ per day
γ	0.0686 per day	0.0686 per day
b_0	3.178×10^{-5} per day	$4.893 \times 10-5$ per day
d_0	2.377×10^{-5}per day	$1.992 \times 10-5$ per day
d_1	2.585×10^{-5} per day	$2.021 \times 10-5$ per day
d_2	2.585×10^{-5} per day	2.021×10^{-5} per day

Table 2. Sensitivity indices of R_0 corresponding to all parameters (case I in United State) for $p_s = 0.7, p_e = 0.6$ and $p_i = 0.6$ [30] in various values of vaccination (ν)

Parameter	$\nu = 0$	$\nu = 0.1$	$\nu = 0.5$	$\nu = 0.8$	$\nu = 1.0$
α	1.4859×10^{-4}	0.2117	0.4157	0.4570	0.4726
β	0.5	0.5	0.5	0.5	0.5
γ	-0.4998	-0.2590	-6.8985×10^{-2}	-3.4586×10^{-2}	-2.1866×10^{-2}
b_0	0.5	0.5	0.5	0.5	0.5
d_0	-0.5	-1.6972×10^{-4}	-3.3955×10^{-5}	-2.1222×10^{-5}	-1.6978×10^{-5}
d_1	-1.4859×10^{-4}	-9.1172×10^{-5}	-3.5814×10^{-5}	-2.4608×10^{-5}	-2.0360×10^{-5}
d_2	-1.8834×10^{-4}	-1.0381×10^{-4}	-3.7136×10^{-5}	-2.5063×10^{-5}	-2.0599×10^{-5}
p_s	0	-0.5375	-0.7692	-1.1363	-1.6667
p_e	0	-0.1932	-0.3795	-0.4172	-0.4315
p_i	0	-0.2244	-0.4014	-0.4335	-0.4453

3.3. Sensitivity analysis

The most optimal approach for reducing the number of infected individuals is to identify several factors that contribute to the transmission of the virus and its prevalence. It is necessary to calculate the sensitivity index for each parameter of the model, which is correlated with the basic reproduction number, R_0. This index provides information on the importance of each parameter in the model that represents the transmission of COVID-19. The index is used to identify the parameters that have the most significant influence on R_0, which will later be used as intervention targets. Parameters with a high impact at R_0 indicate that they have a dominant influence on the COVID-19 pandemic. This statement is supported by the opinion of experts in [10, 29–31].

Sensitivity analysis of the basic reproduction number can also be used to design mitigation strategies to slow the spread of the pandemic by reducing R_0. Sensitivity analysis [32] for basic reproduction numbers mainly helps to find parameters that have a high impact on the R_0 value, and should therefore be targeted for designing intervention strategies. In addition, sensitivity analysis helps to determine the level of change required for the input parameters to determine the desired predictor parameter value. For this analysis we apply the definition given in [33] as given below:

$$\Gamma_p^{R_0} = \frac{\partial R_0}{\partial p}\frac{p}{R_0}.$$

By definition in [21], the sensitivity indices of R_0 analytical formulas can be determined by the parameter values and initial populations of the United States (Case I) and India (Case II) in Table 1 from Wintachai and Prathom [28].

Furthermore, the sensitivity indices for the two countries are obtained as provided in Table 2 for the United State (case I) and Table 3 for India (case II), respectively.

Table 2 lists the values of the sensitivity index for R_0. Here, ν is defined as the vaccination rate of the population in the USA. $\nu = 0$ represents the population without vaccination, whereas $\nu = 0.1, 0.5, 0.8$ and 1.0 represent vaccination rates of 10%, 50%, 80% and 100%, respectively. Based on the sensitivity index, parameters with a positive index have a high impact on the burden of disease in society if their value increases. Likewise, parameters for which the sensitivity index is negative have the effect of minimizing the burden of disease in society because its value increases, while others are constant. In other words, as the value increases, the number of reproductions decreases, which minimizes the endemicity of the disease in the community. Table 2 shows the sensitivity index of the parameter for Case I (USA) for different v values. From Table 2, parameters p_e and α, which are the effectiveness of vaccination in S and transmission rates from E to I, have negative sensitivity indices. It has been shown that the parameter has the effect of minimizing the burden of disease in society. Similarly, parameter β, which is the effective transmission rate of COVID-19, has a positive sensitivity index. This means that if its value increases, it has a high impact on the burden of disease in society. This condition also occurred in case II (India), see Table 3. Table 3 shows that parameter β has a positive sensitivity index, whereas p_s has a negative sensitivity index. It can be concluded that, as the value increases, the number of reproductions decreases, which minimizes the endemicity of the disease in the community.

3.4. Optimal Control Analysis

In this section, we present a successful prevention plan for stopping this pandemic. The transmission of illnesses, particularly COVID-19, can be partially reduced using control methods. In this study, three control strategies to stop the epidemic from spreading, time-dependent control factors such as u_1 provide a method of controlling COVID-19 by educating the public about

Table 3. Sensitivity indices of R_0 according to each parameter (case 2 in India) for $p_s = 0.7, p_e = 0.6$ and $p_i = 0.6$ (Wintachai and Prathom [28]) in various values of vaccination (ν)

Parameter	$\nu = 0$	$\nu = 0.1$	$\nu = 0.5$	$\nu = 0.8$	$\nu = 1.0$
α	1.1618×10^{-4}	0.2117	0.4157	0.4570	0.4726
β	0.5	0.5	0.5	0.5	0.5
γ	-0.4998	-0.2590	-6.8986×10^{-2}	-3.4586×10^{-2}	-2.1866×10^{-2}
b_0	0.5	0.5	0.5	0.5	0.5
d_0	-0.5	-1.4224×10^{-4}	-2.8456×10^{-5}	-1.7785×10^{-5}	-1.4228×10^{-5}
d_1	-1.1618×10^{-4}	-7.1283×10^{-5}	-2.8000×10^{-5}	-1.9239×10^{-5}	-1.5919×10^{-5}
d_2	-1.4726×10^{-4}	-8.1162×10^{-5}	-2.9034×10^{-5}	-1.9595×10^{-5}	-1.6105×10^{-5}
p_s	0	-0.5375	-0.7692	-1.1363	-1.6667
p_e	0	-0.1932	-0.3795	-0.4172	-0.4315
p_i	0	-0.2244	-0.4014	-0.4335	-0.4453

health issues, u_2 indicates a control method that involves using personal defenses, such as seclusion, and u_3 depicts the care provided to COVID-19 patients in hospitals or isolation facilities to reduce their suffering from the disorders. By focusing on the dominant parameters, we modified the model to investigate the impact of preventative actions on future situations. After substitution of three controls, the model in [28] presents the following form:

$$\frac{dS}{dt} = b_0 - (\nu p_s + d_0)S - \beta * (1 - \nu p_s)SI - u_1 S, \quad (1)$$

$$\frac{dE}{dt} = \beta(1 - \nu p_s)SI - (d_1 + \alpha + (1 - \alpha)\nu p_e)E - u_2 E, \quad (2)$$

$$\frac{dI}{dt} = \alpha E - (d_2 + \gamma + (1 - \gamma)\nu p_i)I - (u_2 + u_3)I, \quad (3)$$

$$\frac{dR}{dt} = \nu p_s S + u_1 S + \nu p_e(1 - \alpha)E + u_2 E + (\gamma + (1 - \gamma)\nu p_i I + u_3 I - d_0 R. \quad (4)$$

The goal is to develop an optimal control for each of the three control schemes while minimizing their relative cost. Pontryagin's maximal principle [34] was used to determine the necessary and sufficient conditions for optimal control. In the time interval $[0, T]$, the objective function is defined as follows

$$J(u_1, u_2, u_3) = \int_0^T \left(A_1 E + A_2 I + \frac{b_1}{2}u_1^2 + \frac{b_2}{2}u_2^2 + \frac{b_3}{2}u_3^2 \right) dt, \quad (5)$$

where A_1 and A_1 represent the positive weights and b_1, b_2 and b_3 determines the relative cost of the intervention strategies being considered. The aim is to find the optimal value $u^* = (u_1^*, u_2^*, u_3^*)$ that minimizes the equation of the objective function (5) in the interval $[0, T]$. Consequently, the corresponding solution paths (S^*, E^*, I^*, R^*) that depend on u^* are the optimal solutions of the state system of eqs. (1) to (4).

Optimal control problem

Find optimal control (u_1^*, u_2^*, u_3^*) such that

$$J(u_1^*, u_2^*, u_3^*) = min\{J(u_1, u_2, u_3), (u_1, u_2, u_3) \in U\}. \quad (6)$$

The control set U is assumed to be Lebesgue measurable function defined by

$$U = \{(u_1, u_2, u_3) \mid u_i(t) \text{ is Lebsgue Measurable on } [0, T],\ 0 \le u_i(t) \le 1, i = 1, 2\}.$$

According Pontryagin's Maximum Principle, if the controls (u_1^*, u_2^*, u_3^*) and the corresponding state (S^*, E^*, I^*, R^*) are an optimal pair, respectively, necessarily there exist a costate variable or adjoint variable $\lambda(t)$ such that the function reaches its minimum on the set U point $u^* = (u_1^*, u_2^*, u_3^*)$. Here, a Hamiltonian $(H((S, E, I, R), t, U, \lambda))$ with respect to $u_i(t)$, where $H((S, E, I, R), t, U, \lambda)$ is defined as

$$H((S, E, I, R), t, U, \lambda) = A_1 E + A_2 I + \sum_{i=1}^{4} h_i \lambda_i(t) + \sum_{i=1}^{3} \frac{b_i}{2} u_i^2. \quad (7)$$

where

$$\begin{aligned}
h_1 &= b_0 - (\nu p_s + d_0)S - \beta(1 - v p_s)SI - u_1 S,\\
h_2 &= \beta(1 - \nu p_s)SI - (d_1 + \alpha + (1 - \alpha)\nu p_e)E - u_2 E,\\
h_3 &= \alpha E - (d_2 + \gamma + (1 - \gamma)\nu p_i)I - (u_2 + u_3)I,\\
h_4 &= \nu p_s S + u_1 S + \nu p_e(1 - \alpha)E + u_2 E\\
&\quad + (\gamma + (1 - \gamma)\nu p_i)I + u_3 I - d_0 R.
\end{aligned}$$

Theorem 1. *Given an optimal control $u^* = (u_1^*, u_2^*, u_3^*)$ and a solution to the corresponding state eqs. (1) to (4), (S^*, E^*, I^*, R^*), then there exists a non-trivial vector function $\lambda(t) = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ satisfying the following costate equations*

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= d_0\lambda_1 - u_1\lambda_4 + u_1\lambda_1 - \beta I\lambda_2 + \beta\lambda_1 I - \lambda_4 p_s \nu\\
&\quad + \lambda_1 \nu p_s + \beta\nu I\lambda_2 p_s - I\lambda_1 p_s \beta\nu,\\
\frac{d\lambda_2}{dt} &= A_1 + d_1\lambda_2 + \lambda_2 u_2 - \lambda_4 u_2 + \lambda_2\alpha - \lambda_3\alpha\\
&\quad + \lambda_2 p_e \nu - \lambda_4 p_e \nu - \lambda_2 p_e \alpha\nu + \lambda_4 p_e \alpha\nu,\\
\frac{d\lambda_3}{dt} &= -A_2 + d_2\lambda_3 + \gamma\lambda_3 - \gamma\lambda_4 + \lambda_3 u_2 + \lambda_3 u_3\\
&\quad - \lambda_4 u_3 - \lambda_2 S\beta + \lambda_1 S\beta + \lambda_3 p_i \nu - \gamma\lambda_3 p_i \nu\\
&\quad - \lambda_4 p_i \nu + \gamma\lambda_4 p_i \nu + \lambda_2 p_s S\beta\nu - \lambda_1 p_s S\beta\nu,\\
\frac{d\lambda_4}{dt} &= d_0\lambda_4.
\end{aligned} \quad (8)$$

where λ_1, λ_2, λ_3, and λ_4 are the adjoint variables and $y = (S, E, I, R)$. The minimization condition

$$H((S, E, I, R), t, u^*, \lambda) = \min\{H((S, E, I, R), t, U, \lambda) \mid (u_1, u_2, u_3) \in U\},$$

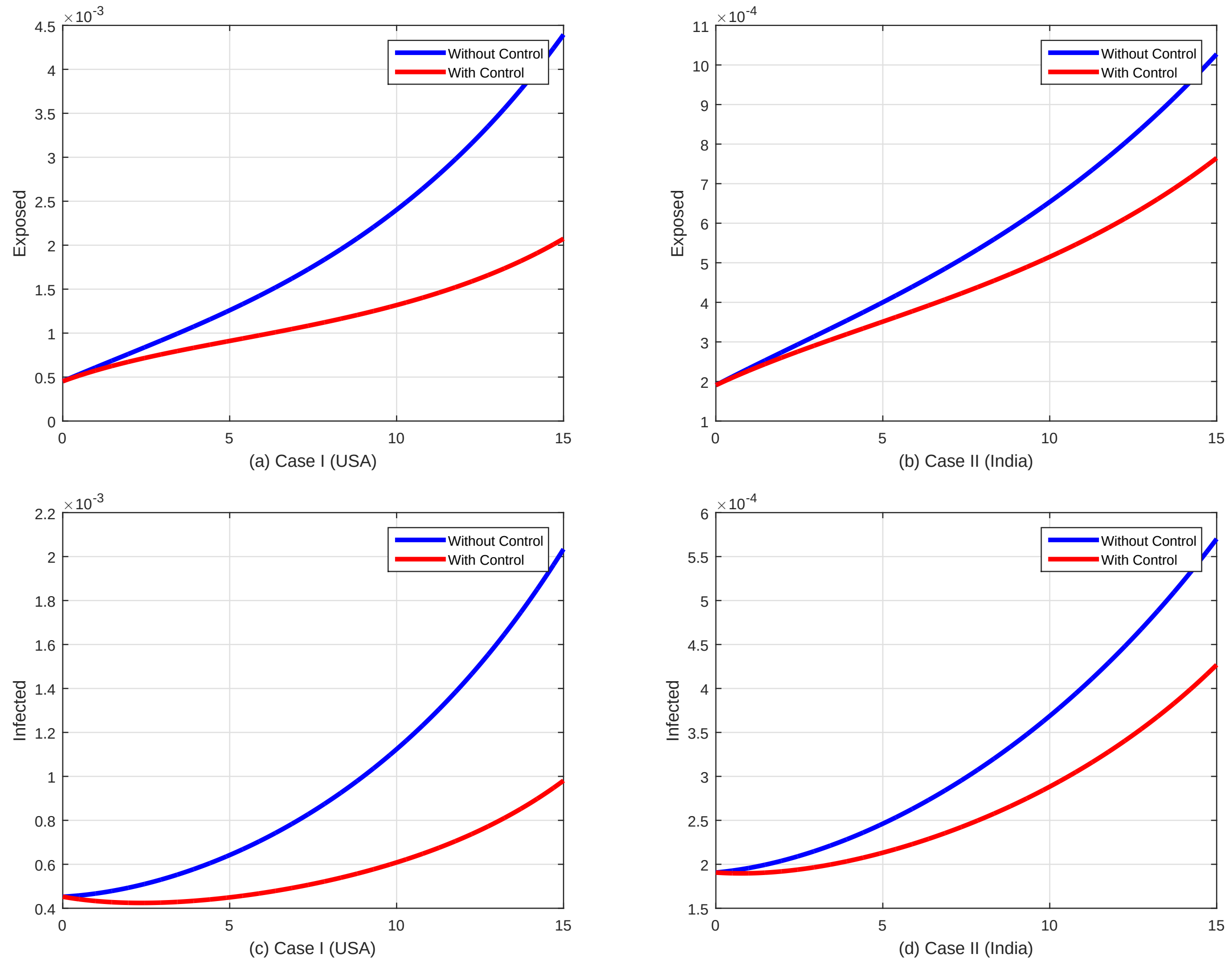

Figure 1. Display the dynamics of fraction of fraction of Exposed and Infected populations as well reservoirs with and without controls for both countries, respectively

holds almost everywhere on $[0,T]$. Furthermore, the optimal controls u_1^*, u_2^*, u_3^* are represented as

$$u_1^* = \max\left\{\min\left\{\frac{(\lambda_1-\lambda_4)S}{b_1},1\right\},0\right\}, \tag{9}$$

$$u_2^* = \max\left\{\min\left\{\frac{(\lambda_2-\lambda_4)E+\lambda_3 I}{b_2},1\right\},0\right\}, \tag{10}$$

$$u_3^* = \max\left\{\min\left\{\frac{(\lambda_3-\lambda_4)I}{b_3},1\right\},0\right\}, \tag{11}$$

Proof. The co-state equations can be found by

$$\frac{d\lambda_1}{dt} = \frac{\partial H}{\partial S},\ \lambda_1(T)=0,$$
$$\frac{d\lambda_2}{dt} = \frac{\partial H}{\partial E},\ \lambda_2(T)=0,$$
$$\frac{d\lambda_3}{dt} = \frac{\partial H}{\partial I},\ \lambda_3(T)=0,$$
$$\frac{d\lambda_4}{dt} = \frac{\partial H}{\partial R},\ \lambda_4(T)=0.$$

where $H = H((S,E,I,R),t,U,\lambda)$. Evaluated at the optimal controls and the corresponding states. For the transversally conditions to hold, we assume that the final states $S(T)$, $E(T)$, $I(T)$, and $R(T)$ are free so that $\frac{d\lambda(t)}{dt} = -\frac{\partial H(y,t,U,\lambda)}{\partial y} = 0$ where $y = (S,E,I,R)$. This results in $\lambda(T) = 0$. To characterize the controls, we use the minimality condition of the Pontryagin's Minimum Principle. The minimality condition in the interior of U and $t \in [0,T]$ is $\frac{\partial H}{\partial u_1} = \lambda_4 S - \lambda_1 S + u_1 b_1 = 0$, $\frac{\partial H}{\partial u_2} = -E\lambda_2 - I\lambda_3 + E\lambda_4 + u_2 b_2 = 0$ and $\frac{\partial H}{\partial u_3} = -I\lambda_3 + I\lambda_4 + u_3 b_3 = 0$. At u_1^*, u_2^*, u_3^* on this set

$$u_1 = \frac{(\lambda_1-\lambda_4)S}{b_1},$$
$$u_2 = \frac{(\lambda_2-\lambda_4)E+\lambda_3 I}{b_2},$$
$$u_3 = \frac{(\lambda_3-\lambda_4)I}{b_3},$$

Since the bound on u_1, u_2, u_3 are $0 \le u_1, u_2, u_3 \le 1$, the optimal controls u_1^*, u_2^*, u_3^* are represented as

$$u_1^* = \max\left\{\min\left\{\frac{(\lambda_1-\lambda_4)S}{b_0},0\right\},1\right\}, \tag{12}$$

$$u_2^* = \max\left\{\min\left\{\frac{(\lambda_2-\lambda_4)E+\lambda_3 I}{b_2},0\right\},1\right\}, \tag{13}$$

$$u_3^* = \max\left\{\min\left\{\frac{(\lambda_3-\lambda_4)I}{b_3},0\right\},1\right\}. \tag{14}$$

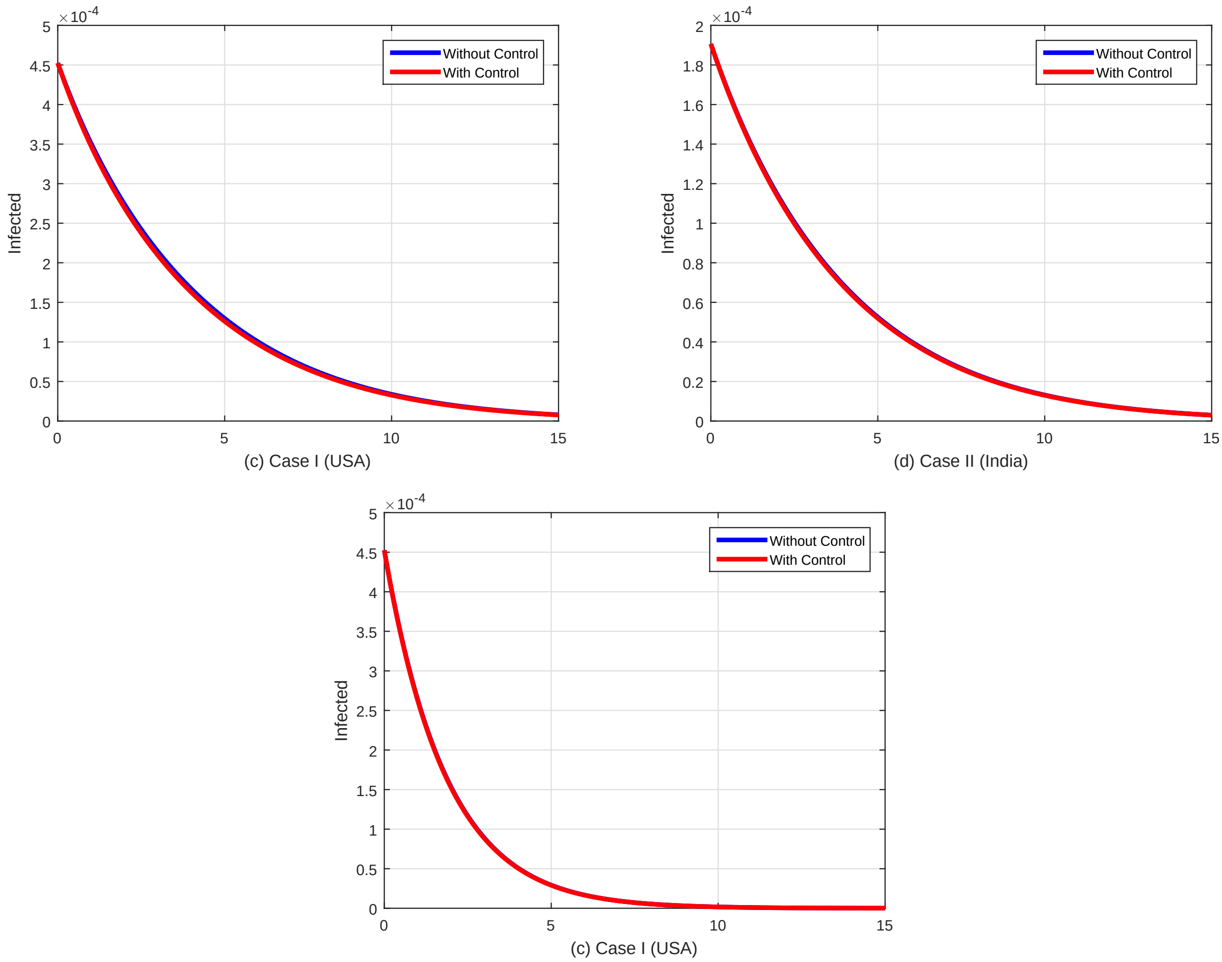

Figure 2. Display the dynamics of infected populations as well reservoirs with and without controls for both countries

Based on the Theorem 1, To solve the optimal control problem, we apply both the initial and boundary conditions with the characterization of the optimal controls $u^* = (u_1^*, u_2^*, u_3^*)$. In addition, the Hessian matrix of the Hamiltonian with respect to (u_1, u_2, u_3) is positive definite, which shows that the optimal control problem is minimum at the controls $u^* = (u_1^*, u_2^*, u_3^*)$. □

4. Numerical Illustration and Discussion

In this section, the optimal control and parameter sensitivity to reproduction number are analyzed. We used the forward-backward sweep method [35] in MATLAB 2015a to find the solution. The procedure starts by guessing the control variables, and then the system in eqs. (1) to (4) are solved simultaneously forward in time, and the estimated control variables and the yielding of a solution are substituted into the adjoining system, which is solved backwards in time using the transverse condition. The control is then updated using a convex combination of previous control and characterization values [36]. The parameters used for the simulations were described in [28].

The results of the optimal control of COVID-19 transmission in two countries, the USA and India, were numerically calculated. In accordance with Omede et al. [37], a control variable was assumed. In this study, we chose to determine the variable control 's parameters $\nu = 0, A_1 = 30\%, A_1 = 70\%, b_1 = 60\%, b_2 = 20\%$, and $b_3 = 20\%$, as shown in Figure 1. The Figure 1 demonstrates the impact of implementing various control strategies in both countries (USA and India) over 15 days. Figure 1 shows the exposed population for both countries, the USA in Figure 1(a) and India in Figure 1(b). The infected populations are shown in Figure 1(c) and 1(d). From Figure 1(a,c, we observe that the number of Exposed and Infected humans decreases dramatically when controls are applied. In India, Figure 1(b,d) show that the persons with Corona-virus decreases by applying when controls are applied. There was no considerable adoption by vaccinated individuals. From Figure 1, there is a significant effect of the use of control strategies in the USA compared to India.

Next, Figure 2 shows that the number of infected people will be reduced by implementing control and vaccination (ν). Here, vaccination is given to 50% and 100% of the USA and India, respectively. From Figure 2, by giving the vaccine to 50% of the population in India, the number of infected people can be reduced and the infected population can be approached with controls. Thus, the infected population will decrease if the Indian government distributes vaccines to only 50% of the population. Instead, the United States government must distribute 100% of vaccinations to the population in the USA. Figure 2(c) shows that up to 15 days, the population without control was the same as the population with control.

Finally, we present the profiles of three controls for 15 days

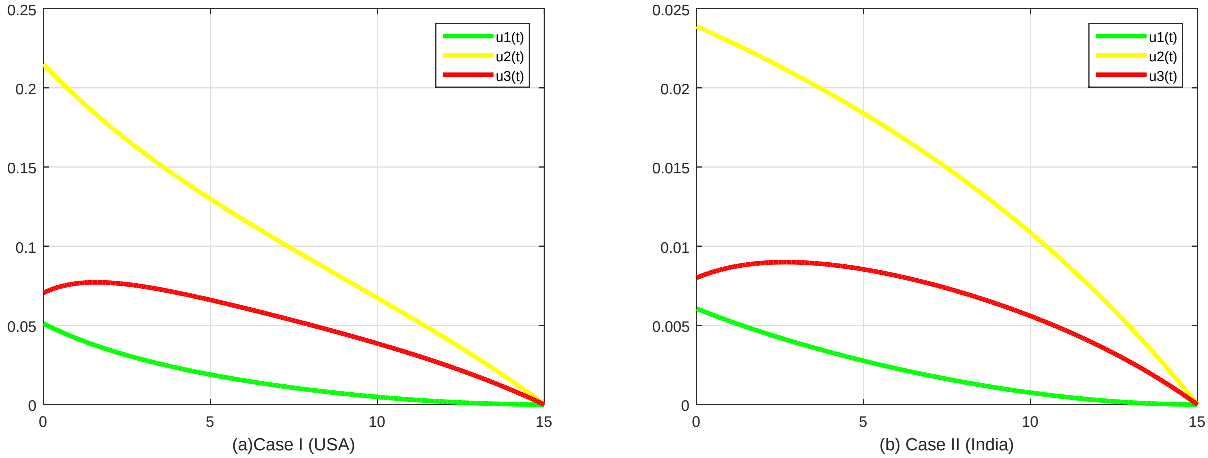

Figure 3. Optimal Function for (a) case I (USA), (b) case II (India)

for two cases: Case I for the spread of COVID-19 in the USA and Case II for the spread of COVID-19 in India for $\nu = 0, A_1 = 70\%, A_2 = 30\%, b_1 = 60\%, b_2 = 20\%$, and $b_3 = 20\%$ in the interval $t \in [0, 15]$, as shown in Figure 3.

From Figure 3, self-control, such as always using a mask, washing hands every day, isolating, increasing immunity, and reducing contact with other people, is in a higher position in the control profile than the care given to COVID-19 patients in hospitals. or isolation facilities to reduce the suffering of COVID-19 and public education on health issues. This situation occurs in both the USA and India. According to the findings, if the three control techniques are implemented ideally since the start of the pandemic, the number of cases in the compartment will be greatly reduced as long as there are no infected cases or hospitalizations.

5. Conclusion

This paper focuses on sensitivity analysis and optimal control with examples from the United States and India. One of the researchers' efforts was to provide alternative solutions to reduce the number of COVID-19 patients. Mathematical models are required to estimate disease transmission, recovery, and mortality. It is necessary to determine the parameters that are most sensitive to the transmission rate of a mathematical model by considering the sensitivity index. The results obtained reveal that the parameters that are the basis for reducing the number of COVID-19 infections for the two countries, the USA and India, are effective transmission rates from S to $E, (\beta)$, transmission rate from E to $I, (\alpha)$, and transmission rate from S to $R, (p_s)$ are the main parameters to watch for growth for Basic Reproduction rates (R_0). In addition, we implemented optimal control measures to reduce the number of people who have been infected individuals. To demonstrate the effect of the three control strategies on the selected model, a numerical simulation was performed. From the simulation results using the Forward-Backward Sweep method, by adopting the control variable in the SEIR model as previously described, with the control variable being the number of sufferers in both countries, The US and India are expected to be reduced. Using this control, the spread rate can be suppressed. From the results obtained, only implementing three types of controls and giving 50% of the vaccine to the people of India, the population decreased in seven days. In the USA, with the implementation of three types of control and administration of vaccines to 100% of the population, transmission can be overcome for 15 days. This control can be an alternative to reduce the spread of other similar diseases.

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Data availability. Data source is adopted from [28].

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