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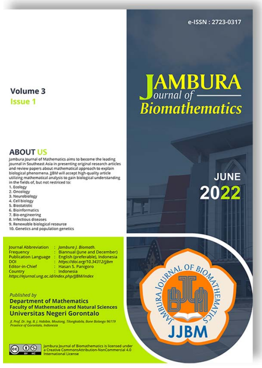

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Research Article

Two isolation treatments on the COVID-19 model and optimal control with public education

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ABSTRACT. *This study examines a COVID-19 mathematical model with two isolation treatments. We assume that isolation has two treatments: isolation with and without treatment. We also investigated the model using public education as a control. We show that the model has two equilibria based on the model without control. The basic reproduction number influences the local stability of the equilibrium and the presence of an endemic equilibrium. Therefore, the optimal control problem is solved by applying Pontryagin's Principle. In the 100th day following the intervention, the number of reported diseases decreased by 85.5% when public education was used as the primary control variable in the simulations.*

1. Introduction

The COVID-19 virus spreads quickly to different nations via people with a history of travel to Wuhan [1, 2]. The symptoms typically manifest gradually. In general, fever, a dry cough, and feeling tired are all signs of COVID-19. There are also other signs, like pain and tenderness in the chest, stuffy nose, headache, diarrhea, loss of taste or smell, etc [3]. The symptoms are typically minor and develop gradually. Also, people can get sick and have mild and severe symptoms, such as fever, cough, chest pain, and trouble breathing or shortness of breath [2, 3].

A strategy to contain the COVID-19 epidemic is necessary in light of the rising number of reported cases. When a new disease outbreak happens in an area with no vaccine or cure, the best way to handle it is by isolation and individual quarantine [4, 5]. Isolation and quarantine are often used interchangeably. Therefore, to clarify the difference, the WHO definition of isolation and quarantine is given [2], which defines isolation as separating infected individuals from others. As long as nothing exists pressing the necessity of going outside, quarantine limits activities or the separation of susceptible people. Then, the quarantine group also includes those who have previously been to a place where transmission of local occurs and kept themselves apart by remaining at home for the duration of the incubation period (2 weeks). These individuals may also have a history of exposure to COVID-19-infected individuals. Some of the WHO's suggestions for controlling COVID-19 are for people to wear masks in public, for people who may have the disease to track down all of their contacts and then be quarantined if they get sick, and for people who are sick to stay in hospitals or other facilities by themselves [6]. In addition, mathematical modeling is necessary to evaluate the spread of an infection and the effectiveness of control efforts.

Future disease propagation can be accurately predicted by mathematical modeling, which plays a significant part in this by looking at current or past conditions so that later it can provide advice on strategies for controlling the spread of disease. In addition, WHO recognizes that decision-makers in the health field (doctors or other health professionals) and policymakers can benefit from the insights provided (governments) by mathematical modeling [7]. Numerous studies, such as those on the Coronavirus, example caused SARS [8] and MERS [9, 10]. Then the COVID-19 virus emerged from the Coronavirus, which caused a stir in 2020.

COVID-19 spread model [11] utilizing the four subpopulations of the SEIR model, namely susceptible (S), exposed (E), infected (I), and recovered (R). Next there is research [12, 13] which adds subpopulations for quarantine (Q) and isolation (H), dividing seven subpopulations of the population: S, E, I, A, Q, H, and R. Research on COVID-19 [14] also added isolation (H) and quarantine (Q), so the model is built six subpopulations: S, E, I, Q, H, and R. Furthermore, there are many more studies that discuss the mathematical modeling of COVID-19 such as [15–20]. Reducing COVID-19's spread requires regulation (control) of the developed mathematical model. Many researchers have established controls to stop COVID-19 spread, including researcher [21], who lists three controls, namely public information, preventing the spread of the disease (such as wear of masks, wash of hands, and others), and treating affected people in hospitals.

Next, the researcher [22] provides two controls, namely, public education and treatment (in the hospital). The other researchers [23] reduced the number of infected by adding two controls, namely, the control of public education and care of medical.

This study was developed by dividing the isolation into two parts. So, the population is divided into six subpopulations. The model that has been constructed is validated using the *lsqcurve-*

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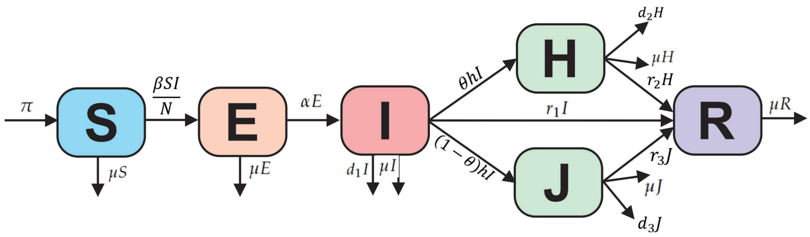

Figure 1. COVID-19 transmission diagram with two isolation treatments

Table 1. Interpretation and parameter values in the model

Notation	Interpretation	Value	Reference
π	Recruitment rate	3783175.865	[26]
K	Environmental carrying capacity		
β	Effective contact rate of S	0.098452	Fitted
α	Progression from E to I	0011828	Fitted
h	Isolation rate	0.0034904	Fitted
r_1	Recovery rate of an infected individual	0.0037098	Fitted
r_2	Recovery rate of isolation with treatment individual	0.95071	Fitted
r_3	Recovery rate of isolation without treatment individual	0.21124	Fitted
θ	Infected individuals proportion who became isolated with treatment	0.99721	Fitted
d_1	Fatalities due to COVID-19 rate in I	8.4073e-07	Fitted
d_2	Fatalities due to COVID-19 rate in H	0.0015666	Fitted
d_3	Fatalities due to COVID-19 rate in J	0.63684	Fitted
μ	Natural death rate	0.0138	[27]

fit function to find out the suitable parameters for the COVID-19 problem in Indonesia. Next, mathematical analysis is carried out, such as identifying 1) positive and boundedness solutions, 2) equilibrium points, 3) basic reproduction numbers, 4) the equilibrium point stability, and 5) optimal control. In the end, a numerical simulation will be carried out based on the parameter values of the parameter estimation results.

2. Mathematical Model

This section describes a COVID-19 model with two isolation treatments. There are six subpopulations within the population: 1) susceptible (S), 2) exposed (E), 3) infected (I), 4) isolated with treatment (H), 5) isolated without treatment (J), and 6) recovered (R). In this model, there are several assumptions that the infected subpopulation can recover without the need for isolation in the presence of immunity. Furthermore, isolation without treatment and isolation with treatment are conditions for patients who are not critical and critical or have comorbidities, respectively. Figure 1 is a diagram of the spread of COVID-19 with two isolation treatments which can be explained as follows.

$$
\begin{aligned}
\frac{dS}{dt} &= \pi - \frac{\beta SI}{N} - \mu S,\\
\frac{dE}{dt} &= \frac{\beta SI}{N} - \alpha E - \mu E,\\
\frac{dI}{dt} &= \alpha E - hI - r_1 I - d_1 I - \mu I,\\
\frac{dH}{dt} &= \theta h I - r_2 H - d_2 H - \mu H,\\
\frac{dJ}{dt} &= (1-\theta) hI - r_3 J - d_3 J - \mu J,\\
\frac{dR}{dt} &= r_1 I + r_2 H + r_3 J - \mu R.
\end{aligned} \tag{1}
$$

The verification of the COVID-19 model using two isolation treatments in eq. (1) is based on data infected with COVID-19 in Indonesia from November 1, 2020, to February 28, 2021, and the interpretation of several notations is presented as in Table 1. Furthermore, the results of fitting parameters obtained $MAPE = 0.010811$ and Figure 2 are very good.

3. Model Analysis

Based on total population $N = S + E + I + H + J + R$. Thus obtained

$$\frac{dN}{dt} \leq \pi - \mu N.$$

Suppose the initial value $N(t) = N(0)$ at $t = 0$ is obtained

$$N(t) \leq \frac{\pi}{\mu} + \left(N(0) - \frac{\pi}{\mu}\right) e^{-\mu t}.$$

Consequently for $t \to \infty$ then $\lim_{t\to\infty} N(t) \leq \frac{\pi}{\mu}$. So, we get a region of the solution is

$$\Omega = \left\{ (S, E, I, H, J, R) \mid N(t) \leq \frac{\pi}{\mu} \right\}.$$

Next is shown the positivity of the solution of the system (1).

Theorem 1. *Suppose S, E, I, H, J,and R are system solutions of eq. (1). If $S(0) \geq 0$; $E(0) \geq 0$; $I(0) \geq 0$; $H(0) \geq 0$; $J(0) \geq 0$; and $R(0) \geq 0$; then all solutions are positive for each $t \geq 0$.*

Proof. Take the first equation on the system (1) as follows

$$\frac{dS}{dt} = \pi - \beta SI - \mu S,$$

let $\eta = \frac{\beta I}{N}$. So, it can be written as

$$\frac{dS}{dt} = \pi - (\eta + \mu) S,$$

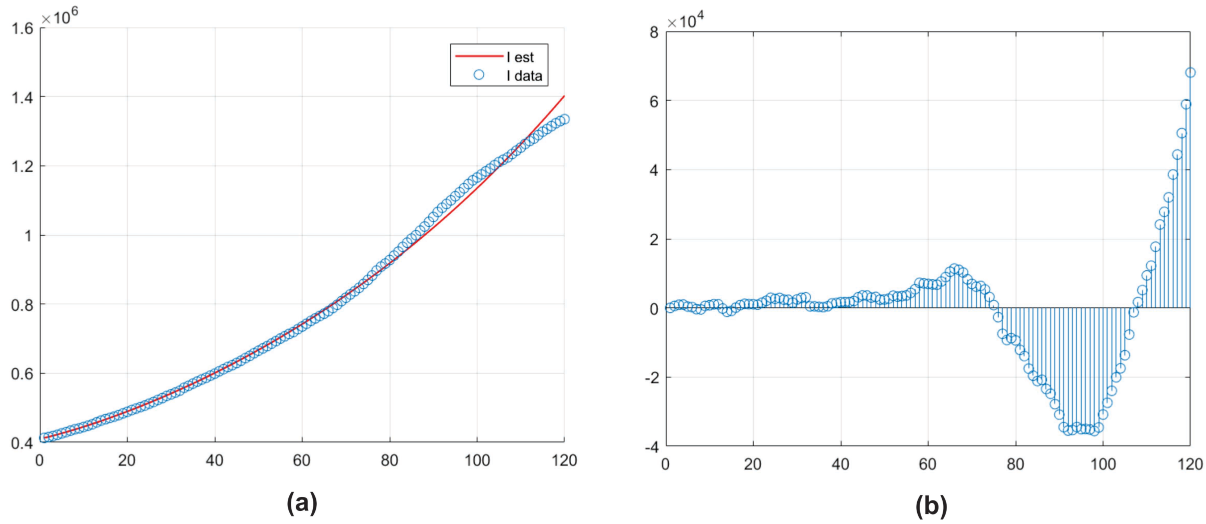

Figure 2. (a) COVID-19 model fittings, (b) residuals from COVID-19 model fittings

$$\frac{d\left(e^{\mu t+\int_0^t \eta\, ds}S\left(t\right)\right)}{dt}=\pi e^{\mu t+\int_0^t \eta\, ds}, \tag{2}$$

then a homogeneous solution is obtained

$$\frac{d\left(e^{\mu t+\int_0^t \eta\, ds}S\left(t\right)\right)}{dt}=0,$$
$$S\left(t\right)=ke^{-\mu t-\int_0^t \eta\, ds}.$$

Thus, a non-homogeneous solution is assumed

$$S\left(t\right)=ke^{-\mu t-\int_0^t \eta\, ds}. \tag{3}$$

Subsequently, by substituting eq. (3) for eq. (2), we get

$$\frac{dk\left(t\right)}{dt}=\pi e^{\mu t+\int_0^t \eta\, ds},$$
$$k\left(t\right)=\int_0^t \pi e^{\mu y+\int_0^y \eta\, dx}dy+K. \tag{4}$$

Substituting eq. (4) into eq. (3) yields

$$S\left(t\right)=\int_0^t \pi e^{\mu y+\int_0^y \eta\, dx}dy\times e^{-\mu t-\int_0^t \eta\, ds}+S(0)e^{-\mu t-\int_0^t \eta\, ds}.$$

So $S(t)$ is positive for $t\geq 0$.

Next, second equation of the system (1) can therefore be taken as

$$\frac{dE}{dt}=\frac{\beta SI}{N}-\alpha E-\mu E\geq -\alpha E-\mu E,$$

or

$$\int\frac{dE\left(t\right)}{E}\geq\int-\left(\alpha+\mu\right)dt,$$
$$E\left(t\right)\geq Ke^{-(\alpha+\mu)t},$$
$$E\left(t\right)\geq E\left(0\right)e^{-(\alpha+\mu)t}.$$

Thus $E(t)$ is positive for $t\geq 0$. Additionally, it can be demonstrated in the same manner starting with $I(t)$, $H(t)$, $J(t)$, and $R(t)$. □

At this point, a disease-free and endemic equilibrium point exists in the system (1). The disease-free equilibrium point is obtained

$$X_0=\left(S_0,E_0,I_0,H_0,J_0,R_0\right)=\left(\frac{\pi}{\mu},0,0,0,0,0\right).$$

The next-generation matrix approach is used to obtain the basic reproduction number, denoted R_0. The matrices F and V at the point X_0 are

$$F\left(X_0\right)=\begin{bmatrix}0 & \beta\\ 0 & 0\end{bmatrix},\text{ and } V\left(X_0\right)=\begin{bmatrix}a_1 & 0\\ -\alpha & a_2\end{bmatrix}.$$

Based on the matrix $F\left(X_0\right)$ and $V^{-1}\left(X_0\right)$, then the next-generation matrix $M=FV^{-1}$ so that we get

$$M=\begin{bmatrix}\frac{\beta\alpha}{a_1a_2} & \frac{\beta}{a_2}\\ 0 & 0\end{bmatrix}.$$

So the basic reproduction number is obtained

$$R_0=\frac{\beta\alpha}{a_1a_2}.$$

Next, the endemic equilibrium point is obtained $X_1=(S^*,E^*,I^*,H^*,J^*,R^*)$ where

$$\begin{aligned}S^*&=\frac{N}{R_0},\\ E^*&=\frac{\pi}{a_1R_0}\left(R_0-1\right),\\ I^*&=\frac{\pi}{\beta}\left(R_0-1\right),\\ H^*&=\frac{\theta h\pi}{a_3\beta}\left(R_0-1\right),\\ J^*&=\frac{\pi h\left(1-\theta\right)}{a_4\beta}\left(R_0-1\right),\\ R^*&=\frac{\pi\left(\left(r_1a_4+r_3h_1\left(1-\theta\right)\right)a_3+a_4h_1r_2\theta\right)}{a_3a_4\beta\mu}\left(R_0-1\right),\end{aligned}$$

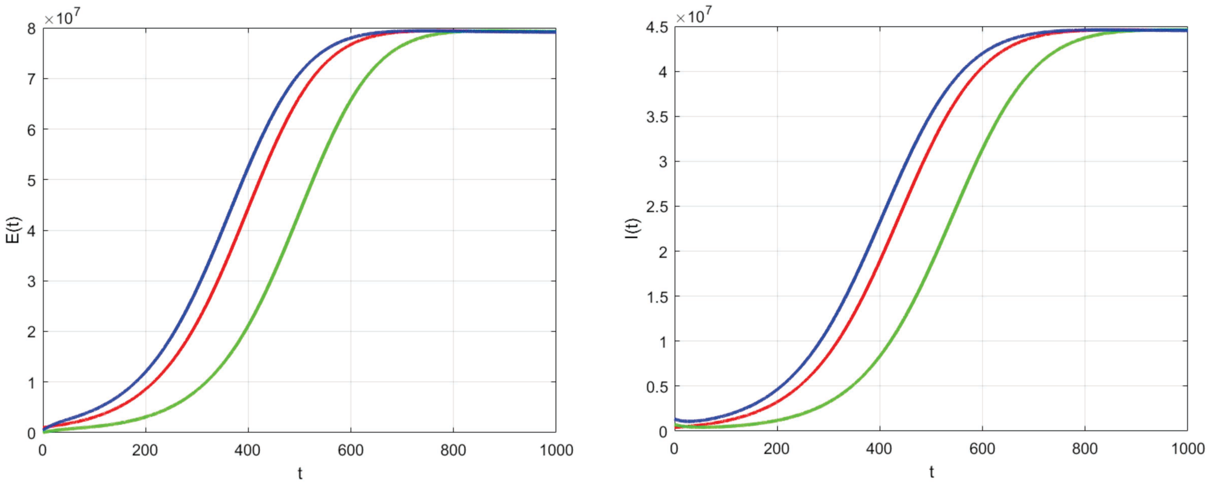

Figure 3. Behavior of the three different initial values

with $a_1 = \alpha + \mu$, $a_2 = h + r_1 + d_1 + \mu$, $a_3 = r_2 + d_2 + \mu$, and $a_4 = r_3 + d_3 + \mu$.

The value of R_0 determines whether the endemic equilibrium point X_1 exists. Furthermore, if $R_0 > 1$ then we get S^*, E^*, I^*, H^*,J^*, and R^* are positive, and there is exactly one endemic equilibrium point X_1. We demonstrate the next theorem's stability for X_0.

Theorem 2. *X_0 as disease-free equilibrium is unstable if $R_0 > 1$ and locally asymptotically stable if $R_0 < 1$.*

Proof. At X_0, the Jacobian matrix is given by

$$J\left(X^0\right) = \begin{bmatrix} -\mu & 0 & -\beta & 0 & 0 & 0 \\ 0 & -a_1 & \beta & 0 & 0 & 0 \\ 0 & \alpha & -a_2 & 0 & 0 & 0 \\ 0 & 0 & h\theta & -a_3 & 0 & 0 \\ 0 & 0 & h\left(1-\theta\right) & 0 & -a_4 & 0 \\ 0 & 0 & r_1 & r_2 & r_3 & -\mu \end{bmatrix}.$$

So the eigenvalues of λ_1, λ_2, λ_3, and λ_4 are negative. Therefore, the following characteristic equation determines how stable the disease-free equilibrium point is:

$$\lambda^2 + x_1\,\lambda + x_2 = 0, \tag{5}$$

with $x_1 = a_1 + a_2 > 0$, and $x_2 = a_1 a_2 - \beta\alpha = a_1 a_2\left(1 - R_0\right) > 0$, if $R_0 < 1$. Applying the Routh-Hurwitz criterion ensures that the real roots of the characteristic equation are negative if $R_0 < 1$. So X_0 is locally asymptotically stable if $R_0 < 1$. □

Theorem 3. *X_1 as endemic equilibrium is unstable if $R_0 < 1$ and locally asymptotically stable if $R_0 > 1$.*

Proof. the Jacobian matrix at the endemic equilibrium X_1 is given by

$$J\left(X_1\right) = \begin{bmatrix} -\mu & 0 & -\beta & 0 & 0 & 0 \\ 0 & -a_1 & \beta & 0 & 0 & 0 \\ 0 & \alpha & -a_2 & 0 & 0 & 0 \\ 0 & 0 & h\theta & -a_3 & 0 & 0 \\ 0 & 0 & h\left(1-\theta\right) & 0 & -a_4 & 0 \\ 0 & 0 & r_1 & r_2 & r_3 & -\mu \end{bmatrix}.$$

So we get the characteristic equation $\lambda^6 + x_1\,\lambda^5 + x_2\lambda^4 + x_3\lambda^3 + x_4\lambda^2 + x_5\lambda + x_6 = 0$. Roots of characteristic equations are difficult to solve analytically because they are related to sixth-order equations. Additionally, with the parameters in Table 1, it will be possible to test the endemic equilibrium's stability numerically, and we will get $R_0 = 2.158 > 1$. Figure 3 shows behavior with three different initial values when $R_0 = 2.158 > 1$ goes to a certain point, namely the endemic equilibrium point. So, the endemic equilibrium point will be asymptotically stable if $R_0 > 1$. □

4. Optimal Control

Public education (u) is used as the control variable. Therefore, the system (1) becomes

$$\begin{aligned} \frac{dS}{dt} &= \pi - (1-u)\,\frac{\beta SI}{N} - \mu S, \\ \frac{dE}{dt} &= (1-u)\,\frac{\beta SI}{N} - \alpha E - \mu E, \\ \frac{dI}{dt} &= \alpha E - hI - r_1 I - d_1 I - \mu I, \\ \frac{dH}{dt} &= \theta hI - r_2 H - d_2 H - \mu H, \\ \frac{dJ}{dt} &= (1-\theta)\,hI - r_3 J - d_3 J - \mu J, \\ \frac{dR}{dt} &= r_1 I + r_2 H + r_3 J - \mu R. \end{aligned} \tag{6}$$

Optimal control in this study aims to minimize the number of infected subpopulations and the objective function. The optimum control issue is expressed as follows:

$$J\left(u_1, u_2\right) = \int_0^T f\left(t, \vec{x}, \vec{u}\right) dt = \int_0^T \left(I + \frac{1}{2}\left(wu^2\right)\right) dt, \tag{7}$$

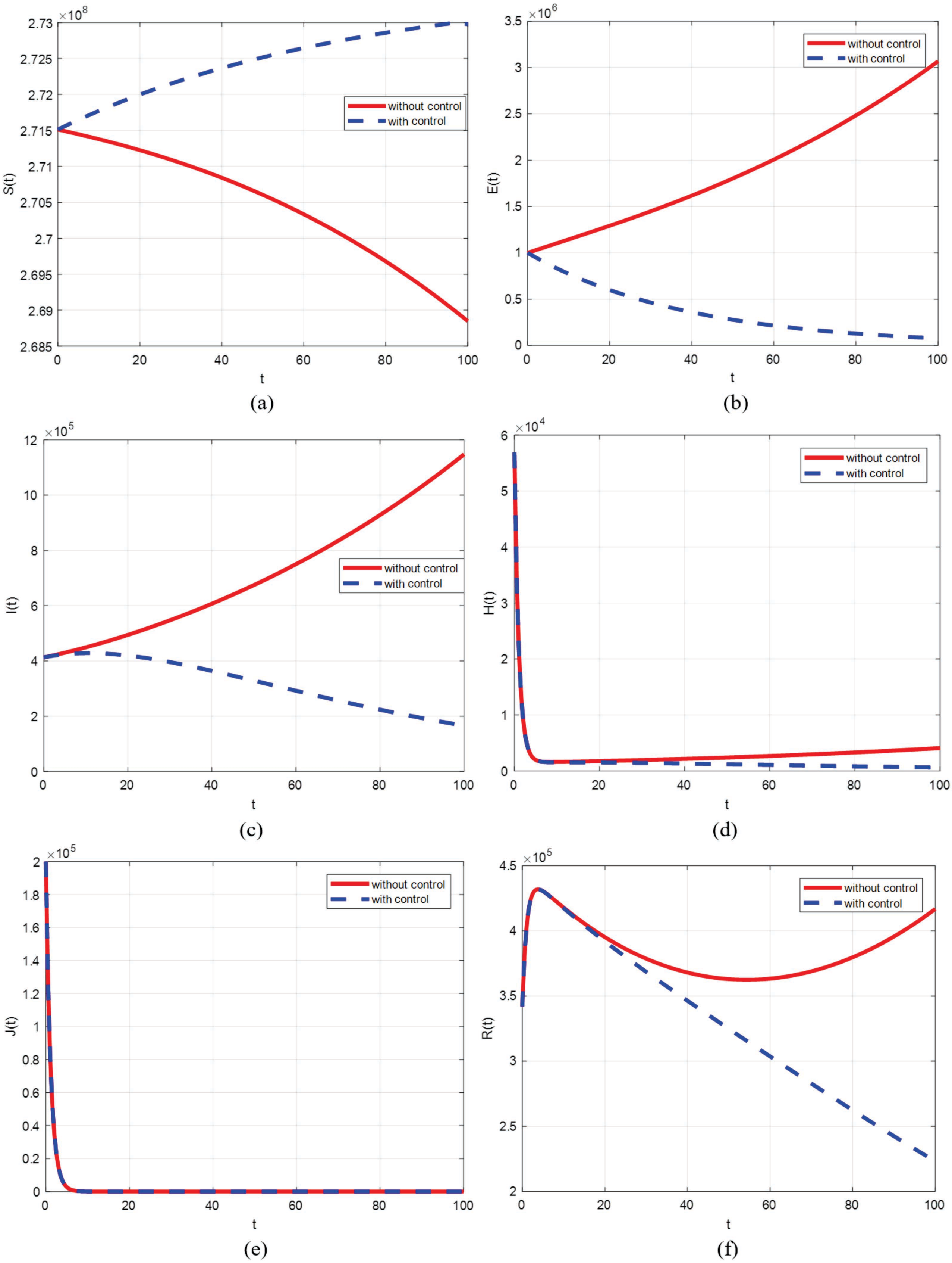

Figure 4. Subpopulation behavior of (a) susceptible, (b) exposed, (c) infected, (d) isolation with treatment, (e) isolation without treatment, (f) recovered

with $u \in [0,1]$, $t \in [0,T]$. Then w is the weight of public education control costs. Following a summary of the Hamilton function.

$$\begin{aligned}\mathcal{H} =& I + \tfrac{1}{2}\left(wu^2\right) + \lambda_1\left(\pi - (1-u)\tfrac{\beta SI}{N} - \mu S\right)\\ &+ \lambda_2\left((1-u)\tfrac{\beta SI}{N} - \alpha E - \mu E\right)\\ &+ \lambda_3\left(\alpha E - hI - r_1 I - d_1 I - \mu I\right)\\ &+ \lambda_4\left(\theta hI - r_2 H - d_2 H - \mu H\right)\\ &+ \lambda_5\left((1-\theta)hI - r_3 J - d_3 J - \mu J\right)\\ &+ \lambda_6\left(r_1 I + r_2 H + r_3 J - \mu R\right).\end{aligned} \tag{8}$$

For each state variable, the derivative of the Hamilton function (8) with a negative value is the costate equation.

$$\begin{aligned}\frac{d\lambda_1}{dt} &= -\frac{\partial \mathcal{H}}{\partial S}\\ &= \tfrac{(1-u)\beta I}{N}(\lambda_1 - \lambda_2) + \tfrac{(1-u)\beta SI}{N^2}(\lambda_2 - \lambda_1) + \mu\lambda_1,\\ \frac{d\lambda_2}{dt} &= -\frac{\partial \mathcal{H}}{\partial E}\\ &= \tfrac{(1-u)\beta SI}{N^2}(\lambda_2 - \lambda_1) + \alpha(\lambda_2 - \lambda_3) + \mu\lambda_2,\\ \frac{d\lambda_3}{dt} &= -\frac{\partial \mathcal{H}}{\partial I}\\ &= \tfrac{(1-u)\beta S}{N}(\lambda_1 - \lambda_2) + (h + d_1 + r_1 + \mu)\lambda_3\\ &\quad - h\theta\lambda_4 - (1-\theta)h\lambda_5 - r_1\lambda_6 - 1,\\ \frac{d\lambda_4}{dt} &= -\frac{\partial \mathcal{H}}{\partial H}\\ &= \tfrac{(1-u)\beta SI}{N^2}(\lambda_2 - \lambda_1) + (d_2 + r_2 + \mu)\lambda_4 - r_2\lambda_6,\\ \frac{d\lambda_5}{dt} &= -\frac{\partial \mathcal{H}}{\partial J}\\ &= \tfrac{(1-u)\beta SI}{N^2}(\lambda_2 - \lambda_1) + (d_3 + r_3 + \mu)\lambda_5 - r_3\lambda_6,\\ \frac{d\lambda_6}{dt} &= -\frac{\partial \mathcal{H}}{\partial R}\\ &= \tfrac{(1-u)\beta SI}{N^2}(\lambda_2 - \lambda_1) + \mu\lambda_6,\end{aligned}$$

with the transverse condition $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = \lambda_5(T) = \lambda_6(T) = 0$..

The optimal control problem (8) has a stationary condition that can be found by

$$u = \frac{\beta SI}{wN^2}(\lambda_2 - \lambda_1).$$

Based on the range of values $0 \le u(t) \le 1$, several possible values for u are obtained so that the optimal control value is obtained:

$$u^* = \max\left\{0, \min\left(\frac{\beta SI}{wN^2}(\lambda_2 - \lambda_1), 1\right)\right\}.$$

5. Numerical Simulation

The COVID-19 model with two isolation treatments is numerically simulated using the Runge-Kutta method. Initial values for each subpopulation: $S = 271,511,990$; $E = 1,000,000$; $I = 412,784$; $H = 56,899$; $J = 200,000$; and $R = 34,192$. Next, Table 1 displays the parameter values used in the simulation, and the observed time is 100 days. The numerical simulation results in Figure 4(a) show that the number of susceptible subpopulations increases due to the control of public education. Then, Figure 4(b)-(f) shows that each subpopulation decreased, especially the infected subpopulation decreased, to 85.5% in the 100th day due to public education.

Table 2 compares the number of subpopulations with and without controls after the intervention. The control used (u) seems quite influential, because it can reduce infections from 1,146,200 to 166,680.

Table 2. Compares subpopulation numbers at the end of the intervention

Subpopulation	Without control	With control
S	268,850,000	272,980,000
E	3,065,400	118,350
I	1,146,200	166,680
H	4,084.6	609.0786
J	12.7926	1.9136
R	416,600	223,180

Figure 5 illustrates the optimal control profile u^*, namely public education for 100 days. From the beginning to $t = 96$, the provision of public education control is given a maximum. Then, control continues to decrease until the end of the period is close to zero, with an optimal cost of 32,041,000.

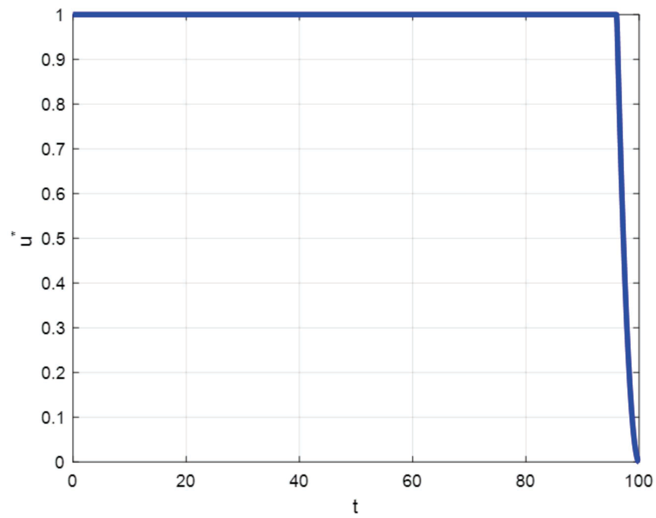

Figure 5. Optimal control profile

6. Conclusion

In this study, a COVID-19 model with two isolation treatments is presented. The control variable is public education. The goal of optimal control is to minimize the number of infected populations. The equilibrium of the model is disease-free and endemic. We also acquired the model's basic reproduction number R_0. While the endemic equilibrium exists and is asymptotically stable if $R_0 > 1$, the disease-free equilibrium is locally asymptotically stable if $R_0 < 1$. The simulation shows that the number of isolation subpopulations with treatment is greater than the isolation subpopulation without treatment. That is, many patients are not critical and few patients are critical or have comorbidities. Furthermore, the simulation results for 100 days show that implementing public education controls is more effective in reducing infectious population than without control.

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