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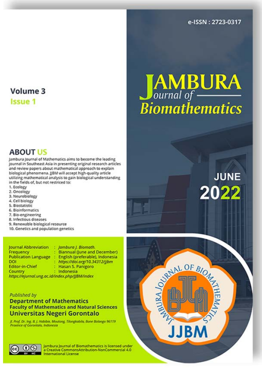

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Research Article

Comparison of Optimal Control Effect from Fungicides and Pseudomonas Fluorescens on Downy Mildew in Corn

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ABSTRACT. *Downy mildew is a disease that continues to infect corn crops in Timor Tengah Utara regency, reducing the amount of crop production and making corn farmers suffer losses. Farmers continue to look for ways to control downy mildew. Two treatments are commonly used by farmers, namely spraying Fungicides and Pseudomonas Fluorescens simultaneously in one unit of time, but have not resulted in optimal production. Therefore, this research is important to get a more optimal way to control find downy mildew. In this paper, we determine the optimal model of downy mildew control in corn plants by comparing the use of Fungicides and Pseudomonas Fluorescens. This research begins by forming a dynamic mathematical model consisting of six populations, namely four corn populations (S_h, F, P, I_h) and two populations of infecting fungi (S_v, I_v). Then they obtained the basic reproduction number (R_0) and two equilibrium points, namely the disease-free equilibrium and the disease-endemic equilibrium which has asymptotic stability. Numerical simulation results based on optimal control analysis with the minimum Pontryagin principle show that using fungicides can reduce the number of plants infected with downy mildew. Therefore, control by using fungicides is necessary and recommended increasing the number of downy mildew infected plants.*

1. Introduction

Corn is a food-crop commodity that plays an important role in the development of the agricultural sector. One of Indonesia's islands, Timor Island in East Nusa Tenggara Province, corn remains a staple food for rural people. Corn is also used as animal feed [1]. More interestingly, corn is used as a raw material for home or small industries in the Timor Tengah Utara (TTU) regency [2]. Corn is used as a material for the tortilla or corncracker agro-industry [3]. Thus, the planting of corn must continue to be improved.

Based on data from Badan Pusat Statistik (BPS) of East Nusa Tenggara Province in 2023, corn productivity in the TTU regency has fluctuated. In 2019, it was 46,621 tons, then in 2020, there was a decrease in the yield of $\pm 10,000$ tons, so that it became 36,406 tons, and in 2021 it increased again by $\pm 6,000$ tons so that it became 42,945 tons. TTU Regency ranks fourth highest in corn productivity in East Nusa Tenggara Province [4].

West Insana District, Bannae village, is a village with high corn productivity [5]. The farmers in the village continue to strive so that corn productivity does not decrease, so that it can fulfill consumer demand and needs. In fact, planting corn to produce corn kernels that are of good quality and does not reduce the quantity for consumption is not easy. Based on direct reports from farmers in Bannae village, there has been a decrease in corn productivity in recent years. The decrease in corn productivity occurred in 2021 to 2022 due to weather changes at the planting time of previous years, so that many diseases attacked corn. One of the diseases encountered in corn is downy mildew. Downy mildew in corn is caused by the fungus Peronosclerospora, which attacks corn leaves so that the growth of corn is inhibited and dwarfed, and the corn kernels produced are abnormal. The fungus spreads through the air. Cool and wet air can accelerate the growth of Peronosclerospora fungus [6]. Bannae Village is an area where the air is cool and wet so that it supports the growth of Peronosclerospora fungus and its rapid spread.

Controlling the spread of downy mildew on corn leaves can be done in several ways, namely by spraying fungicides made from active prodium and Carbio drugs on corn plants that are one to three weeks old. In addition, by using antagonistic microorganisms such as Pseudomonas Fluorescens Spraying. [7]. These two control measures are often practiced by farmers in Bannae village. Combining the two control measures will require more time and the results will not be optimal because each measure has a different effect on the number of downy mildew-free corn plants. Therefore, it is important to conduct this research to find effective control measures. This research begins with constructing a mathematical model of the dynamics of the transmission of downy mildew in corn by adding variables of fungicide treatment and variables of pseudomonas fluorescens treatment.Next, find the basic reproduction number as a center to know how many corn plants can be infected by one downy mildew-affected tree. After that, proceed with optimal control analysis using minimum

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Pontryagin principle and numerical simulation to support the results of the analysis. There is a lot of research on the mathematical model dynamics of the transmission of diseases in plants [8–11]. Two of them are making mathematical models for the transmission of diseases in plants in general, which is not specific to the type of plant and the disease [8], and the next one does research by making mathematical models on corn plants but not specific to the type of disease [12]. Research on the mathematical model of the spread of downy mildew in corn and optimal control as a control effort is rarely. One of the previous research that is close to the research to be carried out is the analysis and optimal control of the mathematical model of the spread of leaf disease in corn plants due to pathogens [13]. The research made a mathematical model and carried out control in the form of prevention efforts by applying fungicides and obtained research results that the provision of control can minimize the number of infected corn plant populations [13]. In the research, only one control treatment was taken, while this research was conducted by taking two control treatments, namely the treatment of fungicides and the treatment of pseudomonas fluorescens.This research has never been conducted by other researchers, so it is very important to do so in order to find out more optimal control for corn farmers. Variable U is the control variable for fungicide treatment and variable V is the control variable for pseudomonas fluorescens treatment. The last step of this research is to compare the two control treatments by looking at the numerical simulation results of each control treatment based on the optimal control analysis results.

2. Methods

This research is quantitative research by comparing the results of numerical simulations based on the results of optimal control analysis on the use of fungicides and Pseudomonas fluorescens to obtain more effective results.The data used from previous journal references and primary data from corn farmers in Bannae village. The steps taken to complete this research are to first create a mathematical model of the transmission of downy mildew in corn by adding variables of downy mildew control measures. Second, find the equilibrium point of the model that has been formed.After that, find the basic reproduction number (R_0), then analyze the equilibrium point obtained by connecting the R_0 value to determine the stability properties of the model created. Next, the optimal control analysis uses the minimum pontryagin principle method for each control variable. Variable U is the control variable for fungicide treatment and variable V is the control variable for pseudomonas fluorescens treatment. The last step is numerical simulation to see the dynamics of downy mildew spread in corn and compare the two control treatments based on the results of the optimal control analysis. The mathematical model formulated in this research is presented in chart form in Figure 1.

Based on Figure 1, the following equation is obtained which will be used in searching for the basic reproduction number. The basic model formed is Susceptible (S_h), which is a corn population that is susceptible to infection by downy mildew, and infected (I_h) is a corn population that has been infected with downy mildew and can spread it to susceptible populations. However, it should be noted in this study, that this basic model is given an additional class given Fungicide (F), namely the corn population given Fungicide and the class given Pseudomonas Fluorescens (P), namely the population given Pseudomonas Fluorescens. In addition, there are two sub populations of vectors that cause downy mildew, namely populations of susceptible fungi and populations of fungi that can infection plants. The purpose of adding this compartment is as a strategy in controlling the transmission of downy mildew in corn. The developed mathematical model can be said to be accurate if it has a stable equilibrium point, in this case the disease-free point is stable and the disease endemic point is stable. The following is a system of non-linear equation

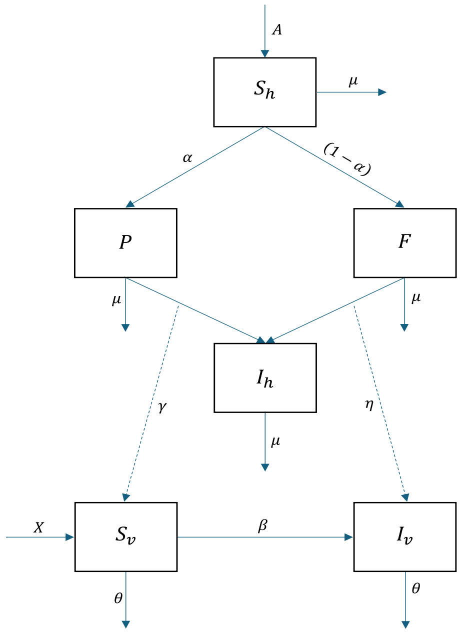


Figure 1. Diagram for the transmission of downy mildew disease in corn

$$
\begin{aligned}
\frac{dS_h}{dt} &= A-\alpha S_h-(1-\alpha)S_h-\mu S_h,\\
\frac{dF}{dt} &= (1-\alpha)S_h-\eta F I_v-\mu F,\\
\frac{dP}{dt} &= \alpha S_h-\gamma P I_v-\mu P,\\
\frac{dI_h}{dt} &= \eta F I_v+\gamma P I_v-\mu I_h,\\
\frac{dS_v}{dt} &= X-\beta S_v I_v-\theta S_v,\\
\frac{dI_v}{dt} &= \beta S_v I_v-\theta I_v.
\end{aligned}
\tag{1}
$$

where A is the rate of corn planting that enters the susceptible corn population, a is the proportion of providing control measures, γ and η is the strength rate of infection by peronosclerospora fungus in corn. X is peronosclerospora fungus natural birth rate which belongs to the group of susceptible fungi, β is the rate of movement of the fungi susceptible to infection fungi. One important formulation in this study is to include control variables in the mathematical model formed. The control variable in question is the U variable as a control variable for the giving of fungicide and the V variable as a control variable for giving Pseudomonas Fluorescens.Optimal control analysis is performed using the Minimum Pontryagin Principle. A comparison of the two control measures will be seen from the graphical results in the numerical simulation based on the results of the optimal control analysis.

3. Results and Discussion

From system (1), two equilibrium points are obtained, namely the disease-free equilibrium and the disease endemic equilibrium.

3.1. Equilibrium Points

a. Disease free equilibrium
The disease-free equilibrium (DFE) assumes that there are no fungal individuals attacking the corn, so there is no infected corn population. The DFE obtained is as follows

$$\begin{aligned} DFE &= (S_h, F, P, I_h, S_v, I_v) \\ &= \left(\frac{A}{\alpha+\mu}, \frac{A(1-\alpha)}{\mu(\mu+1)}, \frac{A\alpha}{\mu(\mu+1)}, 0, \frac{X}{\theta}, 0\right). \end{aligned} \tag{2}$$

b. Disease endemic equilibrium
The disease endemic equilibrium (DEE) is obtained from

$$\frac{S_h}{dt} = \frac{dF}{dt} = \frac{dP}{dt} = \frac{dI_h}{dt}, \frac{dS_v}{dt} = \frac{dI_v}{dt} = 0,$$

so that the DEE is obtained as follows

$$\begin{aligned} S_h^* &= \frac{A}{(\mu+1)}, \\ F^* &= \frac{\beta\theta A(1-\alpha)}{(\beta\eta X + \beta\eta\mu X + \beta\mu\theta + \beta\mu^2\theta - \eta\theta^2 - \eta\mu\theta^2)}, \\ P^* &= \frac{\alpha\beta\theta A}{\beta\gamma X + \beta\gamma\mu X + \beta\mu\theta + \beta\mu^2\theta - \gamma\theta^2 - \gamma\mu\theta^2}, \\ I_h^* &= \frac{(\beta\eta\gamma X + \alpha\beta\gamma\mu\theta + \beta\eta\mu\theta(1-\alpha) - \gamma\eta\theta^2)\, A(\beta X - \theta^2)}{(\mu(\beta\gamma X + \beta\gamma\mu X + \beta\mu\theta + \beta\mu^2\theta - \gamma\theta^2 - \gamma\mu\theta^2))}, \\ S_v^* &= \frac{\theta}{\beta}, \\ I_v^* &= \frac{\beta X - \theta^2}{\beta\theta}. \end{aligned}$$

3.2. Basic Reproduction Number

Basic reproduction number (R_0) is the average of infected individuals in the susceptible population produced by one infected individual. R_0 is obtained using the Next Generation Matrix (NGM) method. In this research, there are two subpopulations that will be the focus in NGM determination, namely subpopulation I_h as the infected subpopulation and subpopulation I_v as the infecting subpopulation. The two selected subpopulations are made based on the decomposition of the Jacobian matrix to obtain the following transmission matrix (F) and transition matrix (V) [14, 15].

$$\begin{aligned} F &= \begin{bmatrix} 0 & \dfrac{\eta(1-\alpha)A + \alpha\gamma A}{\mu(1+\mu)} \\ 0 & \dfrac{X\beta}{\theta} \end{bmatrix}, \\ V &= \begin{bmatrix} \mu & 0 \\ 0 & \theta \end{bmatrix}, \end{aligned}$$

Thus, the NGM is obtained as follows [16].

$$FV^{-1} = \begin{bmatrix} 0 & \dfrac{\eta(1-\alpha)A + \alpha\gamma A}{\theta\mu(1+\mu)} \\ 0 & \dfrac{\beta X}{\theta^2} \end{bmatrix},$$

R_0 obtained based on the spectral radius of the NGM, so it is obtained [17],

$$R_0 = \rho(FV^{-1}) = \frac{\beta X}{\theta^2}.$$

3.3. Stability Analysis of Equilibrium Point

The stability of each equilibrium can be determined based on the eigenvalues obtained from the substitution of the equilibrium point into the Jacobi matrix. If all the eigenvalues obtained are negative, then the point is stable. However, if any eigenvalue is zero or positive, then the point is not asymptotic stable [18, 19]. The following is the Jacobi matrix based on the system (1),

$$J = \begin{bmatrix} -1-\mu & 0 & 0 & 0 & 0 & 0 \\ 1-\alpha & -\eta I_v - \mu & 0 & 0 & 0 & -\eta F \\ \alpha & 0 & -\gamma I_v - \mu & 0 & 0 & -\gamma P \\ 0 & \eta I_v & \gamma I_v & -\mu & 0 & \eta F + \gamma P \\ 0 & 0 & 0 & 0 & -\beta I_v - \theta & -\beta S_v \\ 0 & 0 & 0 & 0 & \beta I_v & \beta S_v - \theta \end{bmatrix} \tag{3}$$

a. Stability analysis of disease-free equilibrium
To get the eigenvalues, the disease-free equilibrium eq. (2) are substituted into the Jacobi matrix eq. (3). So we get something like eq. (4)

$$J_0 = \begin{bmatrix} -1-\mu & 0 & 0 & 0 & 0 & 0 \\ 1-\alpha & -\mu & 0 & 0 & 0 & -\eta F \\ \alpha & 0 & -\mu & 0 & 0 & -\gamma P \\ 0 & 0 & 0 & -\mu & 0 & \eta F + \gamma P \\ 0 & 0 & 0 & 0 & -\theta & -\dfrac{\beta X}{\theta} \\ 0 & 0 & 0 & 0 & 0 & \dfrac{\beta X}{\theta} - \theta \end{bmatrix} \tag{4}$$

So that the eigenvalues obtained as follows

$$\begin{aligned} \lambda_1 &= -1-\mu, & \lambda_4 &= -\mu, \\ \lambda_2 &= -\mu, & \lambda_5 &= -\theta, \\ \lambda_3 &= -\mu, & \lambda_6 &= \frac{\beta X}{\theta} - \theta. \end{aligned}$$

The disease-free equilibrium will be stable if all eigenvalues are negative. The values of $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ have negative eigenvalues, but λ_6 does not always have negative values. Therefore, the manipulation of algebra is done by connecting to R_0, i.e.

$$\begin{aligned} \lambda_6 &< 0 \\ \frac{\beta X}{\theta} - \theta &< 0 \\ \frac{\beta X}{\theta^2} - 1 &< 0 \\ R_0 - 1 &< 0 \\ R_0 &< 1. \end{aligned}$$

Thus, the DFE will be asymptotically stable at $R_0 < 1$.

b. Stability Analysis of Disease Endemic Equilibrium
In the analysis of the stability of the disease endemic equilibrium (DEE), substitution of the DEE section 3.1 into the Jacobi matrix eq. (3) and the eigenvalues were obtained as follows

$$\begin{aligned}\lambda_1 &= -1-\mu,\\ \lambda_2 &= -\eta\left(\frac{\beta X-\theta^2}{\beta\theta}\right)-\mu,\\ \lambda_3 &= -\gamma\left(\frac{\beta X-\theta^2}{\beta\theta}\right)-\mu,\\ \lambda_4 &= -\mu,\\ \lambda_5 &= -\theta,\\ \lambda_6 &= -\left(\frac{\beta X-\theta^2}{\theta}\right).\end{aligned}$$

The DEE will be stable if all eigen values are negative. The values λ_1, λ_4, and λ_5 have negative eigen values, but λ_2, λ_3 and λ_6 it are not sure of a negative value. Therefore, manipulative algebra is done by connecting to R_0, i.e.

i. $\lambda_2 < 0$,

$$\begin{aligned}-\left(\frac{\beta X-\theta^2}{\beta\theta}\right)\eta-\mu &< 0,\\ \left(-\frac{X}{\theta}+\frac{\theta}{\beta}\right)\eta-\mu &< 0,\\ \left(-\frac{\beta X}{\theta^2}+1\right)\eta-\frac{\mu\beta}{\theta} &< 0\\ (1-R_0)\eta-\frac{\mu\beta}{\theta} &< 0 \Leftrightarrow R_0>1.\end{aligned}$$

ii. $\lambda_3 < 0$,

$$\begin{aligned}-\left(\frac{\beta X-\theta^2}{\beta\theta}\right)\gamma-\mu &< 0,\\ \left(-\frac{X}{\theta}+\frac{\theta}{\beta}\right)\gamma-\mu &< 0,\\ \left(-\frac{\beta X}{\theta^2}+1\right)\gamma-\frac{\mu\beta}{\theta} &< 0,\\ (1-R_0)\gamma-\frac{\mu\beta}{\theta} &< 0 \Leftrightarrow R_0>1.\end{aligned}$$

iii. $\lambda_6 < 0$,

$$\begin{aligned}-\left(\frac{\beta X-\theta^2}{\theta}\right) &< 0,\\ \left(-\frac{\beta X}{\theta}+\theta\right) &< 0,\\ \left(-\frac{\beta X}{\theta^2}+1\right) &< 0,\\ (1-R_0) &< 0 \Leftrightarrow R_0>1.\end{aligned}$$

Thus, the DEE will be asymptotically stable at $R_0 > 1$.

3.4. Optimal Control Problem

To minimize the number of infected corn in the dynamic model of downy mildew transmission in corn, fungicides and pseudomonas fluorescens are given as control measures. Both treatments do not produce the maximum amount of production if done together in one unit of time, so an optimal control comparison between the two treatments is needed. The solution to this optimal control, using the minimum Pontryagin Principle method. The first is Optimal Control Problem using Fungicides. The objective function formed is as follows [20–23]:

a. Optimal control problem using fungicides
The objective function formed is as follows:

$$J=\int_0^t (A_1I_h+A_2I_v+A_3U^2)dt.$$

The U variable is the control variable of the given control action. A_1, A_2 are the balanced weight of the infected population, and A_3 is the weight of the balanced cost of control. State variable for model

$$X(t)=\begin{bmatrix}S_h(t)\\F(t)\\P(t)\\I_h(t)\\S_v(t)\\I_h(t)\end{bmatrix},$$

with constrain

$$\begin{aligned}\frac{dS_h}{dt} &= A-\alpha S_h-(1-\alpha)US_h-\mu S_h,\\ \frac{dF}{dt} &= (1-\alpha)US_h-\eta FI_v-\mu F,\\ \frac{dP}{dt} &= \alpha S_h-\gamma PI_v-\mu P,\\ \frac{dI_h}{dt} &= \eta FI_v+\gamma PI_v-\mu I_h,\\ \frac{dS_v}{dt} &= X-\beta S_vI_v-\theta S_v,\\ \frac{dI_v}{dt} &= \beta S_vI_v-\theta I_v,\end{aligned}$$

and boundary conditions $t_0<t<t_1$, $0<t<1$, $S_h(0)\geq 0$, $F(0)\geq 0$, $P(0)\geq 0$, $I_h(0)\geq 0$, $S_v(0)\geq 0$, $I_v(0)\geq 0$. Based on the minimum Pontryagin Principle, the Hamilton function is $H=f(x,U,t)+\Phi' g(x,U,t)$ obtained as follows which is equivalent to

$$\begin{aligned}H = {} & A_1I_h+A_2I_v+A_3U^2+\Phi_1[A-(1-\alpha)US_h-\alpha S_h\\ & -\mu S_h]+\Phi_2[(1-\alpha)US_h-\eta FI_v-\mu F]+\Phi_3[\alpha S_h\\ & -\gamma PI_v-\mu P]+\Phi_4[\eta FI_v+\gamma PI_v-\mu I_h]+\Phi_5[X\\ & -\beta S_vI_v-\theta S_v]+\Phi_6[\beta S_vI_v-\theta I_v],\end{aligned}$$

where Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, and Φ_6 are co-state variables or Lagrange multipliers. The conditions necessary to achieve optimal control must satisfy the following minimum Pontryagin Principles:

i. State Equations

$$\frac{\partial S_h(t)}{dt} = \frac{\partial H}{\partial \Phi_1} = A - (1-\alpha)US_h - \alpha S_h - \mu S_h,$$
$$\frac{\partial F(t)}{dt} = \frac{\partial H}{\partial \Phi_2} = (1-\alpha)US_h - \eta F I_v - \mu F,$$
$$\frac{\partial P(t)}{dt} = \frac{\partial H}{\partial \Phi_3} = \alpha S_h - \gamma P I_v - \mu P,$$
$$\frac{\partial I_h(t)}{dt} = \frac{\partial H}{\partial \Phi_4} = \eta F I_v + \gamma P I_v - \mu I_h,$$
$$\frac{\partial S_v(t)}{dt} = \frac{\partial H}{\partial \Phi_5} = X - \beta S_v I_v - \theta S_v,$$
$$\frac{\partial I_v(t)}{dt} = \frac{\partial H}{\partial \Phi_6} = \beta S_v I_v - \theta I_v.$$

ii. Co-state equation

$$\Phi_1 = -\frac{\partial H}{\partial S_h} = -\left[\Phi_1(-(1-\alpha)U - \alpha - \mu) + \Phi_2(1 - \alpha)U + \Phi_3\alpha\right],$$
$$\Phi_2 = -\frac{\partial H}{\partial F} = -\left[\Phi_2(-\eta I_v - \mu) + \Phi_4(\eta I_v)\right],$$
$$\Phi_3 = -\frac{\partial H}{\partial P} = -\left[\Phi_3(-\gamma I_v - \mu) + \Phi_4\gamma I_v\right],$$
$$\Phi_4 = -\frac{\partial H}{\partial I_h} = -\left[A_1 - \Phi_4 - \mu\right],$$
$$\Phi_5 = -\frac{\partial H}{\partial S_v} = -\left[-\Phi_5(\beta I_v - \theta) + \Phi_6\beta I_v\right],$$
$$\Phi_6 = -\frac{\partial H}{\partial I_v} = -\left[A_2 - \Phi_2\eta F - \Phi_3\gamma P + \Phi_4(\eta F + \gamma P) - \Phi_5(\beta S_v) + \Phi_6\beta S_v - \theta\right].$$

iii. Transversal condition

$$0 = \frac{\partial H}{\partial U},$$
$$0 = 2A_3U - \Phi_1(1-\alpha)S_h + \Phi_2(1-\alpha)S_h,$$
$$0 = 2A_3U - (\Phi_1(1-\alpha) + \Phi_2(1-\alpha))\,S_h,$$
$$2A_3U = (\Phi_1 + \Phi_2)(1-\alpha)S_h,$$
$$U = \frac{(\Phi_1 + \Phi_2)(1-\alpha)S_h}{2A_3}.$$

Since $0 \le U \le 1$, then we get :

$$U = \min\left\{\max\left\{0, \frac{(\Phi_1 - \Phi_2)(1-\alpha)S_h}{2A_3}, 1\right\}\right\}.$$

b. Optimal control problem using pseudomonas fluorescens
The objective function formed is as follows

$$J = \int_0^t (B_1 I_h + B_2 I_v + B_3 V^2)dt$$

The variable V is the control variable of a given treatment. B_1 and B_2 are the balanced weights of the infected population, and B_3 is the weight of the balanced treatment cost.

State variables for the model

$$Y(t) = \begin{bmatrix} S_h(t) \\ F(t) \\ P(t) \\ I_h(t) \\ S_v(t) \\ I_h(t) \end{bmatrix}$$

with constrain

$$\frac{dS_h}{dt} = A - \alpha V S_h - (1-\alpha)S_h - \mu S_h,$$
$$\frac{dF}{dt} = (1-\alpha)S_h - \eta F I_v - \mu F,$$
$$\frac{dP}{dt} = \alpha V S_h - \gamma P I_v - \mu P,$$
$$\frac{dI_h}{dt} = \eta F I_v + \gamma P I_v - \mu I_h,$$
$$\frac{dS_v}{dt} = X - \beta S_v I_v - \theta S_v,$$
$$\frac{dI_v}{dt} = \beta S_v I_v - \theta I_v,$$

and boundary conditions $t_0 < t < t_1$, $0 < t < 1$, $S_h(0) \ge 0$, $F(0) \ge 0$, $P(0) \ge 0$, $I_h(0) \ge 0$, $S_v(0) \ge 0$, $I_v(0) \ge 0$. Based on the minimum Pontryagin Principle, the Hamilton function is $H = f(Y,U,t) + \Psi' g(Y,V,t)$ obtained as follows which is equivalent to

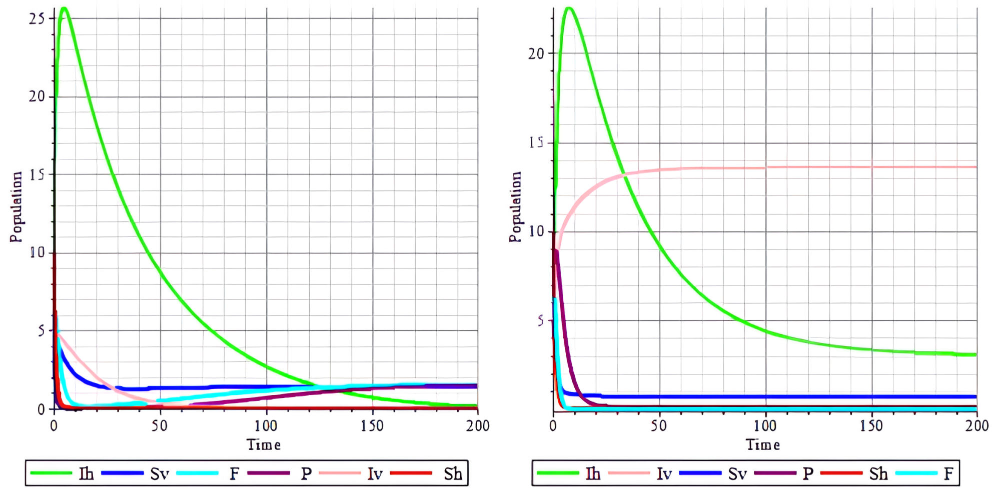
$$H = B_1 I_h + B_2 I_v + B_3 V^2 + \Psi_1[A - (1-\alpha)S_h - \alpha V S_h - \mu S_h] + \Psi_2[(1-\alpha)S_h - \eta F I_v - \mu F] + \Psi_3[\alpha V S_h - \gamma P I_v \mu P] + \Psi_4[\eta F I_v + \gamma P I_v - \mu I_h] + \Psi_5[X - \beta S_v I_v - \theta S_v] + \Psi_6[\beta S_v I_v - \theta I_v],$$

where Ψ_1, Ψ_2, Ψ_3, Ψ_4, Ψ_5, and Ψ_6 are co-state variables or Lagrange multipliers. The conditions necessary to achieve optimal control must satisfy the following minimum Pontryagin Principles:

i. State Equations

$$\frac{\partial S_h(t)}{dt} = \frac{\partial H}{\partial \Psi_1} = A - (1-\alpha)S_h - \alpha V S_h - \mu S_h,$$
$$\frac{\partial F(t)}{dt} = \frac{\partial H}{\partial \Psi_2} = (1-\alpha)S_h - \eta F I_v - \mu F,$$
$$\frac{\partial P(t)}{dt} = \frac{\partial H}{\partial \Psi_3} = \alpha V S_h - \gamma P I_v - \mu P,$$
$$\frac{\partial I_h(t)}{dt} = \frac{\partial H}{\partial \Psi_4} = \eta F I_v + \gamma P I_v - \mu I_h,$$
$$\frac{\partial S_v(t)}{dt} = \frac{\partial H}{\partial \Psi_5} = X - \beta S_v I_v - \theta S_v,$$
$$\frac{\partial I_v(t)}{dt} = \frac{\partial H}{\partial \Psi_6} = \beta S_v I_v - \theta I_v.$$

ii. Co-state equation



(a) $R_0 < 1$ (b) $R_0 > 1$

Figure 2. Population dynamics of corn

$$
\begin{aligned}
\Psi_1 &= -\frac{\partial H}{\partial S_h} = -[\Psi_1(-(1-\alpha) - \alpha V - \mu) + \Psi_2(1 \\
&\quad - \alpha) + \Psi_3 \alpha V], \\
\Psi_2 &= -\frac{\partial H}{\partial F} = -[\Psi_2(-\eta I_v - \mu) + \Psi_4(\eta I_v)], \\
\Psi_3 &= -\frac{\partial H}{\partial P} = -[\Psi_3(-\gamma I_v - \mu) + \Psi_4 \gamma I_v,] \\
\Psi_4 &= -\frac{\partial H}{\partial I_h} = -[B_1 - \Psi_4 - \mu], \\
\Psi_5 &= -\frac{\partial H}{\partial S_v} = -[-\Psi_5(\beta I_v - \theta) + \Psi_6 \beta I_v], \\
\Psi_6 &= -\frac{\partial H}{\partial I_v} = -[B_2 - \Psi_2 \eta F - \Psi_3 \gamma P + \Psi_4(\eta F \\
&\quad + \gamma P) - \Psi_5(\beta S_v) + \Psi_6 \beta S_v - \theta].
\end{aligned}
$$

iii. Transversal condition

$$
\begin{aligned}
0 &= \frac{\partial H}{\partial V}, \\
0 &= 2B_3 V - \Psi_1 \alpha S_h + \Psi_3 \alpha S_h, \\
0 &= 2B_3 V - (\Psi_1 \alpha + \Psi_3 \alpha) S_h, \\
2B_3 V &= (\Psi_1 + \Psi_3) \alpha S_h, \\
V &= \frac{(\Psi_1 + \Psi_3) \alpha S_h}{2B_3}.
\end{aligned}
$$

Since $0 \le V \le 1$, then we get :

$$
V = \min\left\{\max\left\{0, \frac{(\Psi_1 - \Psi_3)\alpha S_h}{2B_3}, 1\right\}\right\}.
$$

3.5. Numerical Simulation

The following is a numerical simulation to see the dynamics of the spread of downy mildew in corn, with and without control using the parameter values in Table 1, with the initial conditions for each compartment being $S_h = 10, F = 6, P = 7, I_h = 8, S_v = 5, I_v = 5$.

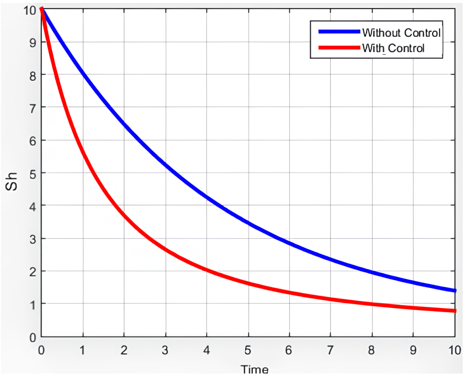
Table 1. Parameter values

Parameter	$R_0 > 1$	$R_0 < 1$	Source
X	$0,1$	1	[24]
α	$0,5$	$0,5$	asumption
A	$0,1$	$0,1$	asumption
μ	$0,03$	$0,03$	[25]
γ	$0,8$	$0,02$	[7]
β	$0,01$	$0,1$	[24]
θ	$0,07$	$0,07$	[24]
η	$0,03$	$0,1$	[13]

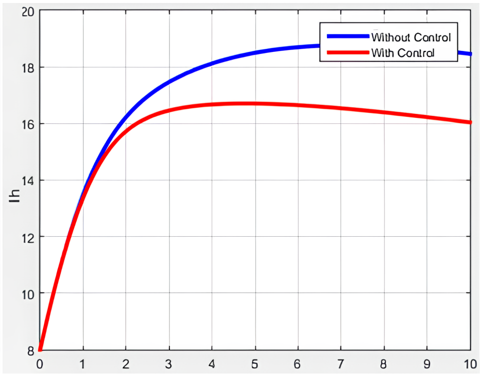
Figure 2a shows that when the value of $R_0 < 1$, the population of infected plants (I_h) will decrease, and then the population will be extinct after 170 days. The infecting vector population (I_v) will decrease, and the population will be extinct after the 50th day. Next, the populations of corn treated with pseudomonas (P), corn treated with fungicide (F), susceptible corn (S_h) and susceptible fungus (S_v) decreased and then increased again and constant at $\pm$ 2 individuals after day 130. Figure 2b, shows the situation when $R_0 > 1$, the infected plant population (I_h) increases then decreases until constant at $\pm$ 3 individuals after day 150. The infected vector population increases and remains constant at $\pm$ 14 after day 50. Furthermore, the population of corn treated with Pseudomonas (P), corn treated with fungicide (F), susceptible corn (S_h) decreased from before day 10, while the susceptible fungus (S_v) decreased and then remained constant after day 20.

Based on these conditions, it can be concluded that if the value in the field is the same as the value listed in Table 1 in the $R_0 < 1$ section, downy mildew will become extinct. Meanwhile, if the value in the field is the same as the value listed in Table 1 in the $R_0 > 1$ section then downy mildew will not become extinct.

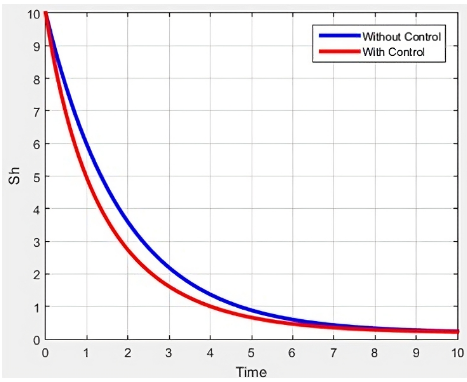
a. Control by fungicide
Figure 3a shows that the infected corn plant population (I_h) grew more slowly in the number of infected when given control compared to when not given control. The population slowly began to decline on day 6 with the total population of infected corn treated with fungicide control being no more



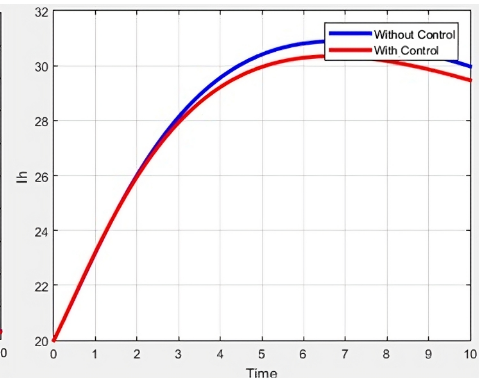
(a) S_h with fungicide control



(b) I_h with fungicide control



(c) S_h with pseudomonas fluorescens control



(d) I_h with pseudomonas fluorescens control

Figure 3. Dynamics of susceptible (S_h) and infected (I_h) populations of corn with and without control

than 16 corn plants. Figure 3b shows that the population of susceptible corn plants (S_h) will decline more quickly when given control. It can be concluded that when fungicide control is given to corn plants, the number of corn plants that can be affected by downy mildew is less than when no control is given.

b. Control by pseudomonas fluorescens
Figure 3d shows that the infected corn plant population (I_h) grew more slowly in the number of infected when given control compared to when no control was given and the infected corn population (I_h) slowly began to decline on the following day. 8th with the number of infected corn populations given pseudomonas fluorescens control of no more than 30 corn plants. Figure 3c shows that the population of susceptible corn plants (S_h) will decline more quickly when given control. Based on the simulation results for each population during the action of giving pseudomonas fluorescens, it shows that when control is given to corn plants, the number of corn plants that can be affected by downy mildew disease is less than when control is not given.

4. Conclusion

The model constructed in this research is a dynamic mathematical model of the transmission of downy mildew in corn by adding two control populations. The two populations are population F (population of corn treated with fungicide) and population P (population of corn treated with pseudomonas fluorescens). The model consists of six populations, namely four maize populations (S_h, F, P, I_h) and two infecting fungal populations (S_v, I_v). Next, the expression for the basic reproduction number is $R_0 = \frac{X\beta}{\theta^2}$ and two equilibrium points are obtained, namely the disease-free equilibrium which is asymptotically stable at $R_0 < 1$ and the disease endemic equilibrium which is asymptotically stable at $R_0 > 1$. After that, an optimal control analysis of each control measure is carried out using the minimum Pontryagin principle in order to see the comparison between the two control measures. Variable U is the control variable for fungicide treatment and variable V is the control variable for pseudomonas fluorescens treatment. Numerical simulation results based on optimal control analysis with the Minimum Pontryagin principle, when comparing the two control measures, show that using fungicides can reduce the number of downy mildew infected plants. Therefore, control by treatment of fungicides is necessary and recommended to increase the number of healthy or susceptible corn plants.

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