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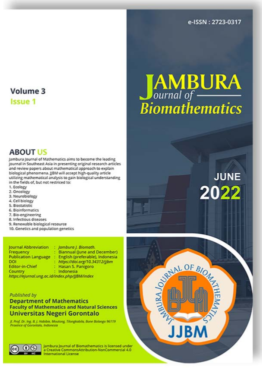

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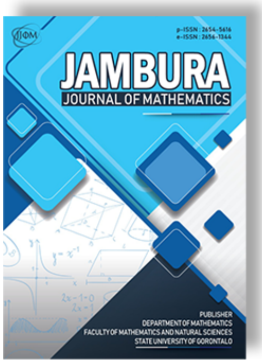

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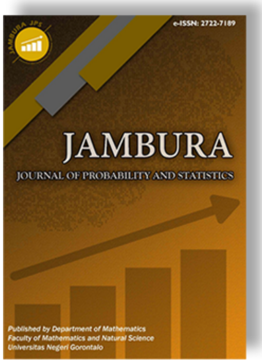

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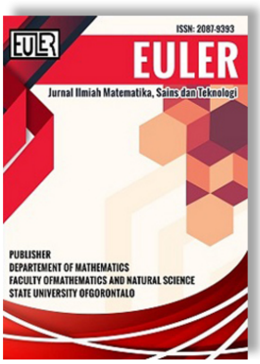

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Research Article

Mathematical Analysis on the Effects of Microplastic Pollution and Ocean Acidification on Coral Reefs in Aquatic Ecosystem

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ABSTRACT. *This study explores the complex interplay between microplastic contamination and ocean acidification in influencing coral reef ecosystems through the development of a mathematical model with time-varying parameters. The model ensures positivity and boundedness to accurately represent ecological dynamics, and stability analyses provide insights into system behavior under various environmental conditions. Numerical simulations validate the theoretical results and reveal that microplastic accumulation in marine environments significantly hinders coral reef establishment while contributing to elevated oceanic carbon dioxide levels. These rising* CO_2 *levels, primarily driven by anthropogenic emissions, lead to accelerated ocean acidification, further degrading coral reefs. Model predictions indicate that, if unchecked, the current trends in microplastic pollution and ocean acidification will result in a 50% reduction in coral reef coverage within four decades. However, the findings suggest that limiting microplastic input into aquatic ecosystems could mitigate these adverse effects, preserving reef health and slowing acidification. By quantifying the relationship between microplastic pollution, ocean acidification, and coral reef dynamics, this study provides a robust framework for understanding and addressing critical threats to marine ecosystems.*

1. Introduction

Plastic pollution and ocean acidification are two interconnected stressors threatening marine ecosystems. Microplastics, originating from various human activities, persist in aquatic environments, harming marine organisms and contributing to acidification through chemical degradation. Simultaneously, increased absorption of atmospheric CO_2 lowers seawater pH and reduces carbonate ion availability, impairing coral calcification. As coral reefs are highly sensitive to both stressors, their combined impact accelerates reef degradation, affecting biodiversity, coastal protection, and livelihoods. This study develops a mathematical model to investigate these joint effects and understand the resulting ecological dynamics.

Plastics are indispensable in modern life, widely used across agriculture, electronics, and packaging. Production rose from 0.5 million tonnes/year to 300 million tonnes by 2013 [1], reaching 320 million in 2018 [2], and may reach 33 billion tonnes by 2050. However, only 6–26% is recycled [2, 3], raising significant concern over microplastics and nanoplastics. Microplastics (< 5 mm) and nanoplastics (< 1 μm), originating from plastic degradation [4], have been detected worldwide [2]. These particles impact 663 marine species by disrupting digestion, growth, predator response, and body condition [1], and pose a particular threat to coral reefs by causing physical damage, disease, and ecological imbalance [5].

Microplastics are water-insoluble synthetic solids or polymer matrices, typically 1 μm to 5 nm in size, classified as primary (e.g., microbeads in toothpaste and cosmetics [4]) or secondary (formed by the breakdown of larger plastics under UV, heat, or mechanical forces [4, 6]). A single 6 kg wash of synthetic clothes releases 700,000 microfibers [7], and industrial pellets further contribute to contamination. They enter oceans via sewage, industrial runoff, and agricultural practices (e.g., microcapsule fertilizers) [4, 8]. Tire wear particles and scratched microcapsules also contribute significantly [3, 9, 10]. Buoyant microplastics persist in the ocean, traveling through currents and entering food chains directly or indirectly. They cause oxidative stress, abnormal behavior, immune suppression, and affect reproductive health in marine organisms [6, 10, 11]. Furthermore, they contribute to ocean acidification by releasing CO_2 and harmful chemicals during degradation, especially under sunlight [12–14].

Marine ecosystems consist of biotic (e.g., fish, coral, bacteria) and abiotic (e.g., pH, temperature, nutrients) components. These systems are being destabilized by human activity, particularly through changes in seawater chemistry. Since the 1980s, ocean surface acidity has increased by 95%, as oceans absorb 30% of anthropogenic CO_2 [15]. Since the 19th century, the concentration of CO_2 in the oceanic atmosphere has been increasing rapidly, as illustrated in Table 1.

This process lowers pH, reduces carbonate ion concentration $[CO_3^{2-}]$, and alters the carbonate buffering system [15, 17–19]. Seawater pH has dropped from 8.25 (1750) to 8.14 (2004), with a projected decrease of 0.2–0.3 units by 2100 [14, 17]. The

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Table 1. Concentration of carbon dioxide increases over the years [16].

Year	1010	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100
Concentration of CO_2 (PPM)	283.6	283.4	283.7	283.1	282.95	283.48	276.4	276.2	280.01	297.6	369.5	788.5

key chemical reactions driving acidification are:

$$\begin{aligned} CO_2 + H_2O &\rightarrow HCO_3^- + H^+, \\ pH &= -\log_{10}[H^+], \\ CO_3^{2-} + H^+ &\rightarrow HCO_3^-. \end{aligned}$$

These reactions increase $[H^+]$ and $[HCO_3^-]$, but reduce pH and $[CO_3^{2-}]$ [20]. Rising CO_2 concentrations (exceeding 410 ppm in 2023 [19]) are primarily due to fossil fuel combustion, industrial processes, and land-use changes. In addition to atmospheric sources, microplastics also release CO_2 during photodegradation (e.g., polystyrene via radical-induced Norrish reactions) and microbial respiration from biodegradable plastics [11, 21].

Among the most biodiverse ecosystems, coral reefs occupy less than 0.1% of ocean area but support over 4,000 fish species, 800 hard coral species, and millions of invertebrates [22]. They provide food, oxygen, climate regulation, and economic value through fishing and tourism ($\sim$ 1 billion from fisheries alone [23]). Reefs also serve as natural coastal barriers, absorbing 97% of wave and storm energy and protecting infrastructure and human lives. However, their sensitivity to environmental changes makes them highly vulnerable to microplastic exposure, acidification, and climate stress [5, 24–27].

Microplastics cause coral ingestion, necrosis, immune suppression, growth reduction, bleaching, and the transport of toxic chemicals and pathogens. Simultaneously, OA reduces the availability of carbonate ions, hindering the formation and maintenance of coral calcium carbonate ($CaCO_3$) skeletons. By 2100, $[CO_3^{2-}]$ in tropical seas may decrease by 100 μmol/kg, half of pre-industrial values, raising the risk of net reef dissolution [27]. The solubility and stability of coral structures depend on:

$$\begin{aligned} CaCO_3(s) &\leftrightarrow {CO_3}^{2-} + Ca^{2+}, \\ \Omega &= \frac{[{CO_3}^{2-}][Ca^{2+}]}{K_{sp}}, \end{aligned}$$

where $\Omega < 1$ indicates conditions favoring dissolution. Since K_{sp} varies with temperature and salinity, increasing $[H^+]$ from acidification further drives Ω below safe thresholds [27, 28]. Coral reef degradation due to microplastics and acidification threatens not only marine biodiversity but also coastal communities and global economies.

This study focuses on modeling the combined effects on coral reefs. To establish a foundation, Section 2 provides a comprehensive review of relevant literature and highlights existing gaps that this work aims to address. Building upon these insights, The formulation of the proposed mathematical model is outlined in Section 3. Subsequently, Section 4 delves into the analytical analysis of the model to explore its theoretical properties. This is followed by Section 5, which presents the numerical simulations, additionally examines the dynamic interactions among the state variables. In Section 6, the results are discussed and the main key findings are summarized. Finally, Section 7 gives the conclusion of this study.

2. Literature Review

The prevalence, origin, and environmental dispersion of microplastics have been the subject of extensive research. The concept of how microplastics are introduced to the ocean was proposed by Qing et al. [29], who identified key factors contributing to microplastic contamination in the marine atmosphere and suggested potential solutions. Merlin et al. [4] emphasized the negative impact of microplastics on aquatic ecosystems, while Tadele [30] demonstrated that surgical face masks may significantly contribute to microplastic pollution in marine environments. Carlo et al. [1] highlighted the growing threat microplastics pose to marine life, and Anthony [9] further explored the mechanisms of microplastic generation and their potential impacts on ocean ecosystems.

Over the past six decades, plastic production has surged from approximately 0.5 Mt/yr in the 1960s to over 300 Mt/yr by 2013, reaching 320 Mt/yr in 2018, and is projected to hit 33 Gt/yr by 2050, with only 6–26% recycled [31, 32]. This rapid growth has led to widespread microplastic pollution in aquatic ecosystems, where more than 663 marine species have experienced physiological stress, disrupted feeding, and reproductive issues [33]. Microplastics are particularly harmful to coral reef ecosystems [5, 25]. Olga [34] presented findings indicating that microplastics negatively affect not only corals but also other reef-associated organisms. Microplastics exacerbate coral breakdown through physical damage and pathogen transmission [35].

Another critical consequence of microplastic pollution is ocean acidification [13]. Cristina et al. [11], through laboratory experiments, showed that abiotic leaching of microplastics contributes to the acidification of marine waters. Ocean acidification poses numerous threats to marine ecosystems [14, 15, 17, 18, 36]. Coral reefs, in particular, are highly vulnerable to acidification stress [26, 28]. Paola et al. [37] suggested that adaptation to ocean acidification may result in changes in coral skeletal porosity. Acidification further impairs coral calcification and structural integrity [38]. Rising levels of atmospheric and marine CO_2 are a significant driver of ocean acidification [20, 21, 36]. Mondal et al. [16, 39] conducted a mathematical and numerical analysis that elucidated the harmful effects of increasing CO_2 concentrations on the ocean's natural ecology. Similarly, John et al. [40] expressed comparable views on the impact of ocean acidification. The theoretical and statistical impacts of ocean acidification on marine ecosystems have also been studied by Geri et al. [17].

Coral reefs, recognized as hotspots of marine biodiversity, are increasingly threatened by a multitude of stressors, including microplastic pollution, CO_2-induced ocean acidification, coral bleaching, and the proliferation of diseases. Recent advances in ecological modeling have attempted to capture the complexity of reef ecosystems, such as the interplay between fish, corals, and pathogens [41], or the role of zooplankton in mediating disease

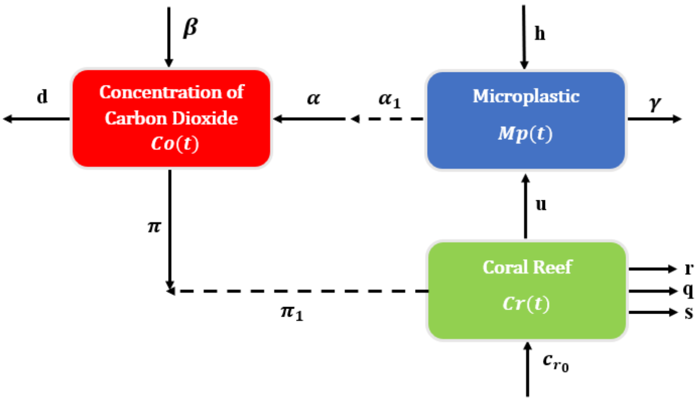


Figure 1. A flowchart illustrates the model.

outbreaks within coral reef environments [42]. Despite notable advancements, the current literature still lacks a holistic framework that simultaneously integrates the interactions among microplastic pollution, atmospheric CO_2 levels, and the health of coral reef ecosystems. Addressing this gap is crucial for developing a holistic understanding of the compounded impacts facing coral reef systems in the Anthropocene.

Several mathematical models have been developed to study coral reef dynamics. For instance, Mumby et al. [43] proposed a classical model focusing on coral–macroalgae interactions and herbivory, revealing the potential for regime shifts and bistability. Anthony et al. [38] extended this framework by incorporating the dual stressors of ocean warming and acidification, demonstrating potential coral cover collapse under combined thermal and pH pressures.

The model presented in this study distinguishes itself by integrating microplastic accumulation (Mp) and carbon dioxide concentration (Co) as stressors, in addition to coral cover (Cr). Unlike previous models that considered these environmental stressors in isolation, the model captures the dynamic feedbacks between atmospheric CO_2, acidification-driven plastic degradation, and their compounded effects on coral decline. This offers a more realistic representation of modern multi-stressor marine environments, making the model both novel and timely. In this study, we present a novel three-compartment model tracking microplastic concentration (Mp), CO_2 levels (Co), and coral cover (Cr). Unlike previous models, this model integrates plastic degradation pathways and acidification feedbacks, and aligns with emerging evidence of multi-stressor tipping points.

The parameter values used in this model are aligned with empirical data. For example, the annual coral degradation rate ($u = 0.012\,\text{yr}^{-1}$) reflects observed reef loss rates in polluted and overfished regions such as the Caribbean [44]. The rates of microplastic accumulation and degradation (π, α_1) are informed by both field observations and laboratory studies on plastic fragmentation under acidic marine conditions [45]. Moreover, the model exhibits threshold behavior and tipping points, qualitatively consistent with ecosystem-scale experiments and field evidence [38, 46].

These consistencies with empirical trends suggest that the proposed model captures essential ecological dynamics and provides a robust framework for scenario analysis involving the dual impacts of microplastic pollution and ocean acidification on coral reefs.

3. Model Formulation

In the model, $Co(t)$, $Mp(t)$, and $Cr(t)$ denote the concentration of CO_2 contributing to ocean acidification, the amount of microplastics present in the water, and the extent of coral reef coverage in the marine ecosystem, respectively, at any given time t. The interconnections among these three state variables are visualised in the diagram (Figure 1).

The variation in carbon dioxide concentration within the marine environment is influenced by several interacting processes. To begin with, atmospheric CO_2 dissolves into seawater at a rate of βCo. In addition, natural mechanisms work to reduce this concentration, modeled by the term $-dCo$. What is more, microplastic pollution contributes to an increase in CO_2, represented by the interaction term $\alpha MpCo$. Lastly, a decline in coral reef coverage leads to a decrease in carbon dioxide through the term $-\pi CrCo$. These combined effects yield the differential equation:

$$\frac{dCo}{dt} = \beta Co - dCo + \alpha MpCo - \pi CrCo.$$

Here, β denotes the rate at which atmospheric CO_2 dissolves into the seawater. The parameter d represents the natural decay rate of carbon dioxide. Furthermore, coefficient α reflects the rate at which microplastics elevate CO_2 levels in seawater, and π characterizes the rate at which carbon dioxide is reduced due to coral reef degradation.

To add to that, the dynamics of microplastic density in seawater are governed by multiple factors. The continuous input of microplastics into the ocean is denoted by h, while their removal from the system occurs at a rate of γMp. Additionally, microplastics that play a role in elevating CO_2 levels are simultaneously reduced, captured by the term $\alpha_1 MpCo$. These components together lead to the following differential equation:

$$\frac{dMp}{dt} = h - \gamma Mp - \alpha_1 MpCo.$$

In this formulation, h represents the external addition of microplastics into the marine environment. The parameter γ quantifies the elimination of microplastics from the ocean water. Lastly, α_1 signifies the fraction of microplastics reduced due to their contribution to carbon dioxide accumulation in the marine system.

The last aspect to be discussed is that the dynamic variation in the coverage area of coral reefs can also be mathematically expressed through multiple contributing factors. The positive contribution arises from the natural growth or increase of coral reefs in the aquatic ecosystem, denoted by Cr_o. Conversely, several negative influences contribute to coral reef degradation. These include loss due to climate change, represented by qCr, as well as the impact of pathogenic diseases, expressed as rCr. In addition, destructive fishing practices result in further coral reef depletion, modeled by the term sCr.

Moreover, the adverse effects of microplastic contamination are incorporated into the system through the interaction term $-uCrMp$, which quantifies coral reef decline resulting from microplastics present in the marine environment. Likewise, coral decay induced by ocean acidification, attributed to elevated concentrations of carbon dioxide (CO_2), is captured by the term $-\pi_1 CrCo$.

Collectively, these factors yield the following differential equation that governs the temporal evolution of coral reef coverage:

$$\frac{dCr}{dt} = Cr_o - rCr - qCr - sCr - uCrMp - \pi_1 CrCo.$$

Here, the parameters q, r, s, and π_1 correspond to the respective rates of coral reef degradation caused by climate change, pathogenic diseases, destructive fishing, and ocean acidification. The parameter u characterizes the rate of decline attributable to microplastic pollution. Therefore, we get the following system of equations:

$$\begin{aligned}\frac{dCo(t)}{dt} &= \beta Co(t) - dCo(t) + \alpha Mp(t)Co(t) - \pi Cr(t)Co(t),\\ \frac{dMp(t)}{dt} &= h - \gamma Mp(t) - \alpha_1 Mp(t)Co(t),\\ \frac{dCr(t)}{dt} &= Cr_o - (r+q+s)Cr(t) - uCr(t)Mp(t)\\ &\quad - \pi_1 Cr(t)Co(t).\end{aligned} \tag{1}$$

4. Analytical Analysis of the Model

Coral reefs, vital for coastal protection and marine biodiversity, face severe threats from microplastics and ocean acidification, compounded by their slow reproduction rate. This study analyzes a mathematical model (1) to explore the mechanisms driving these issues and their impact on coral reef decline.

4.1. Positivity and Boundedness

We must establish that the solutions to the model (1) are positive and bounded for all values of t to make the model feasible.

Lemma 1. *If $Co(0) > 0$, $Mp(0) > 0$, $Cr(0) > 0$ then the solutions $Co(t)$, $Mp(t)$ and $Cr(t)$ of the model (1) are all positive.*

Proof. To prove the Lemma 1, we will use the model (1). Taking all parameters of the system and all initial values to be positive, we have to prove that $Co(t)$, $Mp(t)$ & $Cr(t)$ will be positive for all $t \in [0, T]$ in R_3^+. To find the positivity of $Co(t)$,

$$\begin{aligned}\frac{dCo}{dt} &= \beta Co - dCo + \alpha MpCo - \pi CrCo,\\ \frac{dCo}{Co} &= \beta dt - ddt + \alpha Mpdt - \pi Crdt,\\ \ln Co &= \beta t - dt + \alpha \int Mpdt - \pi \int Crdt,\\ Co &= e^{\beta t - dt + \alpha \int Mpdt - \pi \int Crdt} > 0.\end{aligned}$$

$\therefore$ Hence, $Co(t) > 0 \, \forall \, t \geq 0$.
Again for $Mp(t)$, we have

$$\begin{aligned}\frac{dMp}{dt} &= h - \gamma Mp - \alpha_1 MpCo,\\ \frac{dMp}{dt} &\geq -\gamma Mp - \alpha_1 MpCo,\\ \frac{dMp}{Mp} &\geq -\gamma dt - \alpha_1 Codt,\\ \ln Mp &\geq \gamma t - \alpha_1 \int Codt,\\ Mp &\geq e^{\gamma t - \alpha_1 \int Codt} > 0.\end{aligned}$$

$\therefore Mp(t) > 0 \; \forall t \geq 0$.
Now to prove the positivity of $Cr(t)$, we have:

$$\begin{aligned}\frac{dCr}{dt} &= Cr_o - rCr - qCr - sCr - uCrMp - \pi_1 CrCo,\\ \frac{dCr}{dt} &\geq Cr(-r - q - s - uMp - \pi_1 Co),\\ \frac{dCr}{Cr} &\geq (-r - q - s - uMp - \pi_1 Co)dt,\\ \ln Cr &\geq -t(r+q+s) - u \int Mpdt - \pi_1 \int Codt,\\ Cr &\geq e^{-t(r+q+s) - u \int Mpdt - \pi_1 \int Codt}.\end{aligned}$$

$\therefore$ Hence, $Cr(t) > 0 \; \forall t \geq 0$. □

Lemma 2. *The solution trajectories $(Co(t), Mp(t), Cr(t))$ of the model (1) are bounded.*

Proof. Adding the first two equations from the model (1), we get

$$\begin{aligned}\dot{Co}(t) + \dot{Mp}(t) &= \beta Co(t) - dCo(t) + \alpha Mp(t)Co(t)\\ &\quad - \pi Cr(t)Co(t) + h - \gamma Mp(t) - \alpha_1 Mp(t)Co(t),\\ \dot{Co}(t) + \dot{Mp}(t) &\leq Co(t)(\beta - d) + h - \gamma Mp(t).\end{aligned}$$

Taking $\alpha = \min\{\alpha, \alpha_1\}$], then

$$\dot{Co}(t) + \dot{Mp}(t) \leq h - Co(t)(d - \beta) - \gamma Mp(t).$$

Taking $\tau = \min\{d - \beta, \gamma\}$, we get

$$\dot{Co}(t) + \dot{Mp}(t) \leq h - \tau\{Co(t) + Mp(t). \tag{2}$$

Taking the limit supremum, we get

$$\lim_{t\to\infty} \sup\{Co(t) + Mp(t)\} \leq \frac{h}{\tau}. \tag{3}$$

In the same way, adding the model's first, second, and third equation gives us

$$\begin{aligned}\dot{Co}(t)+\dot{Mp}(t)+\dot{Cr}(t)&=\beta Co(t)-dCo(t)-\pi Cr(t)Co(t)\\&\quad+h-\gamma Mp(t)+Cr_0-Cr(t)(r+q\\&\quad+s)-uCr(t)Mp(t)-\pi Cr(t)Co(t),\\\dot{Co}(t)+\dot{Mp}(t)+\dot{Cr}(t)&\leq h+Cr_0-Co(t)(d-\beta)-\gamma Mp(t)\\&\quad-Cr(t)(r+q+s).\end{aligned}$$

Taking $\alpha=\min\{\alpha,\alpha_1\}$, then

$$\begin{aligned}\dot{Co}(t)+\dot{Mp}(t)+\dot{Cr}(t)&\leq h+Cr_0-Cr(t)(r+q+s)\\&\quad-\phi\{Co(t)+Mp(t)\}.\end{aligned}$$

Furthermore, taking $\phi=\min\{d-\beta,\gamma\}$, we have

$$\dot{Cr}(t)\leq Cr_o+(\tau+\phi)\{Co(t)+Mp(t)\}-Cr(t)(r+q+s).$$

Using equation (2), we obtain

$$\dot{Cr}(t)\leq Cr_0+\frac{(\tau+\phi)h}{\tau}-Cr(t)(r+q+s).$$

Taking the limit supremum, we get

$$\lim_{t\to\infty}\sup\{Cr(t)\}\leq\frac{\phi}{(r+q+s)}+\frac{(\tau+\phi)Co}{\tau(r+q+s)}\tag{4}$$

So, all the solutions $(Co(t),(Mp(t),Cr(t))$ of the model (1) are bounded. □

4.2. Existence and Uniqueness of The Model

Lemma 3. *Let $\mathcal{D}$ represent a domain in which the Lipschitz continuity conditions hold. Under this assumption, it follows that for any given non-negative initial condition, the model (1) admits a unique solution. Furthermore, this solution not only exists but also persists for all time $T\geq 0$, remaining continuously well-defined within the domain $\mathcal{D}$.*

Alternatively stated, the fulfillment of Lipschitz criteria within $\mathcal{D}$ guarantees both the existence and uniqueness of the solution trajectory of the model (1), ensuring that it does not escape the domain as time progresses.

Proof. As established by Sowole et al., [47], the proposed lemma necessitates adherence to the Lipschitz condition within a designated region $\mathcal{D}$, thereby ensuring the existence and uniqueness of the solution to the system under consideration. Let

$$\begin{aligned}f(Co,Mp,Cr)&=\frac{dCo(t)}{dt}=\beta Co(t)-dCo(t)+\alpha Mp(t)Co(t)\\&\quad-\pi Cr(t)Co(t),\\g(Co,Mp,Cr)&=\frac{dMp(t)}{dt}=h-\gamma Mp(t)-\alpha_1Mp(t)Co(t),\\h(Co,Mp,Cr)&=\frac{dCr(t)}{dt}=Cr_o-(r+q+s)Cr(t)\\&\quad-uCr(t)Mp(t)-\pi_1Cr(t)Co(t).\end{aligned}\tag{5}$$

Using the system of equations described above, the partial derivatives of the functions f,g,h, corresponding to the components Co,Mp,Cr, with respect to the compartments Co,Mp,Cr are computed as follows:

$$\begin{aligned}\frac{\partial f}{\partial Co}&=\beta-d-aMp-\pi Cr,\\\therefore\left|\frac{\partial f}{\partial Co}\right|&=|\beta-d-aMp-\pi Cr|\leq\frac{h}{\tau}<\infty.\\\frac{\partial f}{\partial Mp}&=aCo,\\\therefore\left|\frac{\partial f}{\partial Mp}\right|&=|aCo|\leq\frac{h}{\tau}<\infty.\\\frac{\partial f}{\partial Cr}&=-\pi Co,\\\therefore\left|\frac{\partial f}{\partial Cr}\right|&=|-\pi Co|\leq\frac{\varphi}{(r+q+s)}+\frac{(\tau+\varphi)Co}{\tau(r+q+s)}.\\\frac{\partial g}{\partial Co}&=-\alpha_1Mp,\\\therefore\left|\frac{\partial g}{\partial Co}\right|&=|-\alpha_1Mp|\leq\frac{h}{\tau}<\infty.\\\frac{\partial g}{\partial Mp}&=-\gamma-\alpha_1Co,\\\therefore\left|\frac{\partial g}{\partial Mp}\right|&=|-\gamma-\alpha_1Co|\leq\frac{h}{\tau}<\infty.\\\frac{\partial g}{\partial Cr}&=0,\\\therefore\left|\frac{\partial g}{\partial Cr}\right|&=0\leq\frac{\varphi}{(r+q+s)}+\frac{(\tau+\varphi)Co}{\tau(r+q+s)}<\infty.\\\frac{\partial h}{\partial Co}&=-\pi_1Cr,\\\therefore\left|\frac{\partial h}{\partial Co}\right|&=|-\pi_1Cr|\leq\frac{h}{\tau}<\infty.\\\frac{\partial h}{\partial Mp}&=uCr,\\\therefore\left|\frac{\partial h}{\partial Mp}\right|&=|uCr|\leq\frac{h}{\tau}<\infty.\\\frac{\partial h}{\partial Cr}&=-(r+q+s)-uMp-\pi_1Co,\\\therefore\left|\frac{\partial h}{\partial Cr}\right|&=|-(r+q+s)-uMp-\pi_1Co|\\&\leq\frac{\varphi}{(r+q+s)}+\frac{(\tau+\varphi)Co}{\tau(r+q+s)}<\infty.\end{aligned}$$

Hence, it has been demonstrated that all the partial derivatives of the functions involved are continuous and uniformly bounded within the domain of interest. Therefore, the Lipschitz continuity condition is satisfied. Consequently, by invoking the existence and uniqueness theorem as presented in [47], we conclude that the model (5) admits a unique solution in the region $\mathcal{D}$.

□

4.3. *Equilibrium Analysis*

By solving the following equations, we can obtain the equilibrium point.

$$\begin{aligned}\beta Co - dCo + \alpha MpCo - \pi CrCo &= 0,\\ h - \gamma Mp - \alpha_1 MpCo &= 0, \qquad (6)\\ Cr_0 - rCr - qCr - sCr - uCrMp - \pi_1 CrCo &= 0.\end{aligned}$$

Solving the eq. (6) we get two non-negative equilibrium points:

$$E_0\left(0, \frac{h}{\gamma}, \frac{\gamma Cr_o}{\gamma(r+q+s)+uh}\right) \text{ and } E(Co^*, Cr^*, Mp^*),$$

where,

$$\begin{aligned}Co^* &= \frac{\alpha h - \gamma\pi Cr^* - \gamma\beta - \gamma d}{\alpha_1\pi Cr^* - \alpha_1\beta + \alpha_1 d},\\ Mp^* &= \frac{\pi Cr^* - \beta + d}{\alpha},\\ Cr^* &= \sqrt[3]{\varpi + \sqrt{\varpi^2 + \varrho^3}} + \sqrt[3]{\varpi - \sqrt{\varpi^2 + \varrho^3}} - \vartheta.\end{aligned}$$

Taking

$$\begin{aligned}\varpi &= \frac{-N^3}{27M^3} + \frac{NO}{6M^2} - \frac{P}{2M},\\ \varrho &= \frac{O}{3M} - \frac{N^2}{9M^2},\\ \vartheta &= \frac{N}{3M},\end{aligned}$$

Such that

$$\begin{aligned}M &= -u\pi^2,\\ N &= -\pi(w - 2\alpha_1\beta u + 2du + \gamma\pi_1),\\ O &= \pi Cr_0 + \alpha_1\beta w - dw - u\beta^2 + 2u\beta d - ud^2 - \alpha\pi_1 h\\ &\quad + \alpha_1\gamma\beta\pi + \alpha_1\gamma\pi d,\\ P &= Cr_0(d - \alpha_1\beta),\\ w &= r + q + s.\end{aligned}$$

4.4. *Stability Analysis*

To investigate the behaviour of the stability, the Jacobian matrix will be used, and after that, the characteristic equation will be examined.

> **Theorem 1.** *The equilibrium point E_o is asymptotically stable if $\beta + \frac{\alpha h}{\gamma} < d + \frac{Cr_0\gamma\pi}{w\gamma + uh}$.*

Proof. To prove Theorem 1, at first we consider

$$\begin{aligned}f &= \beta Co - dCo + \alpha MpCo - \pi CrCo, &(7)\\ g &= h - \gamma Mp - \alpha MpCo, &(8)\\ h &= Cr_o - rCr - qCr - sCr - uCrMp - \pi CrCo. &(9)\end{aligned}$$

For the eq. (7), we get the following jacobian matrix.

$$J = \frac{\partial(f, g, h)}{\partial(Co, Mp.Cr)} = \begin{bmatrix}\frac{\partial f}{\partial Co} & \frac{\partial f}{\partial Mp} & \frac{\partial f}{\partial Cr}\\ \frac{\partial g}{\partial Co} & \frac{\partial g}{\partial Mp} & \frac{\partial g}{\partial Cr}\\ \frac{\partial h}{\partial Co} & \frac{\partial h}{\partial Mp} & \frac{\partial h}{\partial Cr}\end{bmatrix},$$

$$\therefore J_{E_0} = \begin{bmatrix} j_1 & \alpha Co & -\pi Co\\ -\alpha_1 Mp & -\gamma - \alpha_1 Co & 0\\ -\pi_1 Cr & -uCr & j_2\end{bmatrix}, \qquad (10)$$

$$\begin{aligned}j_1 &= \beta - d + \alpha Mp - \pi Cr,\\ j_2 &= -r - q - s - uMp - \pi_1 Co.\end{aligned}$$

At the equilibrium point $E_0\left(0, \frac{h}{\gamma}, \frac{\gamma Cr_o}{\gamma w + uh}\right)$, eq. (10) becomes

$$J_{E_0} = \begin{bmatrix}\beta - d - \frac{Cr_0\gamma\pi}{\gamma w + hu} + \frac{\alpha h}{\gamma} & 0 & 0\\ -\frac{\alpha_1 h}{\gamma} & -\gamma & 0\\ -\frac{Cr_0\gamma\pi_1}{\gamma w + hu} & -\frac{Cr_0\gamma u}{\gamma w + hu} & -w - \frac{hu}{\gamma}\end{bmatrix}.$$

By solving $|J_{E_0} - \lambda I| = 0$, we get three eigenvalues: $\lambda_1 = -w - \frac{hu}{\gamma}$, $\lambda_2 = -\gamma$, $\lambda_3 = \beta - d - \frac{Cr_0\gamma\pi}{w\gamma + uh} + \frac{\alpha h}{\gamma}$. Hence, it is clear that, if $\beta + \frac{\alpha h}{\gamma} < d + \frac{Cr_0\gamma\pi}{w\gamma + uh}$, the equilibrium point $E_0\left(0, \frac{h}{\gamma}, \frac{\gamma Cr_o}{\gamma w + uh}\right)$ is asymptotically stable. ☐

> **Theorem 2.** *The equilibrium point $E(Co^*, Mp^*, Cr^*)$ is stable if $d > \beta$, $u > \alpha$, $Co^*\pi > Mp^*\alpha$, and $du > \alpha w$.*

Proof. The jacobian matrix at the equilibrium point $E(A^*, Mp^*, Cr^*)$ is

$$J(A^*, Mp^*, Cr^*) = \begin{bmatrix}\frac{\partial f(Co^*,Mp^*,Cr^*)}{\partial Co} & \frac{\partial f(Co^*,Mp^*,Cr^*)}{\partial Mp} & \frac{\partial f(Co^*,Mp^*,Cr^*)}{\partial Cr}\\ \frac{\partial g(Co^*,Mp^*,Cr^*)}{\partial Co} & \frac{\partial g(Co^*,Mp^*,Cr^*)}{\partial Mp} & \frac{\partial g(Co^*,Mp^*,Cr^*)}{\partial Cr}\\ \frac{\partial h(Co^*,Mp^*,Cr^*)}{\partial Co} & \frac{\partial h(Co^*,Mp^*,Cr^*)}{\partial Mp} & \frac{\partial h(Co^*,Mp^*,Cr^*)}{\partial Cr}\end{bmatrix},$$

$$\therefore J_E = \begin{bmatrix} j_3 & \alpha Co^* & -\pi Co^*\\ -\alpha_1 Mp^* & -\gamma - \alpha_1 Co^* & 0\\ -\pi_1 Cr^* & -uCr^* & j_4\end{bmatrix},$$

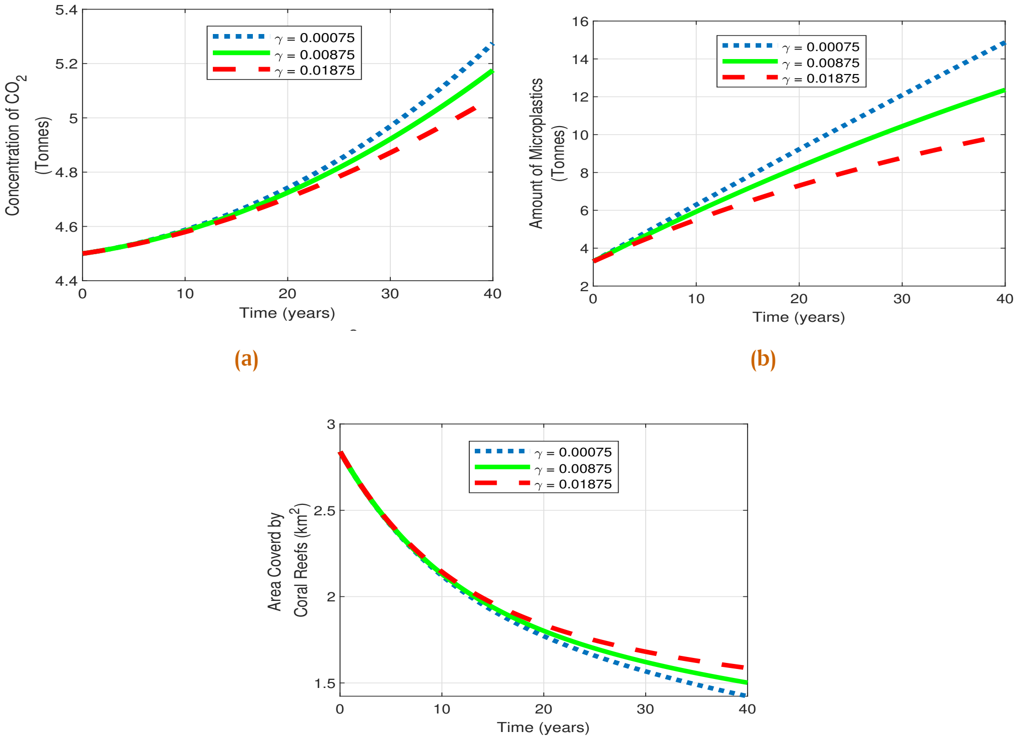
$$\begin{aligned}j_3 &= \beta - d + \alpha Mp^* - \pi Cr^*,\\ j_4 &= -r - q - s - uMp^* - \pi_1 Co^*.\end{aligned}$$

From $|J_E - \lambda I| = 0$, we get the following characteristic equation.

$$\psi_0\lambda^3 + \psi_1\lambda^2 + \psi_2\lambda + \psi_3 = 0,$$

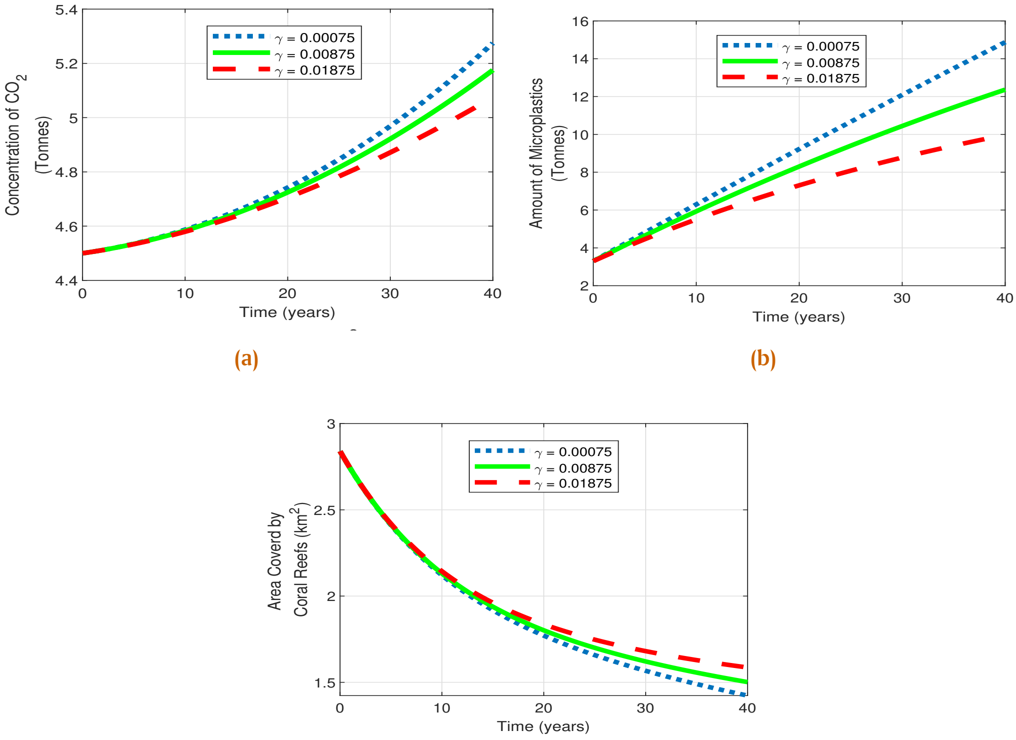
where,

$$\begin{aligned}\psi_0 &= 1,\\ \psi_1 &= d - \beta + \gamma + w + Co^*\alpha_1 - \alpha Mp^* + Co^*\pi + \pi Cr^* + uMp^*,\\ \psi_2 &= \gamma d - \beta\gamma - \beta w + dw + \gamma w - Co^*\alpha_1\beta + Co^*\alpha_1 d - Co^*\beta\pi_1\\ &\quad + Co^* d\pi_1 - Mp^*\alpha\gamma + Co^*\gamma\pi_1 + Co^*\alpha_1 w + Cr^*\gamma\pi\\ &\quad - Mp^*\beta u + Mp^* du - Mp^*\alpha w + Mp^*\gamma u + Cr^*\pi w\\ &\quad + {Co^*}^2\alpha_1\pi_1 + {Mp^*}^2\alpha u + Co^*Cr^*\alpha_1\pi - Co^*Mp^*\alpha\pi_1\\ &\quad + Co^*Mp^*\alpha_1 u + Cr^*Mp^*\pi u,\\ \psi_3 &= d\gamma w - \beta\gamma w - {Co^*}^2\alpha_1\beta\pi_1 + {Co^*}^2\alpha_1 d\pi_1 + {Mp^*}^2\alpha\gamma u\\ &\quad - Co^*\beta\gamma\pi_1 + Co^* d\gamma\pi_1 - Co^*\alpha_1\beta w + Co^*\alpha_1 dw - Mp^*\beta\gamma u\\ &\quad + Mp^* d\gamma u + Mp^*\alpha\gamma w + Cr^*\gamma\pi w - Co^*Mp^*\alpha_1\beta u\\ &\quad + Co^*Mp^*\alpha\gamma\pi_1 + Co^*Mp^*\alpha_1 du + Co^*Cr^*\alpha_1\pi w\\ &\quad + Cr^*Mp^*\gamma\pi u + 2Co^*Cr^*Mp^*\alpha_1\pi u.\end{aligned}$$



(a) **(b)** **(c)**

Figure 2. (a) Concentration of CO_2 is decreasing when the reducing rate of microplastics in the ocean γ is increasing. (b) For the increase of γ, amount of microplastics in the ocean is decreasing. (c) When γ increases, the area covered by coral reefs increases.



(a) **(b)** **(c)**

Figure 3. (a) CO_2 concentration is changing considerably for variation of β. (b) The increase of β barely affects the change in the amount of microplastics in the ocean. (c) The area covered by coral reefs is decreasing for an increase in β.

Table 2. Validated parameters

Parameter	Value	Explanation/Source
d	0.0005575	Natural CO_2 reduction rate in ocean; approx. 0.05–0.1%/yr.
β	0.06050713	Dissolution rate of CO_2 from atmosphere into oceanwas derived from the mixed-layer CO_2 exchange flux reported by [48], validated against IPCC AR6 global carbon budget (Sect. 5.2).
γ	0.00875	Rate of microplastic removal from surface (estimated 10+ year residence time).
h	0.31	Microplastic accumulation rate in seawater based on global production [49].
q	0.06375	The coral decay rate $q = 0.06375\,\text{yr}^{-1}$ was estimated based on observed coral mortality (30–60% over 1–2 years) reported by NOAA Coral Reef Watch [46].
r	3.75×10^{-3}	Coral reef decay rate due to disease [50].
s	7.5×10^{-3}	The parameter $s = 0.0075\ \text{yr}^{-1}$, accounting for coral decline due to destructive fishing, is based on ReefBase data indicating annual coral loss of 0.5–1% in affected regions [51].
u	0.00375	We set $u = 0.00375\,\text{yr}^{-1}$, representing additional coral loss due to microplastic contact, based on findings by [35], showing plastic presence increased disease likelihood dramatically (4% $\rightarrow$ 89%).
π	0.000929	The acidification-induced decay rate $\pi = 0.000929\,\text{yr}^{-1}$ was chosen to reflect the 14% annual decline in coral calcification observed on the Great Barrier Reef since 1990 [52].
π_1	0.00553	Coral reef mortality due to ocean acidification [53].
Cr_0	0.215	Coral reef regrowth rate under favorable conditions [54].
α	3.07×10^{-5}	The parameter $\alpha = 3.07 \times 10^{-5}\,\text{yr}^{-1}$ represents the CO_2 release from microplastic degradation, based on sorption/desorption rates of organic contaminants measured over several months in the field [45].
α_1	0.0003007	$\alpha_1 = 3.007 \times 10^{-4}\,\text{yr}^{-1}$ reflects accelerated microplastic decay under acidic conditions, inspired by laboratory observations of acid-enhanced polymer degradation [45].

It is clear that, if $d > \beta$, $u > \alpha$, $Co^*\pi > Mp^*\alpha$, and $du > \alpha w$, then $\psi_1 > 0, \psi_2 > 0$ and $\psi_1\psi_2 - \psi_3 > 0$. Hence according to Routh-Hurwitz criteria the equilibrium point $E(Co^*, Mp^*, Cr^*)$ is stable. □

4.5. Characteristics of Equilibrium Points with Respect toγ

The characterization of the equilibrium values of microplastic pollution and the area covered by coral reefs with respect to γ will be discussed below. From the model (1), two functions of Mp^*, Cr^*, γ are obtained as follows:

$$\begin{aligned} f(Mp^*, Cr^*, \gamma) &= h - \gamma Mp, \\ g(Mp^*, Cr^*, \gamma) &= Cr_0 - wCr - uCrMp, \\ w &= r + q + s. \end{aligned}$$

Hence,

$$\begin{aligned} \frac{dMp^*}{d\gamma} &= \frac{\Omega_1}{\Omega_2}, \\ &= \frac{-Mp^*}{\gamma}, \\ \Omega_1 &= \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial Cr^*} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial \gamma} \\ &\quad - \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial \gamma} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial Cr^*}, \\ \Omega_2 &= \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial Mp^*} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial Cr^*} \\ &\quad - \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial Cr^*} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial Mp^*}. \end{aligned}$$

It is clear that the numerator is negative and the denominator is positive. Therefore, $\frac{dMp^*}{d\gamma} < 0$ which indicates that Mp^* is decreasing when γ is increasing.

Again,

$$\begin{aligned} \frac{dCr^*}{d\gamma} &= \frac{\Omega_3}{\Omega_4}, \\ &= \frac{uMp^*Cr^*}{\gamma(w + uMp^*)}, \\ \Omega_3 &= \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial \gamma} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial Mp^*} \\ &\quad - \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial Mp^*} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial \gamma}, \\ \Omega_4 &= \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial Mp^*} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial Cr^*} \\ &\quad - \frac{\partial f(Mp^*, Cr^*, \gamma)}{\partial Cr^*} \times \frac{\partial g(Mp^*, Cr^*, \gamma)}{\partial Mp^*}. \end{aligned}$$

Here $\frac{dCr^*}{d\gamma} > 0$ which shows that Cr^* is increasing when γ is increasing. Now,

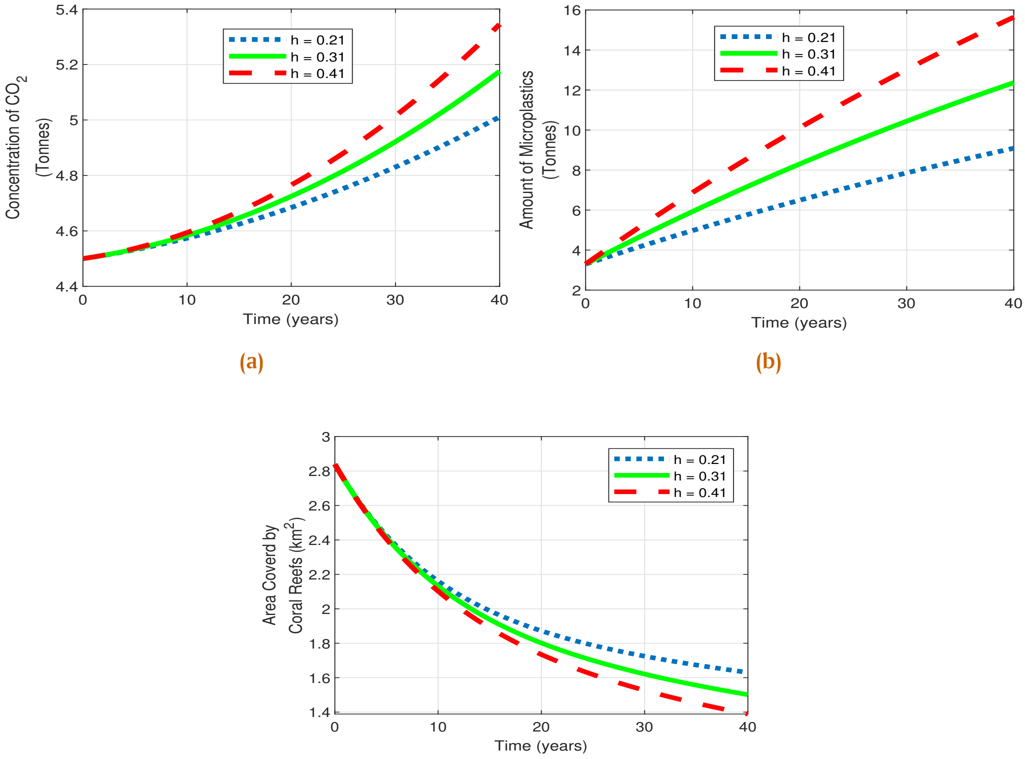
$$\frac{dCo^*}{d\gamma} = \frac{\pi Co^* \frac{dCr^*}{d\gamma} - \alpha Co^* \frac{dMp^*}{d\gamma}}{\beta - d + \alpha Mp^* - \pi Cr^*}$$

Since, $\frac{dMp^*}{d\gamma} < 0$ and $\frac{dCr^*}{d\gamma} > 0$, then if $\beta - d < \pi Cr^* - \alpha Mp^*$, we can say $\frac{dCo^*}{d\gamma} > 0$ that indicates Co^* decreases with respect to increase of γ.

5. Numerical Simulations

We numerically solved the model using several parameter values and examined the model's stability. Relevant to this work is a collection of logical parameter values derived from the above Table 2. For mathematical model analysis, we have used MATLAB (R2022a). The system has been solved using the Table 2 parameter values and the fourth-order Runge-Kutta method.

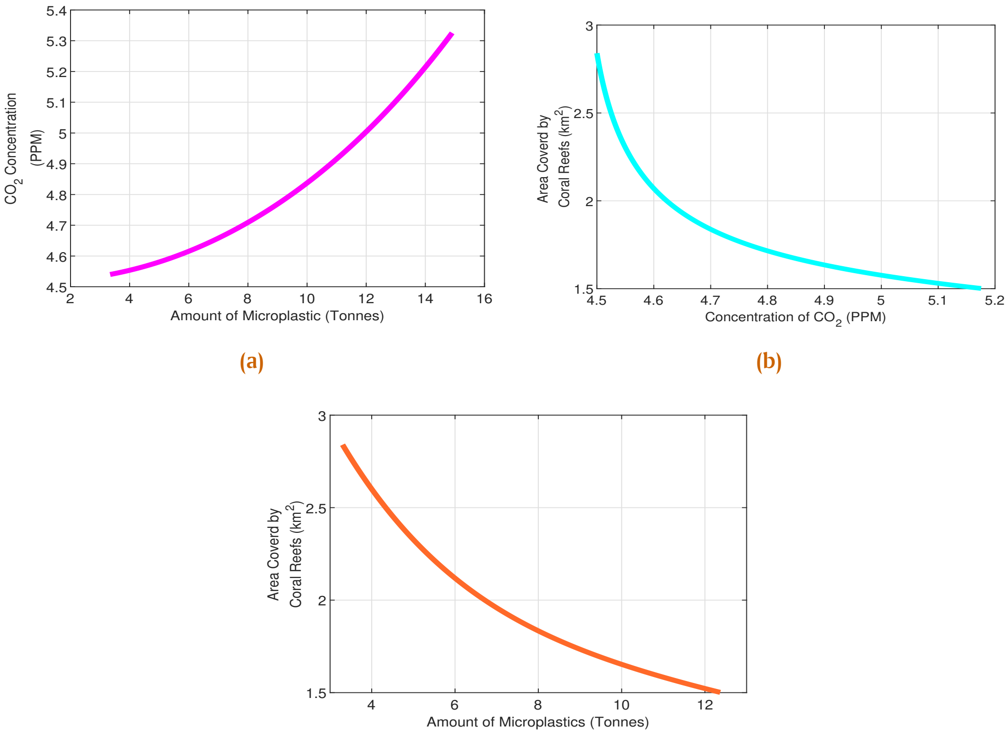
It has been expected that the initial value of carbon dioxide concentration $Co(0)$ will be 450 ppm, which is the current concentration in the aquatic environment [16]. Again, between $82,000$ and $578,000$ metric tons of microplastics are thought to be floating about in the sea [2, 4, 55]. As an initial point, we have estimated that there are $330,000$ metric tons of microplastics in the ocean which means $Mp(0) = 330,000$. Moreover, it is estimated that coral reefs cover $284,000$ square kilometers of the ocean floor [22–24]. Therefore, $Cr(0) = 284,000$ has been used



(a) (b)

(c)

Figure 4. (a) For variation in h, concentration of carbon dioxide changes dramatically. (b) Changes in h causes amount of microplastics to vary significantly. (c) Because of increasing in h, the area covered by coral reefs is decreasing.



(a) (b)

(c)

Figure 5. (a) This graph shows the relationship between concentration of carbon dioxide $Co(t)$ and amount of microplastics $Mp(t)$. (b) Relation between concentrations of carbon dioxide $Co(t)$ and area covered by coral reefs $Cr(t)$ is reflected by this graph. (c) This graph illustrates the connection between amount of microplastics $Mp(t)$ and area covered by coral reefs $Cr(t)$.

as the baseline measure of coral reef coverage. We will perform numerical analysis on the model over a period of 40 years.

To begin, Figure 2a depicts how the concentration of CO_2 varies as the rate of elimination of microplastics from the ocean γ increases. Up to 12 years, γ fluctuation does not influence carbon dioxide content. When the γ value is increased from year 12 to year 20, the curves begin to divide. The CO_2 content then begins to fall dramatically in year 20 as γ values rise. There is a notable variance in CO_2 concentration for different gamma values in year 40.

The effect of increasing the microplastic reduction rate gamma on the quantity of microplastics is seen in Figure 2b. Three of the curves are nearly identical from year 1 to year 5. However, after 5 years, the quantity of microplastics in seawater begins to change for various γ values. It becomes apparent between years 20 and 40 when the three curves move away from each other.

Figure 2c shows that γ variations have little impact on the area covered by coral reefs. However, when γ grew, they varied from year 15 to 40, increasing the area covered by coral reefs.

Figure 3a depicts the influence of β fluctuations on CO_2 concentration, where β represents the rate of CO_2 dissolution in the marine environment. The three curves for the various three values of β begin to change from year 5 to 40 and climb with higher β values.

There is essentially little consequence of β variation on the number of microplastics in the aquatic environment which is shown in Figure 3b. However, the curves continue to go up, indicating that the amount of microplastic in the marine ecosystem is increasing dramatically.

The area covered by coral reefs is considerably influenced by an increase in β value, as illustrated in Figure 3c. It falls dramatically from the start, but the three curves for the three beta values begin to separate between years 5 and 40. The curve is most adversely affected when the value of β is the largest.

Figures 4a to 4c demonstrate the impact of the microplastic input rate h on the three state variables in the marine environment. Figure 4a shows how the change in h influences the concentration of CO_2 in seawater. As the h value grows, the three curves begin to separate in year 8. There is a notable difference between the three curves in year 40; the curve rises higher when the h value is greatest. However, CO_2 concentrations continue to climb dramatically.

The effect of h variation on the number of microplastics is seen in Figure 4b. As h represents the intake rate of microplastics in the marine environment, the three curves for different h values begin to separate considerably from the start. The graph clearly illustrates that lowering the value considerably reduces the number of microplastics in the marine environment.

Variation in the h value has a small but dominant influence on the regions covered by coral reefs. Figure 4c demonstrates that the three curves remain constant until year 9 before splitting as the value of h fluctuates. After year 10, the distinctions between the curves become more apparent. However, the area covered by coral reefs will fall by about half in the next 40 years.

5.1. Interaction Between State Variables

Simulations of the model reveal the non-linear relationships between microplastics, carbon dioxide (CO_2), and coral reef coverage. Figure 5a shows that rising ocean microplastic levels (currently $\sim 330,000$ metric tons) contribute to increased CO_2 concentrations, projected to reach 520 ppm and 1.2 million metric tons of microplastics in 40 years.

Figure 5b highlights the inverse correlation between CO_2 levels and coral reef coverage, currently at $284,000$ km^2, representing 0.1% of the global ocean area. Figure 5c confirms that increasing microplastics and CO_2 concentrations may halve coral reef coverage within the next four decades.

These findings underscore the urgency of implementing robust actions at both international and national levels to mitigate the twin challenges of microplastic pollution and ocean acidification. Further research is necessary to understand how microplastics infiltrate marine organisms and to elucidate their cascading effects on the ecosystem. Simultaneously, raising public awareness about environmental microplastics can spur technological innovations to reduce plastic usage, enhance material collection and reuse, and ultimately minimize the environmental footprint of plastics.

6. Results and Discussion

Marine biodiversity encompasses a wide array of oceanic elements that function in a delicate balance. The disruption of any single component can significantly disturb this equilibrium. Among these elements, coral reefs stand out as one of the most vital ecosystems, supporting a vast range of marine life. However, the progression of industrial civilization has led to a continual rise in carbon dioxide emissions, particularly affecting oceanic environments through ocean acidification and warming. Additionally, the proliferation of microplastics, primarily driven by industrial activities, poses a growing threat to marine ecosystems. Although there is a global shift toward sustainable and green energy solutions, the transition remains limited to a few regions, leaving much of the marine environment vulnerable to anthropogenic stressors.

The findings of this research reveal the intricate dynamics of microplastic pollution and ocean acidification, two critical factors that negatively affect coral reefs. Using a novel model based on a system of differential equations, we demonstrate that the solutions to the model are both positive and bounded, which ensures their feasibility and physical relevance. Theoretical and numerical analyses reveal that microplastic pollution significantly contributes to increased carbon dioxide (CO_2) levels in marine environments, thereby intensifying ocean acidification, a major driver of coral reef degradation. Moreover, the model underscores ocean acidification as a dominant factor in coral reef decline.

The analysis reveals that the removal rate of microplastics, denoted as γ, plays a multifaceted role in shaping the dynamics of marine ecosystems. An increase in γ leads to a reduction in microplastic concentration, which aligns with expectations. However, an interesting observation emerges wherein elevated values of γ are associated with increased levels of oceanic carbon dioxide (CO_2). This counterintuitive result highlights the complex feedback mechanisms between microplastic pollution and carbon cycling in marine environments. Furthermore, despite the reduction in microplastics, coral reef coverage tends to decline with increasing γ, suggesting indirect ecological disturbances resulting from shifts in environmental balance.

Similarly, the natural dissolution rate of carbon dioxide, represented by β, exhibits a significant influence on marine CO_2 concentrations. Higher values of β contribute to increased CO_2 accumulation in the ocean, which, in turn, exerts adverse effects on coral reef ecosystems. The dissolution process appears to operate independently of microplastic concentrations, implying that the chemical dynamics of CO_2 are decoupled from particulate pollution in certain contexts. The resultant acidification of ocean water due to elevated CO_2 levels is a critical driver of coral reef degradation.

The microplastic input rate, h, also emerges as a key factor exacerbating environmental stress. Increased values of h contribute to a rise in both microplastic and CO_2 concentrations. This dual escalation poses a compounded threat to coral reef health, as the ecosystem is subjected to simultaneous chemical and physical stressors. The decline in coral reef coverage under elevated microplastic input underscores the urgency of mitigating anthropogenic pollutants at their source.

7. Conclusion

The combined effects of microplastic pollution and ocean acidification on coral reef ecosystems are evaluated by developing a nonlinear differential equation model. Effective intervention requires simultaneous management of both chemical emissions and plastic waste to preserve the ecological integrity of the oceans and ensure the long-term survival of coral reef ecosystems. The results indicate that microplastic pollution contributes to the alteration of marine chemical processes, which can intensify the impacts of ocean acidification. However, ocean acidification arises from multiple sources, with higher atmospheric carbon dioxide absorption being the dominant factor. Within this context, microplastics act as an additional stressor, further weakening coral resilience and amplifying the risks of ecosystem degradation. The present work contributes to a deeper understanding of the interconnected roles of anthropogenic pollution, carbon dynamics, and reef sustainability. By examining the nonlinear interactions within this system, the study highlights the complexity of marine ecosystems and underscores the need for comprehensive approaches in both policy and conservation practices. Future research may extend this framework through optimal control strategies to identify effective pathways for protecting coral health against the combined pressures of acidification and pollution.

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Data availability. The manuscript contains the data that support the findings of this study.

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