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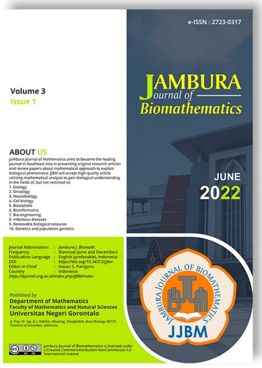

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Research Article

Implementation of non-standard finite difference on a predator prey model considering cannibalism on predator and harvesting on prey

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ABSTRACT. *The type of interaction between two different species in the same ecosystem plays an important role in the coexistence between these species. One type of interaction between species is predator-prey interaction. Several important factors are crucial to guarantee the existence of predator and prey in the same ecosystem, such as the carrying capacity of the ecosystem for the survival of prey, the intensity of predation, cannibalism in the predator population, and many other factors. External factors such as human intervention, such as harvesting, increase the complexity of the problem. Here in this article, we discuss a predator-prey model that takes predation and harvesting in prey populations into account. We implement a Non-Standard Finite Difference (NSFD) numerical scheme to solve our model due to it good performance on stability and approximation. Mathematical analysis on the existence and stability of equilibrium points from the discrete model was analyzed in detail. We implement a Nonstandard Finite Difference (NSFD) scheme to ensure numerical stability across various simulation scenarios. It is shown that NSFD has a better numerical stability compared to the standard numerical scheme like Euler or fourth-order Runge-Kutta method. From the sensitivity of autonomous simulation, we have shown that increases of cannibalism in predator populations will reduce predator populations, and as a result, the population of prey will increase due to the lack of number of predators. We also showed that increasing harvesting in prey populations may cause extinction in prey and predator populations. Furthermore, we have shown how periodic harvesting on prey populations may cause a critical condition on the existence of prey populations that takes a longer period to get recovered.*

1. Introduction

Predator-prey models are crucial for understanding ecological dynamics, specifically species interactions within an ecosystem. These models usually incorporate characteristics such as predation rates, population growth, and carrying capacity. However, real-world ecosystems exhibit more complex behaviours, such as predator cannibalism and human-induced interventions like prey species harvesting. Incorporating such variables into mathematical models increases their utility and provides additional insight into population dynamics [1].

A recent study underlined the relevance of functional responses in predator-prey systems, particularly when cannibalism occurs. Functional responses, such as the Holling Type-II model, describe the nonlinear interactions between species, including satiation effects at enormous prey densities, such mechanisms provides insights into how predator populations self-regulate through cannibalism, limiting resource depletion and competition with limited prey supply [2, 3]. Studies have revealed how prey harvesting strategies influence system dynamics, underlining the importance of setting sustainable harvesting levels to avoid catastrophic collapses. These advancements emphasize the need for numerical methods that can preserve the qualitative characteristics of these complicated models to give accurate simulations of real-world ecosystems. This study examines a predator-prey system that incorporates predator cannibalism and prey harvesting. Predator cannibalism, defined by a Holling Type-II functional response, is a self-limiting component of predator population development, whereas harvesting exerts external pressure on the prey population [4, 5]. The model builds on the framework presented in previous publications [6], encompassing these important ecological processes.

Another important thing that was discussed by authors in the predator-prey interaction is the existence of disease in prey or predator populations. Authors in [7] involved disease in predator population in their model. The existence and local stability analysis of their model were conducted in detail. Beay et al. [8] introduced a three-dimensional ordinary differential equation to model predator-prey interaction that involves harvesting in both populations and disease in the predator population. The effect of some important parameters on the dynamics of the population is discussed through numerical experiments. Furthermore, authors

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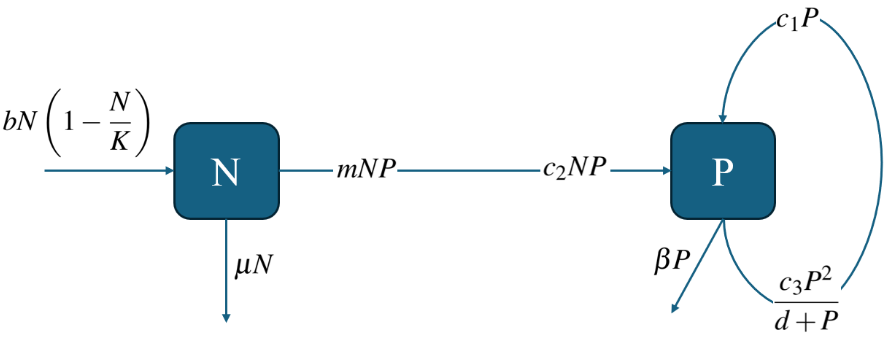


Figure 1. Interaction diagram between predator and prey for model in (1)

in [9] incorporated diseases in the prey population with the intervention of quarantine in their proposed model. The calculation of the reproduction number, which determines the persistence of disease, was discussed by the authors.

In addition to theoretical advances, predator-prey models are increasingly being applied in practical ecological management and conservation. To balance ecosystem sustainability and resource exploitation, integrated harvesting policies have been devised [10], considering both economic benefits and ecological restrictions. These models are especially useful in marine environments, where overfishing regularly produces predator-prey imbalances that can propagate across food webs. Cannibalism has been proven to provide a buffer against overharvesting, allowing predator populations to sustain themselves even when prey is scarce [23]. For readers who are interested a further reading on predator prey model can check [11–15] for predator-prey model with harvesting, [16–19] for predator prey model with diseases, and [20–22] for a more complex model. However, the relationship between harvesting techniques and self-regulation mechanisms such as cannibalism is poorly understood, and it is the primary subject of this study.

To understand this phenomenon, we modify the predator-prey model introduced by the authors in [24] and later modified by the authors in [25]. We incorporate harvesting in the prey population, which we assume may affect the dynamics of the predator population. To investigate the dynamics of this expanded model, we use a Non-Standard Finite Difference (NSFD) scheme. NSFD approaches are effective in retaining qualitative aspects of continuous systems, especially stability and positivity under different situations [26]. Our analysis shows that the NSFD scheme has better numerical stability than typical numerical methods such as Euler or fourth-order Runge-Kutta. Furthermore, we conduct a thorough examination of equilibrium points, their stability, and the effects of critical parameters, such as cannibalism and harvesting rates, on system behaviour.

This study advances ecological modelling by introducing a robust numerical approach for simulating predator-prey interactions under realistic ecological conditions. Our findings highlight the need of sustainable harvesting procedures, as well as the impact of intrinsic behaviours like cannibalism on population balance and ecosystem stability.

2. Model construction

Let us denote N and P represent the number of prey and predator population, respectively. The interaction diagram of the model is represented in Figure 1. Model construction is given as follow. We assume that the prey population growth follows a logistic equation with b and K representing intrinsic growth rate and carrying capacity, respectively. On the other hand, the predator population growth following negative exponential function in the absence of prey population, where β is the natural death rate of predator. We assume that there are no alternative food predator except the prey population. Hence, with the absence of predation and cannibalism, we have the dynamic of prey given by $\frac{dN}{dt}=bN\left(1-\frac{N}{K}\right)$ and for predator is given by $\frac{dP}{dt}=-\beta P$. With a predation of predator to prey with a specific successful contact rate is given by m, then the prey population will decreases due to predation with a rate of mNP. Assuming the conversion energy from predation activity, then we assume that the predator population will increases due to predation with a constant energy conversion of c_0. Hence, the predator increases in a rate of c_0mNP. Since m and c_0 are constant, then we simplify c_0m into c_2. Furthermore, due to cannibalism, the predator population will also decreases with a specific rate of c_3. We use a Functional response Holling type-2 to describe the cannibalism term, with d representing the saturation term. Hence, the cannibalism term is given by $\frac{c_3P^2}{d+P}$. Due to this cannibalism, then predator population will increasss with a specific constant rate c_1. In several special circumstances, it is possible that human play an important role in the balance of ecosystem. One of the intervention by human is population control with targeted harvesting. Hence, we include a prey-harvesting in our model with a specific constant rate μ. Based on the above assumption and interaction diagram in Figure 1, our mathematical model of predator-prey interaction considering cannibalism and prey harvesting is given by:

$$\begin{aligned}\frac{dN}{dt}&=bN\left(1-\frac{N}{K}\right)-mNP-\mu N,\\ \frac{dP}{dt}&=c_1P+c_2NP-\beta P-\frac{c_3P^2}{d+P},\end{aligned}\tag{1}$$

with a non-negative initial condition $N(0)>0, P(0)>0$. Implementing the Non Standard Finite Difference (NSFD) on eq. (1), then we have:

$$N_{n+1}=\frac{N_nK(b\phi+1)}{K(1+\phi(\mu+mP_n)+b\phi N_n},$$

$$P_{n+1} = P_n + \phi\left(c_1 P_n + c_2 N_{n+1} P_n - \beta P_{n+1} - \frac{c_3 P_{n+1} P_n}{P_n + d}\right), \tag{2}$$

where $\phi = \phi(h)$ is the denominator function such that $\phi(h) = h + O(h^2)$ with h represent the discretized time step size. This model had been constructed in [24], [25], without harvesting and involving predator cannibalism and/or refugees.

3. Model Analysis

By direct calculation, there are four type of equilibrium points of model in (2).

1. **Extinction of predator and prey.** This equilibrium represent a condition where both predator and prey extinct from the population after long period of time, and is given by

$$E_0 = (N_0, P_0) = (0, 0). \tag{3}$$

To analyze the local stability of E_0, we analyzed it using the Jacobian matrix of our model, and linearized it around E_0. This calculation gives us:

$$J(E_0) = \begin{bmatrix} \dfrac{\phi b + 1}{\phi\mu + 1} & 0 \\ 0 & \dfrac{\phi c_1 + 1}{\phi\beta + 1} \end{bmatrix}.$$

For a two-dimension system, we have the equilibrium point is locally asymptotically stable if $|\lambda_i| < 1$ for $i = 1, 2$. The eigenvalues of $J(E_0(0,0))$ is given by it diagonal entries. Hence, $J(E_0(0,0))$ is locally asymptotically stable if $b < \mu$ and $c_1 < \beta$.

2. **Extinction of prey.** Different with E_0, this equilibrium, denoted by E_1 represent a condition when only prey population extinct from the population, while predator may still exist. This equilibrium is given by:

$$E_1 = (N_1, P_1) = \left(0, \frac{d(c_1 - \beta)}{c_3 + \beta - c_1}\right). \tag{4}$$

To have a biological interpretation, E_1 should be in $\mathbb{R}^2_{\geq 0}$. Hence, P_1 should be positive and fulfilled only if $0 < c_1 - \beta < c_3$. From a direct calculation, the Jacobian matrix of system (2) evaluated at E_1 is given by:

$$\begin{aligned} J(E_1) &= \begin{bmatrix} J_1 & 0 \\ J_2 & J_3 \end{bmatrix}, \\ J_1 &= \frac{b\phi + 1}{-d\frac{(\beta - c_1)m\phi}{\beta - c_1 + c_3} + \mu\phi + 1}, \\ J_2 &= \frac{d(\beta - c_1)\phi c_2(b\phi + 1)}{L_1}, \\ J_3 &= \frac{(c_1^2 - 2\beta c_1 + \beta(\beta + c_3))\phi + c_3}{c_3(\phi c_1 + 1)}, \\ L_1 &= (((-dm + \mu)c_1 + (dm - \mu)\beta - \mu c_3)\phi - \beta + c_1 \\ &\quad - c_3)(\phi c_1 + 1). \end{aligned}$$

It was found that E_1 will locally asymptotically stable if

$$0 < (1 + L_1)(L_3) < 4, \tag{5}$$

where

$$\begin{aligned} L_2 &= \frac{(1 + b\phi)(\beta - c_1 + c_3)}{(1 + \mu\phi)(\beta - c_1 + c_3) - dm\phi(\beta - c_1)}, \\ L_3 &= \frac{((\beta - c_1)^2 + c_3\beta)\phi + c_3(\phi c_1 + 1)}{c_3(\phi c_1 + 1)}. \end{aligned}$$

3. **Extinction of predator.** This equilibrium represent a condition where predator extinct from the population after long period of time, and is given by

$$E_2 = (N_2, P_2) = \left(\frac{K(b - \mu)}{b}, 0\right). \tag{6}$$

$E_2 \in \mathbb{R}_2^+$ if $b > \mu$ is satisfied. The Jacobian matrix of our model evaluated in E_2 gives us:

$$J(E_2) = \begin{bmatrix} \dfrac{\mu\phi + 1}{b\phi + 1} & -\dfrac{\phi m(b - \mu)K}{(b\phi + 1)b} \\ 0 & \dfrac{((c_2 K + c_1)b - K\mu c_2)\phi + b}{(\beta\phi + 1)b} \end{bmatrix}.$$

Hence, E_2 is locally asymptotically stable if

$$0 < \left(\frac{\phi(\mu + b) + 2}{b\phi + 1}\right)\left(1 + \frac{(Kc_2(b - \mu) + bc_1)\phi + b}{b(1 + \beta\phi)}\right) < 4. \tag{7}$$

4. **Existence of predator and prey.** This equilibrium represent a condition where both predator and prey still exist in the population after a long period of time. The equilibrium called *coexistence point*, and is given by

$$E_3 = (N_3, P_3) = \left(\frac{K(b - \mu - mP_3)}{b}, P_3^+\right), \tag{8}$$

where P_3^+ is taken from the positive roots of

$$a_2 P_3^2 + a_1 P_3 + a_0 = 0, \tag{9}$$

with $a_2 = Kmc_2, a_1 = Kdmc_2 + K\mu c_2 - Kbc_2 + \beta b - bc_1 + bc_3$, and $a_0 = Kd\mu c_2 - Kdbc_2 + \beta db - c_1 db$. Assuming $b > \mu+, P_3^+ > 0$, then system (2) has

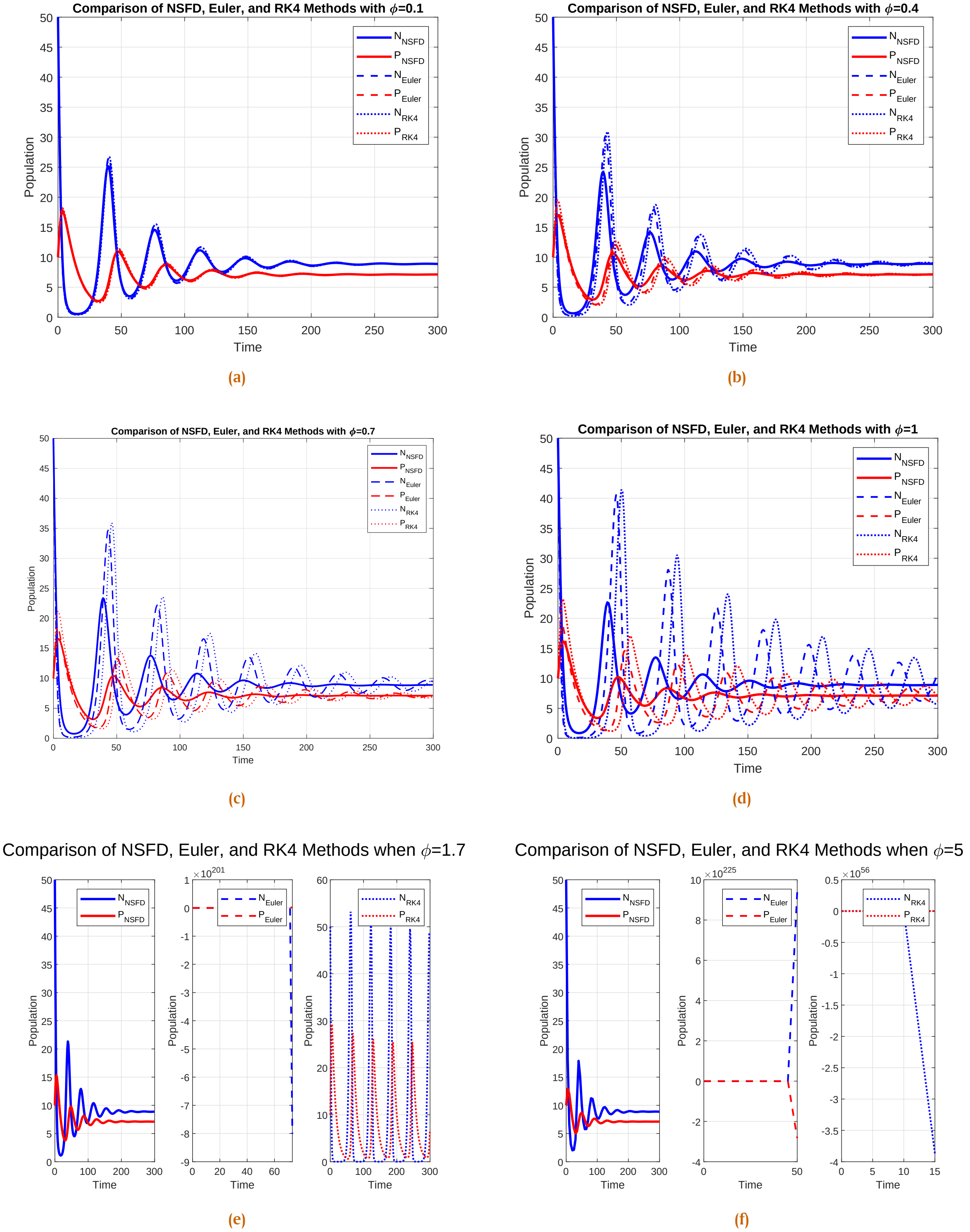
   (a) unique coexistence equilibrium if $a_0 < 0$,
   (b) multiple coexistence equilibrium if $a_0 > 0, a_1 < 0$, and $a_1^2 - 4a_2 a_0 \geq 0$.
   (c) no coexistence equilibrium otherwise.

Due to complexity expression of E_3, the local stability analysis conducted numerically using the following parameter values: $b = 10, K = 10, c_2 = 3, \mu = 0.4, \beta = 1.5, c_1 = 1, c_3 = 1.5, d = 1, m = 1, \phi(h) = 1$. With this parameter values, we have $E_3 = (0.61, 8.98)$ and the corresponds eigenvalues are $\lambda_{1,2} = 0.81 \pm 0.503i$, which indicates the stability of E_3.

4. Numerical experiments

We conduct our numerical simulations in this section using the NSFD method, and compared it with the Euler and the fourth order Runge-Kutta method. The numerical experiments conducted in several scenarios as follows. We choose $\phi(h) = h$ for our NSFD method, and use the following parameter values except it is stated differently:

$$K = 100,\ b = 0.5,\ m = 0.05,\ \mu = 0.1, c_1 = 0.02,\ c_2 = 0.01,$$
$$\beta = 0.1,\ c_3 = 0.01, d = 1, h = 0.01.$$



(a)

(b)

(c)

(d)

(e)

(f)

Figure 2. Comparison between NSFD, Euler and fourth order Runge-Kutta method using various value of ϕ: (a) 0.1, (b) 0.4, (c) 0.7, (d) 1, (e) 1.7, and (f) 5 .

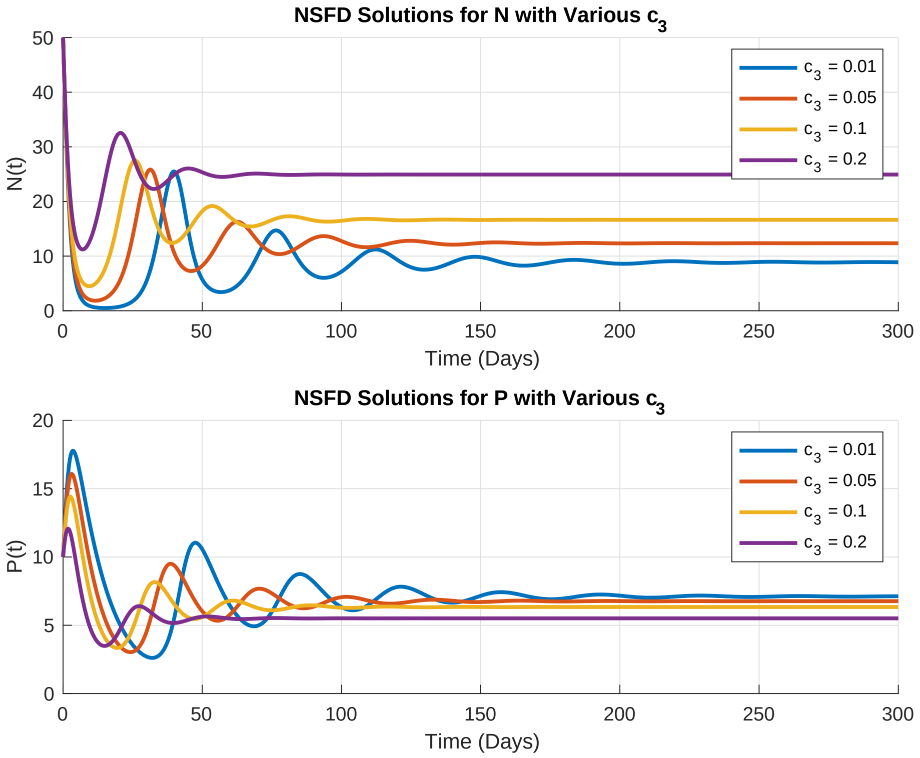


Figure 3. Autonomous simulation of system (2) for prey population (top) and predator population (bottom), with various value of cannibalism rate (c_3).

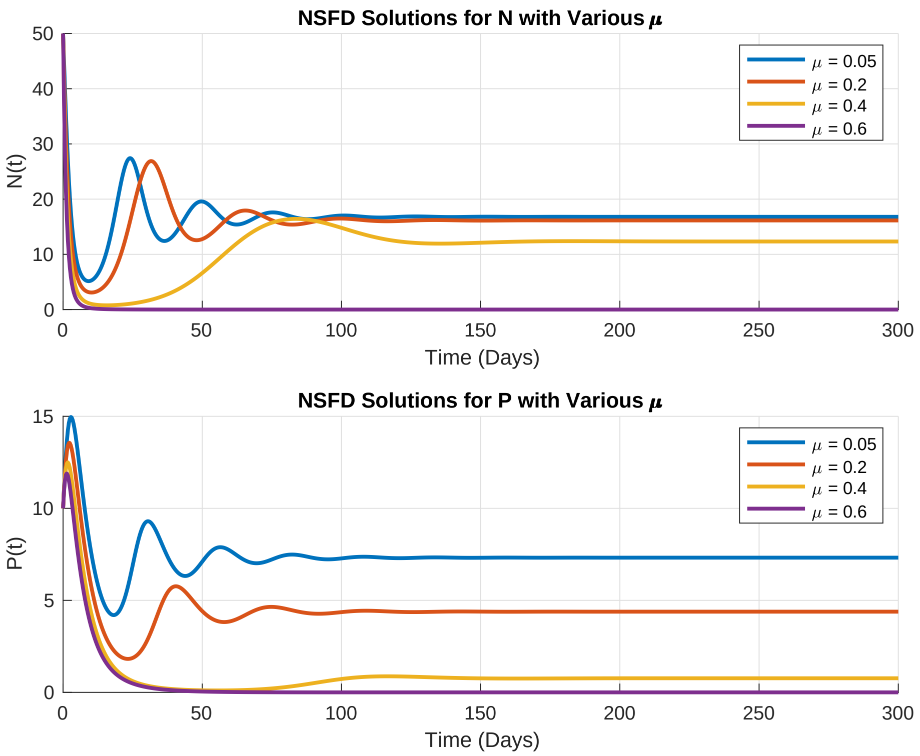


Figure 4. Autonomous simulation of system (2) for prey population (top) and predator population (bottom), with various value of harvesting rate (μ).

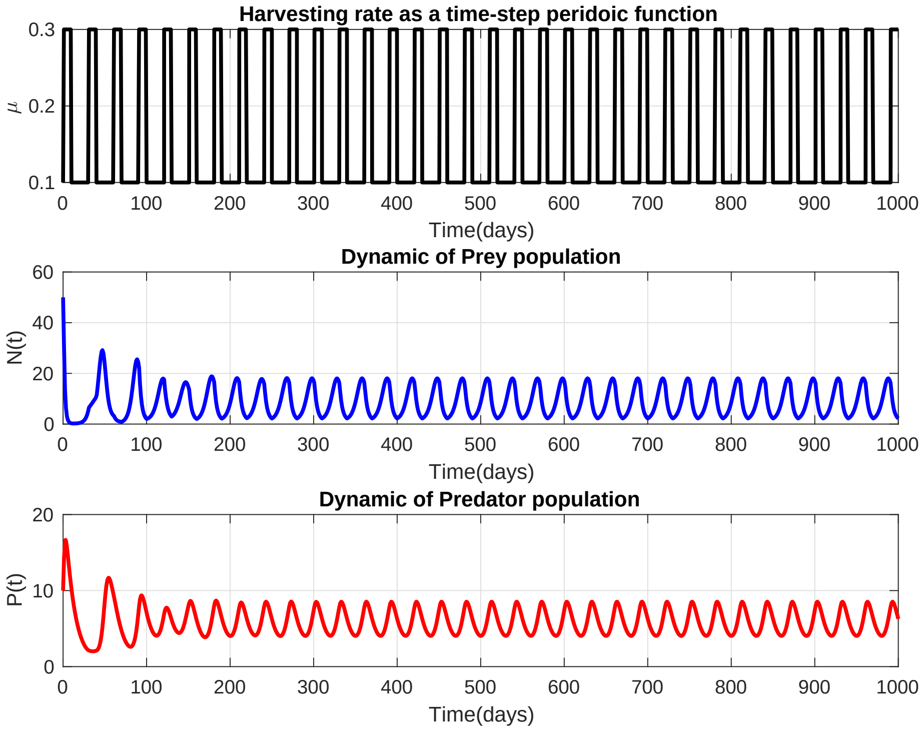


Figure 5. Autonomous simulation of system (2) for prey population (middle) and predator population (bottom), with time-dependent harvesting rate ($\mu(t)$) shown at top figure.

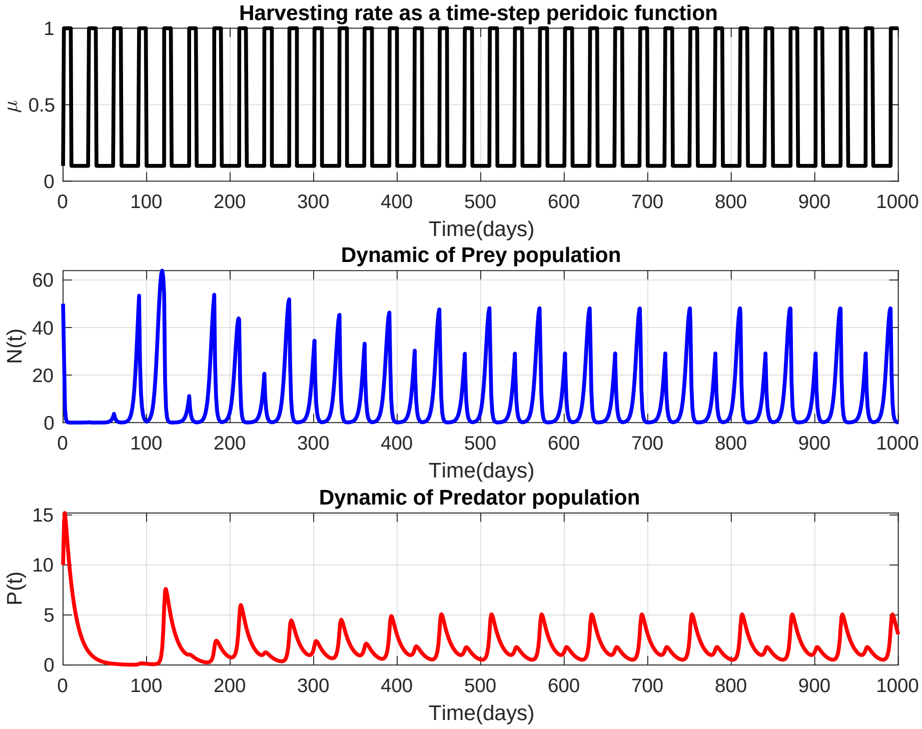


Figure 6. Autonomous simulation of system (2) for prey population (middle) and predator population (bottom), with time-dependent harvesting rate ($\mu(t)$) shown at top figure.

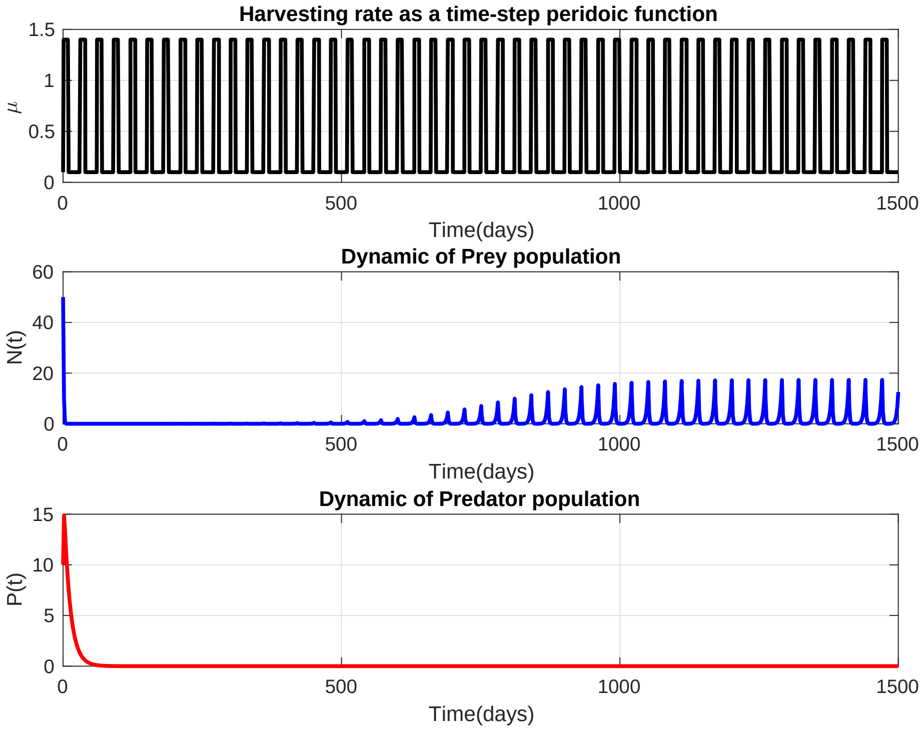


Figure 7. Autonomous simulation of system (2) for prey population (middle) and predator population (bottom), with time-dependent harvesting rate ($\mu(t)$) shown at top figure.

Each simulations conducted for 300 days.

1. **Comparison between the NSFD, Euler, and fourth order Runge-Kutta method**. For the first simulation, we aim to compare the performance of the NSFD, Euler, and fourth-order Runge-Kutta numerical schemes. We use the previously mentioned parameter values, with ϕ varying as $0.1, 0.4, 1, 1.7$, and 5. The results are presented in Figure 2. When $\phi = 0.1$, as shown in Figure 2a, the results from each numerical scheme are consistent, with all curves converging to the same equilibrium points. However, as ϕ increases to 0.4, differences in the solutions begin to emerge, becoming more pronounced at $\phi = 1$, as shown in Figure 2b. At this stage, the NSFD method reaches the equilibrium faster than the other methods. When ϕ is increased to 1.7, as depicted in Figure 2d, the Euler method fails, while the NSFD and fourth-order Runge-Kutta methods avoid divergence. When φ is increased to 1.7, as shown in Figure 2d, the simulation becomes more sensitive due to the increased nonlinearity or stiffness in the system dynamics. Under this condition, the Euler method—which is a first-order method with limited stability—fails to produce accurate results and diverges from the expected behavior, meaning its numerical solution grows without bound or becomes erratic. In contrast, the Nonstandard Finite Difference (NSFD) scheme and the fourth-order Runge-Kutta (RK4) method are more robust and stable, allowing them to maintain numerical accuracy and avoid divergence, even under this more extreme parameter setting. Furthermore, at $\phi = 5$ as shown in Figure 2e, the fourth-order Runge-Kutta method fails to converge to the equilibrium points, whereas only the NSFD method produces consistent solutions, tending toward the coexistence equilibrium.
   From this numerical experiments, we can see that the NSFD technique outperforms the Euler and fourth-order Runge-Kutta methods in terms of stability and consistency. For tiny ϕ, all approaches work similarly, but as ϕ rises, variations become noticeable. Both the fourth-order Runge-Kutta method and the Euler method fail to converge to the equilibrium points at $\phi = 5$ and $\phi = 1.7$, respectively. On the other hand, even at greater ϕ levels, the NSFD technique constantly converges to the coexistence equilibrium, demonstrating its dependability. This demonstrates how reliable the NSFD approach is in situations with higher parameter values of time steps ϕ.
2. **Effect of cannibalism** (c_3). For the second numerical experiment, we conduct simulations to investigate the impact of cannibalism on the predator population by varying the values of c_3. Figure 3 displays the findings for four distinct values of c_3: 0.01, 0.05, 0.1, and 0.2. The NSFD technique is employed for the numerical calculations, and all other parameter values stay the same as in the prior experiment. At equilibrium, an increase in the cannibalism rate (c_3) benefits the prey population while having a detrimental effect on the predator population. The amount of prey increases and the number of predators decreases as cannibalism rates rise. Moreover, as c_3 increases, the time required for both populations to reach their stable equilibrium also increases. This suggest that cannibalism slows down the process of

predator-prey population to reach their stable condition.

3. **Effect of a constant prey harvesting** (μ). The final set of numerical experiments was conducted for various values of the harvesting rate μ. All other parameter values remain the same as in the previous simulations, except for $\mu = 0.05, 0.2, 0.4,$ and 0.6. The results are presented in Figure 4. It is clearly observed that increasing the harvesting rate in the prey population leads to a reduction in the equilibrium levels of both prey and predator populations. A higher value of μ also increases the time required for each population to reach its equilibrium point. When the harvesting rate becomes excessively high ($\mu = 0.9$), extinction occurs not only in the prey population but also in the predator population—with the predator going extinct even earlier. These results highlight the critical role of properly managing the harvesting rate to prevent the collapse of both species and preserve ecosystem stability.
4. **Effect of periodic harvesting rate** $(\mu(t))$ For the last scenario of numerical experiment, we perform a time step periodic harvesting intervention on prey population. The time dependent periodic harvesting is define as the following function:

$$f(d) = \begin{cases} a, & 1 \leq (d \mod 30) \leq 10 \\ b, & 11 \leq (d \mod 30) \leq 30 \end{cases}. \tag{10}$$

We conduct three types of scenarios. The first scenario is for $a = 0.3$ and $b = 0.1$, where the results are shown in Figure 5; the second scenario is when $a = 1$ and $b = 0.1$, where the results are depicted in Figure 6; and the third scenario is when $a = 1.4$ and $b = 0.1$, where the results are given in Figure 7. The first scenario is implemented when the lower bound for the harvesting rate is 0.1, and the highest rate is only 0.3. With these types of scenarios, we can see that the dynamic of prey and predator can reach periodic behaviour much faster. The peaks of the prey and predator populations remain the same in each repeated periodic cycle. If we compare to scenario 2, where the upper bound for the harvesting rate increased up to $a = 1$ for the first 10 days, the dynamics are more interesting. The periodic solutions are more varied compared to the first scenario in Figure 5. Notably, there are two distinct peaks for the prey and predator populations in each repeated cycle. Additionally, harvesting prey has a more pronounced effect on the predator population, as the minimum predator population is lower than in the first scenario. The more extreme cases are shown by Scenario 3 in Figure 7. The upper bound of the harvesting rate is taken much higher, which is $a = 1.4$. Here, the prey population becomes critically low, nearing extinction by day 500. Consequently, the predator population also approaches extinction. As time increases, the prey population starts to recover, as well as the predator population. However, the predator population needs a longer time to get recovered from the over-harvesting in the prey population.

5. Conclusion

In this article, we have modified the predator-prey model introduced by the authors in [25] by incorporating a harvesting factor in the prey population. The mathematical analysis of the existence and local stability of equilibrium points has been carried out both analytically and numerically. We identified four types of equilibrium: extinction of both predator and prey populations, extinction of only the prey population, extinction of only the predator population, and coexistence of predator and prey in the ecosystem.

For numerical simulations, we employed the non-standard finite difference (NSFD) numerical scheme and compared it with the Euler method and the fourth-order Runge-Kutta method. Our numerical experiments demonstrated that the NSFD scheme is more stable and reliable, even with larger time steps. Additionally, we observed that as cannibalism increases within the predator population, the prey population begins to recover due to the reduced pressure from predators. Moreover, the system reaches equilibrium faster with higher rates of cannibalism.

We also analyzed numerically how the harvesting rate impacts not only the prey population but also the predator population over time. Our findings indicate that excessive and uncontrolled harvesting of the prey population can lead to a significant decline, potentially causing extinction. The extinction of the prey population, in turn, poses a severe threat to the predator population, increasing the likelihood of its extinction as well. These results underscore the critical importance of sustainable harvesting management for maintaining a balanced ecosystem.

Author Contributions. **Luis, P.**: Conceptualization, Methodology, Software, Formal analysis, Writing-original draft. **Kamalia, P. Z.**: Conceptualization, Methodology, Formal analysis, Writing-Review and Editing, Supervision. **Peter, O. J.**: Writing-Review and Editing. **Dipo, A.**: Conceptualization, Methodology, Software, Validation, Writing-original draft, Writing-Review and Editing, Supervision, Project administration, Funding Acquisition.

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