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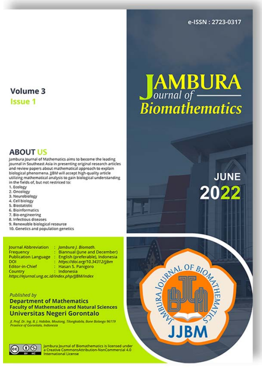

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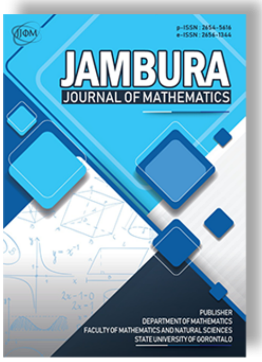

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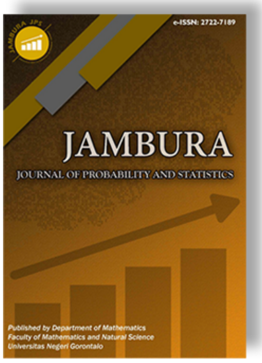

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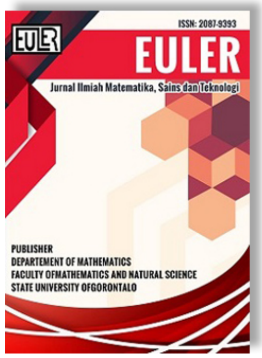

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Research Article

Mathematical Modeling of Teenage Pregnancy Focused on Awareness and Behavioral Change

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ABSTRACT. *Teenage pregnancy remains a significant public health concern, particularly in the Philippines. This study extends a previous SIT model by introducing a behavioral relapse pathway (Ω) that represents the rate at which informed adolescents revert to risky sexual behavior. The model divides the population into susceptible, corrupted, and aware compartments, incorporating contraceptive use and sex education. Analytical results show that the corruption-free equilibrium is locally asymptotically stable when $R_0 < 1$, while corruption persists when $R_0 > 1$. Numerical simulations reveal that increasing Ω from 0.01 to 0.2 raises the long-term corrupted population fraction from approximately 8% to more than 25% with transient peaks up to 22%, even with high awareness levels. A local sensitivity analysis further reveals that the recruitment rate (ω), voluntary cessation rate (π), and natural death rate (μ) exert the greatest influence on long-term outcomes. These findings highlight that sustained awareness campaigns must be coupled with strategies that minimize relapse into risky behavior, such as continuous sex education, peer mentorship, and counter-misinformation initiatives.*

1. Introduction

Teenage pregnancy is a global public health concern with significant social, economic, and health implications. It is associated with higher risks of maternal and infant mortality, as well as adverse health outcomes for both mothers and children. According to the World Health Organization (WHO), approximately 21 million girls aged 15–19 in developing regions become pregnant annually, with around 12 million continuing to childbirth [1]. While the global adolescent birth rate (ABR) has decreased substantially - from 64.5 births per 1,000 women aged 15–19 in 2000 to 41.3 in 2023 [1] - the rate of decline has been uneven across regions and within countries.

In the Philippines, teenage pregnancy remains a pressing concern. The country has one of the highest adolescent birth rates in Southeast Asia, with about one in ten Filipino girls aged 15–19 years being pregnant or already a mother [2]. The 2022 National Demographic and Health Survey [2] reports a decline in teenage pregnancies among 15–19-year-olds from 8.6% in 2012 to 5.4% in 2022, yet regional disparities persist. Northern Mindanao recorded the highest rate at 10.9%, followed by Davao (8.2%), Central Luzon (8.0%), and Caraga (7.7%). Alarmingly, total live births among girls under 15 years increased by 35.13% from 2021 to 2022. Recognizing the urgency of the problem, Executive Order No. 141 was issued to address the root causes of rising teenage pregnancies. Government initiatives such as the Social Protection Program for Teenage Mothers and their Children, and strengthened sex education campaigns, have sought to mitigate the trend. However, the Commission on Population and Development (CPD) declared teenage pregnancy a national emergency in 2022, emphasizing that access to accurate and reliable sexual health information remains a critical challenge.

While numerous studies have investigated the sociocultural and economic determinants of teenage pregnancy [3–6], mathematical modeling offers a valuable tool for understanding its dynamics and evaluating intervention strategies. Compartment-based epidemic models, for instance, have been extensively used to analyze biological and social systems due to their ability to capture transmission mechanisms and intervention outcomes. Related modeling studies include fractional-order approaches to teenage pregnancy rehabilitation [7], relapse-driven dynamical systems [8], and information- or awareness-related behavioral models, and microsimulation-based evaluations of health coaching interventions to prevent teen pregnancy [9]. In the Kenyan study of Nathan [10], an SIT compartmental model was used, where the population is divided into Susceptible (uninformed and uncorrupted), Infected/Corrupted (sexually active in risky ways), and Treatment/Aware (informed about sexual health) classes. The original model assumed that once informed, adolescents would not revert to risky behaviors.

However, research indicates that awareness alone does not guarantee sustained safe behavior. Behavioral regression, or relapse into unsafe sexual practices, has been observed even among youth who have received formal sex education. For instance, Habito et al. [11] reported that despite participation in awareness programs, 12–18% of adolescents engaged in unprotected intercourse within six months, with peer pressure and re-

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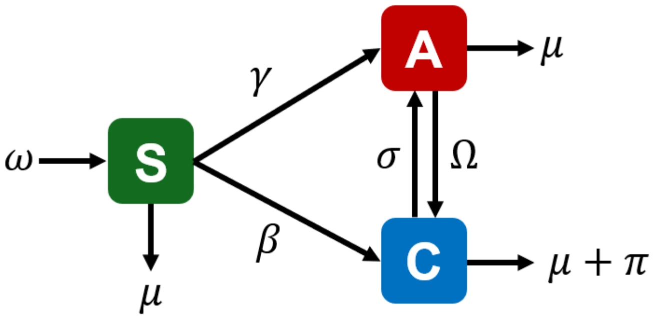

Figure 1. Schematic Representation of the Model

Table 1. Description of the Parameters

Parameter	Description
ω	Recruitment rate
μ	Natural death rate
β	Rate of susceptibility influencing class C
γ	Rate of susceptibility influencing class A
σ	Corrupted class guided to join class A
π	Corrupted class who voluntarily cease engaging in risky sexual behavior
Ω	Transition from the awareness class back to the corrupted class

lationship influences cited as key drivers. Internationally, Kirby et al. [12] found that while school-based programs improve knowledge and attitudes, these effects can diminish over time without reinforcement, leading to renewed risky behavior. Moreover, the CPD behavioral surveillance data revealed that misinformation from online platforms can counteract formal awareness campaigns, resulting in conflicting beliefs about contraceptive use and sexual health.

TThese findings support the introduction of the new pathway parameter Ω in our model. This parameter represents the rate at which informed adolescents revert to risky behavior, allowing the model to capture the cyclical nature of adolescent decision-making and the influence of persistent sociocultural and environmental pressures. The use of relapse pathways, nonlinear behavioral transitions, and awareness-feedback mechanisms has also been examined in various dynamical-systems frameworks [13–18]. By including Ω, our extended SIT framework more accurately reflects the complex interplay between awareness, environmental influences, and behavioral choices among adolescents. For analytical tractability, we first study the special case $\Omega = 0$. In the later sections, we relax this assumption and use numerical simulations to examine the effect of $\Omega > 0$ on the system dynamics.

2. Mathematical Model Formulation

In this work, we extend the SIT framework of Nathan [10] by adding a behavioral relapse pathway (Ω) to capture the rate at which informed adolescents revert to risky behavior. The total adolescent population, $N(t)$, is divided into three compartments:

- **Susceptible** (S) - Adolescents who are uninformed and have not engaged in risky sexual activity.
- **Corrupted** (C) - Adolescents engaged in unhealthy sexual activities or influencing others to do so.
- **Awareness** (A) - Adolescents who are informed about safe sexual practices.

At any time t, the total population is

$$N(t) = S(t) + C(t) + A(t).$$

The model assumes:

1. New individuals enter the S class at a constant recruitment rate ω.
2. Susceptible individuals may transition to the awareness class at rate γ or corrupted at rate β
3. Awareness does not guarantee permanent safe behavior; informed individuals may revert to C at a rate Ω.
4. Corrupted individuals may be guided into awareness (σ) or voluntarily quit unhealthy behavior (π).
5. All compartments are subject to a natural death rate μ.

The resulting system of equations is:

$$\begin{aligned} \frac{dS}{dt} &= \omega - \beta SC - \gamma S - \mu S, \\ \frac{dC}{dt} &= \beta SC + \Omega AC - (\sigma + \mu + \pi)C, \\ \frac{dA}{dt} &= \gamma S + \sigma C - \Omega AC - \mu A, \end{aligned} \tag{1}$$

with the positive initial conditions:

$$S(0) \geq 0, \quad C(0) \geq 0, \quad \text{and} \quad A(0) \geq 0. \tag{2}$$

Parameter descriptions are given in Table 1, and the schematic diagram in Figure 1 illustrates the possible transitions between compartments.

3. Analytical Results

Well-posedness, nonnegativity, and boundedness of solutions of the proposed model can be shown using basic theory of dynamical systems as described in [19–22]. By adding all the equations in system (1) we obtain,

$$\begin{aligned} \frac{dN}{dt} &= \frac{dS}{dt} + \frac{dC}{dt} + \frac{dA}{dt}, \\ &= \omega - \beta SC - \gamma S - \mu S + \beta SC + \Omega AC - (\sigma + \mu + \pi)C \\ &\quad + \gamma S + \sigma C - \Omega AC - \mu A, \\ &= \omega - \mu S - (\mu + \pi)C - \mu A, \\ &= \omega - \mu(S + C + A) - \pi C, \\ &= \omega - \mu N - \pi C, \\ &< \omega - \mu N. \end{aligned}$$

Applying the Gronwall inequality we have,

$$\begin{aligned} N(t) &\leq e^{-\mu t} N(0) + (-\mu)^{-1} \left(e^{-\mu t} - 1\right) \omega \\ &= e^{-\mu t} N(0) - \frac{1}{\mu} \left(e^{-\mu t} - 1\right) \omega \\ &= e^{-\mu t} N(0) - \frac{\omega}{\mu} \left(e^{-\mu t} - 1\right) \\ &= e^{-\mu t} N(0) - \frac{\omega}{\mu} e^{-\mu t} + \frac{\omega}{\mu} \\ &= e^{-\mu t} \left(N(0) - \frac{\omega}{\mu}\right) + \frac{\omega}{\mu} \end{aligned}$$

From which $N(t) \leq N_m$, whenever $N(0) \leq N_m$ where, $N_m = \frac{\omega}{\mu}$.
Therefore, it follows that the biologically feasible region for system (1) is

$$\mathfrak{D} = \left\{ (S, C, A) \in R^3_{\geq 0} : N(t) \leq \frac{\omega}{\mu} \right\}. \quad (3)$$

3.1. Corruption-free equilibrium point

The corruption-free equilibrium point of the system (1) is a point where the corruption of population at puberty is not present. It is obtained by setting the derivatives to zero and putting the infected compartments to zero.

Theorem 1. *The system (1) admits corruption-free equilibrium point (CFE) of the system (1) given by* $\mathcal{E}^0 = \left(\frac{\omega}{\gamma+\mu}, 0, 0 \right)$.

Proof. Let $\mathcal{E}^0 = (S^0, C^0, A^0)$ be the corruption-free equilibrium point of the model (1), that is

$$\omega - \beta S^0 C^0 - \gamma S^0 - \mu S^0 = 0, \quad (4)$$
$$\beta S^0 C^0 + \Omega A^0 C^0 - (\sigma + \mu + \pi) C^0 = 0, \quad (5)$$
$$\gamma S^0 + \sigma C^0 - \Omega A^0 C^0 - \mu A^0 = 0. \quad (6)$$

Suppose that the $C = 0$ and $A = 0$, we assume that there are no corrupted/influenced teenagers, then from eq. (4), we obtain

$$\begin{aligned} \omega - \beta S^0 C^0 - \gamma S^0 - \mu S^0 &= 0 \\ \omega - \gamma S^0 - \mu S^0 &= 0 \\ S^0(\gamma + \mu) &= \omega \\ S^0 &= \frac{\omega}{\gamma + \mu} \end{aligned}$$

Therefore, the corruption-free equilibrium point is

$$\mathcal{E}^0 = \left(\frac{\omega}{\gamma + \mu}, 0, 0 \right).$$

□

3.2. Basic reproduction number

Next, we will calculate the basic reproduction number of the system (1) using the next generation matrix method [23, 24]. Since this study is not a problem in epidemiology, we redefine the reproduction number based on the definition of [10] which says that the basic reproduction number R_0 is the average number of population at puberty, one sexually corrupt person can recruit in a purely susceptible population throughout the contact period. The disease compartments are C and A compartments.

Consider that $\mathcal{F}$ is the rate of appearance of new corrupted individuals in compartments and $\mathcal{V}$ is the rate of transfer of individuals out of compartment. In this way, the matrices $\mathcal{F}$ and $\mathcal{V}$ associated with system (1) are given by

$$\mathcal{F} = \begin{pmatrix} \beta SC + \Omega AC \\ \Omega AC \end{pmatrix},$$

$$\mathcal{V} = \begin{pmatrix} (\sigma + \mu + \pi)C \\ -\gamma S - \sigma C + \mu A \end{pmatrix}$$

The Jacobian matrices of $\mathcal{F}$ and $\mathcal{V}$ evaluated at the corruption-free equilibrium are F and V, respectively.

$$\begin{aligned} F &= \begin{pmatrix} \beta S + \Omega A & \Omega C \\ \Omega A & \Omega C \end{pmatrix}, \\ V &= \begin{pmatrix} \sigma + \mu + \pi & 0 \\ -\sigma & \mu \end{pmatrix}, \\ FV^{-1}(\mathcal{E}^0) &= \begin{pmatrix} \beta S + \Omega A & \Omega C \\ \Omega A & \Omega C \end{pmatrix} \begin{pmatrix} \frac{1}{\sigma+\mu+\pi} & 0 \\ -\frac{1}{\sigma} & \frac{1}{\mu} \end{pmatrix}, \\ &= \begin{pmatrix} \frac{\beta S}{\sigma+\mu+\pi} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sigma+\mu+\pi} & 0 \\ -\frac{1}{\sigma} & \frac{1}{\mu} \end{pmatrix}, \\ &= \begin{pmatrix} \frac{\beta(\frac{\omega}{\gamma+\mu})}{\sigma+\mu+\pi} & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned}$$

The basic reproduction number is the dominant eigenvalue of FV^{-1} which is $R_0 = \beta(FV^{-1})$. Thus,

$$R_0 = \frac{\beta\omega}{(\gamma + \mu)(\sigma + \mu + \pi)}$$

3.3. Corrupt-persistence equilibrium point ($\Omega = 0$ case)

In teenage pregnancy, the corrupt-persistence equilibrium point refers to a stable state in which the prevalence of a teenage pregnancy issue remains relatively constant within a population over time.

The following results are derived under the assumption $\Omega = 0$. When relapse is present ($\Omega > 0$), the algebraic expressions may become analytically intractable, and we therefore turn to numerical exploration.

Theorem 2. *The system (1) with $\Omega = 0$ admits a unique corrupt-persistence equilibrium point (CPE) given by $\mathcal{E}^* = (S^*, C^*, A^*)$, whenever $R_0 > 1$, where*

$$S^* = \frac{\omega}{R_0(\gamma + \mu)}, \quad C^* = \frac{(\gamma + \mu)(R_0 - 1)}{\beta},$$
$$A^* = \frac{1}{\mu\beta}[\gamma(\sigma + \mu + \pi) + \sigma(\gamma + \mu)(R_0 - 1)].$$

Proof. Let $\mathcal{E}^* = (S^*, C^*, A^*)$ be a corrupt-persistence equilibrium point. Suppose that $S \neq 0$, $C \neq 0$ and $A \neq 0$. To solve for the corrupt-persistence equilibrium point, we set the equations in the system (1) to 0, that is

$$\omega - \beta S^* C^* - \gamma S^* - \mu S^* = 0, \quad (7)$$
$$\beta S^* C^* + \Omega A^* C^* - (\sigma + \mu + \pi) C^* = 0, \quad (8)$$
$$\gamma S^* + \sigma C^* - \Omega A^* C^* - \mu A^* = 0. \quad (9)$$

Then from eq. (7), we obtain

$$\begin{aligned} \omega - \beta S^* C^* - \gamma S^* - \mu S^* &= 0 \\ \omega - S^*(\beta C^* + \gamma + \mu) &= 0 \end{aligned}$$

$$S^*(\beta C^* + \gamma + \mu) = \omega$$
$$S^* = \frac{\omega}{\beta C^* + \gamma + \mu}$$
$$S^* = \frac{\omega}{C^*\beta + \gamma + \mu} \tag{10}$$

From eq. (9) substitute the value of eq. (10) to solve for A^*,

$$\begin{aligned}
0 &= \gamma S^* + \sigma C^* - \Omega A^* C^* - \mu, A^* \\
0 &= \gamma S^* + \sigma C^* - A^*(\Omega C^* + \mu), \\
A^* &= \frac{\gamma S^* + \sigma C^*}{\Omega C^* + \mu}, \\
&= \frac{\gamma\left(\frac{\omega}{\beta C^* + \gamma + \mu}\right) + \sigma C^*}{\Omega C^* + \mu}, \\
&= \frac{\gamma\omega}{(\beta C^* + \gamma + \mu)(\Omega C^* + \mu)} + \frac{\sigma C^*}{(\sigma C^* + \mu)}, \\
&= \frac{\gamma\omega + \sigma C^*(\beta C^* + \gamma + \mu)}{(\beta C^* + \gamma + \mu)(\Omega C^* + \mu)}, \\
&= \frac{\gamma\omega + \sigma C^*\beta + \sigma C^*\gamma + \sigma C^*\mu}{(\beta C^* + \gamma + \mu)(\Omega C^* + \mu)}, \\
A^* &= \frac{C^{*^2}\beta\sigma + C^*\gamma\sigma + C^*\mu\sigma + \gamma\omega}{(C^*\beta + \gamma + \mu)(C^*\Omega + \mu)}
\end{aligned} \tag{11}$$

From equation eq. (8) substitute the value of eqs. (10) and (11) to solve for C^*, we obtain

$$\begin{aligned}
0 &= \beta S^* C^* + \Omega A^* C^* - (\sigma + \mu + \pi)C^*, \\
0 &= C^*(\beta S^* + \Omega A^* - (\sigma + \mu + \pi)), \\
0 &= \beta S^* + \Omega A^* - (\sigma + \mu + \pi) \quad \text{or} \quad C^* = 0, \\
0 &= \Omega\left(\frac{(C^{*^2}\beta\sigma + C^*\gamma\sigma + C^*\mu\sigma + \gamma\omega)}{(C^*\beta + \gamma + \mu)(C^*\Omega + \mu)}\right) \\
&\quad \beta\left(\frac{\omega}{(C^*\beta + \gamma + \mu)}\right) + -(\sigma + \mu + \pi), \\
0 &= \frac{\beta\omega}{C^*\beta + \gamma + \mu} + \frac{\Omega(C^{*^2}\beta\sigma + C^*\gamma\sigma + C^*\mu\sigma + \gamma\omega)}{(C^*\beta + \gamma + \mu)(C^*\Omega + \mu)} \\
&\quad - (\sigma + \mu + \pi), \\
0 &= \beta\omega(C^*\Omega + \mu) + \Omega(C^{*^2}\beta\sigma + C^*\gamma\sigma + C^*\mu\sigma + \gamma\omega) \\
&\quad - (\sigma + \mu + \pi)(C^*\beta + \gamma + \mu)(C^*\Omega + \mu).
\end{aligned}$$

The expanded form would be,

$$\begin{aligned}
0 &= \beta\omega\Omega C^* + \beta\omega\mu + \beta\sigma\Omega C^{*^2} + \gamma\sigma\Omega C^* + \mu\sigma\Omega C^* + \Omega\gamma\omega \\
&\quad - (C^{*^2}\Omega\beta\mu + C^{*^2}\Omega\beta\pi + C^{*^2}\Omega\beta\sigma + C^*\Omega\gamma\mu + C^*\Omega\gamma\pi \\
&\quad + C^*\Omega\gamma\sigma + C^*\Omega\mu^2 + C^*\Omega\mu\pi + C^*\Omega\mu\sigma + C^*\beta\mu^2 \\
&\quad + C^*\beta\mu\pi + C^*\beta\mu\sigma + \gamma\mu^2 + \gamma\mu\pi + \gamma\mu\sigma + \mu^3 \\
&\quad + \mu^2\pi + \mu^2\sigma).
\end{aligned}$$

Now, we can rewrite this as a quadratic equation in terms of C^*:

$$aC^{*^2} + bC^* + c = 0$$

where

$$a = -\Omega\beta(\mu + \pi),$$
$$b = \beta\omega\Omega - \Omega(\gamma + \mu)(\mu + \pi) - \beta\mu(\mu + \pi + \sigma),$$
$$c = \mu(\mu + \pi + \sigma)(\gamma + \mu)\left[R_0 - 1 + \frac{\Omega\gamma\omega}{\mu(\mu + \pi + \sigma)(\gamma + \mu)}\right].$$

We assume that $\Omega = 0$, which means there are no teens who attended the awareness class left because they chose to be corrupted. By substituting $\Omega = 0$ into our quadratic equation, we obtain

$$C^* = \frac{(\gamma + \mu)(R_0 - 1)}{\beta}, \quad S^* = \frac{\omega}{R_0(\gamma + \mu)},$$
$$A^* = \frac{1}{\mu\beta}[\gamma(\sigma + \mu + \pi) + \sigma(\gamma + \mu)(R_0 - 1)].$$

Note that S^*, C^*, and A^* exist whenever $R_0 > 1$. □

3.4. *Stability analysis of equilibrium points*

In this subsection, we will show the local stability of the equilibrium points. Stability analysis provides insight into the long-term behavior of a system near its equilibrium state.

Theorem 3. *The corruption-free equilibrium point,*

$$\mathcal{E}^0 = \left(\frac{\omega}{\gamma + \mu}, 0, 0\right)$$

is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof. The Jacobian matrix of the system (1) is given by

$$J = \begin{pmatrix} -\beta C^0 - \gamma - \mu & -\beta S^0 & 0 \\ \beta C^0 & \beta S^0 + \Omega A^0 - (\sigma + \mu + \pi) & \Omega C^0 \\ \gamma & \sigma - \Omega A^0 & -\Omega C^0 - \mu \end{pmatrix}.$$

Now, the Jacobian matrix evaluated at the corruption-free equilibrium point is of the form

$$J(\mathcal{E}^0) = \begin{pmatrix} -(\gamma + \mu) & \frac{-\beta\omega}{\gamma + \mu} & 0 \\ 0 & \frac{\beta\omega}{\gamma + \mu} - (\sigma + \mu + \pi) & 0 \\ \gamma & \sigma & -\mu \end{pmatrix}.$$

Stability at the corruption-free equilibrium point can be seen from the eigenvalues of the Jacobian matrix $J(\mathcal{E}^0)$. Based on the Routh-Hurwitz criterion, if all real parts of the eigenvalues of the Jacobian matrix are negative at $\mathcal{E}^0$, then $\mathcal{E}^0$ is locally asymptotically stable.

$$\det\left(J\left(\mathcal{E}^0\right) - \lambda I\right) = 0$$

.

Now,

$$\det\begin{pmatrix} -(\gamma + \mu) - \lambda & \frac{-\beta\omega}{\gamma + \mu} & 0 \\ 0 & \frac{\beta\omega}{\gamma + \mu} - (\sigma + \mu + \pi) - \lambda & 0 \\ \gamma & \sigma & -\mu - \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Hence the eigenvalues are,

$$\lambda_1 = -(\gamma + \mu),$$
$$\lambda_2 = \frac{\beta\omega}{\gamma + \mu} - (\sigma + \mu + \pi),$$
$$\lambda_3 = -\mu.$$

Note that,

$$\lambda_1 = -(\gamma+\mu) < 0 \text{ and } \lambda_3 = -\mu < 0.$$

In order for the corruption-free equilibrium to be locally asymptotically stable, λ_2 should be less than 0. Observe that,

$$\begin{aligned} \frac{\beta\omega}{\gamma+\mu} - (\sigma+\mu+\pi) &< 0 \\ (\sigma+\mu+\pi)\left(\frac{\beta\omega}{(\gamma+\mu)(\sigma+\mu+\pi)} - 1\right) &< 0 \\ (\sigma+\mu+\pi)(R_0-1) &< 0 \end{aligned}$$

Therefore, the corruption-free equilibrium is locally asymptotically stable when $R_0 < 1$ and unstable if $R_0 > 1$. □

Theorem 4. *The corrupt-persistence equilibrium point, $\mathcal{E}^* = (S^*, C^*, A^*)$, where*

$$S^* = \frac{\omega}{R_0(\gamma+\mu)}, \quad C^* = \frac{(\gamma+\mu)(R_0-1)}{\beta},$$
$$A^* = \frac{1}{\mu\beta}[\gamma(\sigma+\mu+\pi)+\sigma(\gamma+\mu)(R_0-1)].$$

is locally asymptotically stable if $R_0 > 1$ and unstable if $R_0 < 1$.

Proof. The Jacobian matrix of the system (1) is as follows.

$$J = \begin{pmatrix} -\beta C^* - \gamma - \mu & -\beta S^* & 0 \\ \beta C^* & \beta S^* + \Omega A^* - (\sigma+\mu+\pi) & \Omega C^* \\ \gamma & \sigma - \Omega A^* & -\Omega C^* - \mu \end{pmatrix}$$

Assume that $\Omega = 0$, the Jacobian matrix becomes

$$J = \begin{pmatrix} -\beta C^* - \gamma - \mu & -\beta S^* & 0 \\ \beta C^* & \beta S^* & 0 \\ \gamma & \sigma & -\mu \end{pmatrix}$$

Now, the Jacobian matrix evaluated at the corrupt- persistence equilibrium point is of the form,

$$J(\mathcal{E}^*) = \begin{pmatrix} j_1 & \left(\frac{-\beta\omega}{R_0(\gamma+\mu)}\right) & 0 \\ (\gamma+\mu)(R_0-1) & j_2 & 0 \\ \gamma & \sigma & -\mu \end{pmatrix},$$
$$j_1 = -(\gamma+\mu)(R_0-1) - \gamma - \mu,$$
$$j_2 = \left(\frac{\beta\omega}{R_0(\gamma+\mu)}\right) - \sigma - \mu - \pi.$$

To show that all the eigenvalues of $J(\mathcal{E}^*)$ are negative, we note that the third column of $J(\mathcal{E}^*)$ contains only the diagonal element $-\mu$, indicating that $-\mu$ is a negative eigenvalue. The remaining eigenvalues can be determined from the sub matrix $J_1(\mathcal{E}^*)$, which can be obtained by eliminating the 3rd row and 3rd column. This gives us,

$$J_1(\mathcal{E}^*) = \begin{pmatrix} j_1 & \left(\frac{-\beta\omega}{R_0(\gamma+\mu)}\right) \\ (\gamma+\mu)(R_0-1) & j_2 \end{pmatrix}$$

Stability at the corrupt-persistence equilibrium point can be seen from the eigenvalues of $J_1(\mathcal{E}^*)$. Based on the Routh-Hurwitz criterion, if all real parts of the eigenvalues of the Jacobian matrix are negative at $\mathcal{E}^*$, then $\mathcal{E}^*$ is locally asymptotically stable.

$$\det\left(J_1(\mathcal{E}^*) - \lambda I\right) = 0$$

From $J_1(\mathcal{E}^*)$, the characteristic polynomial is given by

$$\begin{aligned} 0 = &\frac{\pi R_0^2\gamma + \pi R_0^2\mu + R_0^2\gamma\mu + R_0^2\gamma\sigma + R_0^2\mu^2 + R_0^2\mu\sigma - \beta\omega}{R_0} \\ &+ \left(\frac{R_0^2\gamma^2 + 2R_0^2\gamma\mu + R_0^2\mu^2 + \pi R_0\gamma + \pi R_0\mu}{R_0(\gamma+\mu)}\right. \\ &+ \left.\frac{R_0\gamma\sigma + R_0\mu^2 + R_0\mu\sigma - \beta\omega}{R_0(\gamma+\mu)}\right)\lambda + \lambda^2, \end{aligned}$$

which can be rewritten as

$$\lambda^2 + a_1\lambda + a_2 = 0$$

where,

$$\begin{aligned} a_1 &= \frac{R_0^2\gamma^2 + 2R_0^2\gamma\mu + R_0^2\mu^2 + \pi R_0\gamma + \pi R_0\mu}{R_0(\gamma+\mu)} \\ &\quad + \frac{R_0\gamma\sigma + R_0\mu^2 + R_0\mu\sigma - \beta\omega}{R_0(\gamma+\mu)}, \\ a_2 &= \frac{\pi R_0^2\gamma + \pi R_0^2\mu + R_0^2\gamma\mu + R_0^2\gamma\sigma + R_0^2\mu^2 + R_0^2\mu\sigma - \beta\omega}{R_0}. \end{aligned}$$

Based on the Routh-Hurwitz criterion, all roots of a polynomial, say $\lambda^2 + a_1\lambda + a_2 = 0$, have negative real parts if and only if $a_1 > 0$ and $a_2 > 0$.

Now,

$$\begin{aligned} a_1 &= \frac{R_0^2\gamma^2 + 2R_0^2\gamma\mu + R_0^2\mu^2 + \pi R_0\gamma + \pi R_0\mu + R_0\gamma\sigma + R_0\mu^2}{R_0(\gamma+\mu)} \\ &\quad + \frac{R_0\mu\sigma - \beta\omega}{R_0(\gamma+\mu)}, \\ &= R_0(R_0(\gamma+\mu)(\gamma+\mu) + \pi(\gamma+\mu) + \sigma(\gamma+\mu) + \mu^2) - \beta\omega, \\ &= (R_0^2(\gamma+\mu)^2 + \pi(\gamma+\mu) + \sigma(\gamma+\mu) + \mu(\gamma+\mu))R_0 - \beta\omega, \\ &= \frac{(R_0(\gamma+\mu)(R_0(\gamma+\mu) + (\pi+\sigma+\mu)) - \beta\omega)}{R_0(\gamma+\mu)} > 0, \\ &\Rightarrow R_0(\gamma+\mu)(R_0(\gamma+\mu) + (\pi+\sigma+\mu)) - \beta\omega > 0, \\ &\Rightarrow R_0(\gamma+\mu)(R_0(\gamma+\mu) + (\pi+\sigma+\mu)) > \beta\omega, \\ &\Rightarrow \frac{1}{\sigma+\mu+\pi}R_0(R_0(\gamma+\mu) + (\pi+\sigma+\mu)) > \frac{\beta\omega}{(\gamma+\mu)}\frac{1}{\sigma+\mu+\pi}, \\ &\Rightarrow \frac{R_0^2(\gamma+\mu)}{\pi+\sigma+\mu} + R_0 > R_0, \\ &\Rightarrow \frac{R_0^2(\gamma+\mu)}{\pi+\sigma+\mu} > 0, \\ &\Rightarrow R_0(\gamma+\mu) > 0, \\ &\Rightarrow R_0 > 0. \end{aligned}$$

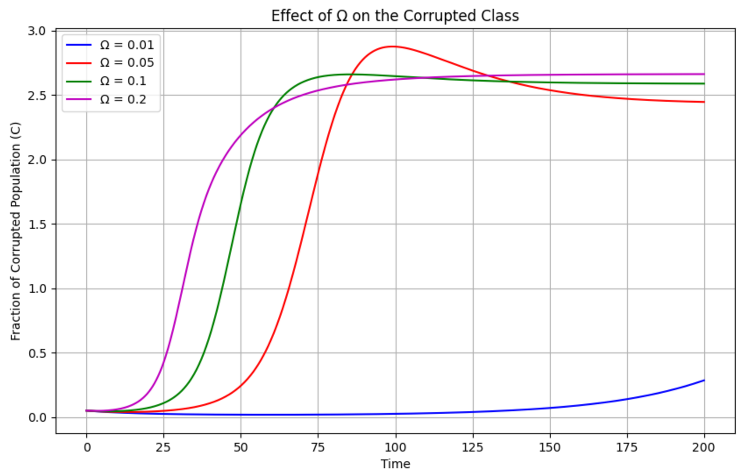

Figure 2. Time evolution of the corrupted fraction $C(t)/N$ for different relapse rates $\Omega(0.01, 0.05, 0.10, 0.20)$. Simulations were conducted over a 100-month period using the parameter values in Table 2, with initial conditions $S(0) = 0.90N, C(0) = 0.05N, A(0) = 0.05N$, and $N = \Omega/\mu$. The curves show that higher Ω values lead to faster growth and higher equilibrium levels of the corrupted class. Intermediate Ω values (0.05 and 0.10) exhibit transient peaks before stabilizing, indicating short-term surges in risky behavior that may undermine awareness interventions.

Similarly,

$$
\begin{aligned}
a_2 &= \frac{(\pi R_0^2\gamma + \pi R_0^2\mu + R_0^2\gamma\mu + R_0^2\gamma\sigma + R_0^2\mu^2 + R_0^2\mu\sigma - \beta\omega)}{R_0}, \\
&= \frac{(R_0(\pi R_0\gamma + \pi R_0\mu + R_0\gamma\mu + R_0\gamma\sigma + R_0\mu^2 + R_0\mu\sigma) - \beta\omega)}{R_0}, \\
&= \frac{(\pi R_0^2 + \pi R_0^2\mu + R_0^2\gamma\mu + R_0^2\gamma\sigma + R_0^2\mu^2 + R_0^2\mu\sigma - \beta\omega)}{R_0}, \\
&= \frac{(\pi\gamma + \pi\mu + \gamma\mu + \gamma\sigma + \mu^2 + \mu\sigma)R_0 - \beta\omega}{R_0}, \\
&= \frac{(\pi(\gamma+\mu) + \sigma(\gamma+\mu) + \mu(\gamma+\mu))R_0 - \beta\omega}{R_0}, \\
&= \frac{(\pi\gamma + \pi\mu + \gamma\mu + \gamma\sigma + \mu^2 + \mu\sigma)R_0 - \beta\omega}{R_0}, \\
&= R_0(\gamma+\mu)(\pi+\sigma+\mu) - \frac{\beta\omega}{R_0} > 0, \\
&= \frac{(R_0^2(\gamma+\mu)(\pi+\sigma+\mu) - \beta\omega)}{R_0} > 0, \\
&= R_0(\gamma+\mu)(\pi+\sigma+\mu) - \beta\omega > 0, \\
&= R_0(\gamma+\mu)(\pi+\sigma+\mu) > \beta\omega \\
&\Rightarrow R_0 > \frac{\beta\omega}{(\gamma+\mu)(\pi+\sigma+\mu)}, \\
&\Rightarrow R_0 > 0
\end{aligned}
$$

We have shown that $a_1, a_2 > 0$. Hence, by Routh-Hurwitz criterion, the characteristic polynomial has negative real parts provided that $R_0 > 1$.Therefore, $\mathcal{E}^*$ is locally asymptotically stable if $R_0 > 1$ and unstable if $R_0 < 1$. □

4. Numerical Results

To illustrate the theoretical results, we conducted numerical simulations using specific model parameter values. The descriptions and sources of these parameters are detailed in Table 2. When exact parameter values were unavailable in the literature, we assigned realistic estimates for the purpose of illustration.

Table 2. Parameter Values

Parameter	Description	Reference
ω	0.09	PSA [25]
μ	0.01	PSA [2]
β	0.02	Assumed
γ	0.05	Assumed
σ	0.03	Assumed
π	0.02	Assumed

4.1. Numerical simulations

We examined the impact of Ω, the rate at which aware teens revert to the corrupted class C. To analyze this effect, we considered various values of Ω specifically 0.01, 0.05, 0.1, and 0.2. The results of these simulations are illustrated in Figure 2, which highlights the influence of Ω on the dynamics of the corrupted class.

Figure 2 illustrates the time evolution of the corrupted fraction $C(t)/N$ for various relapse rates $\Omega(0.01, 0.05, 0.10, 0.20)$. At $\Omega = 0.01$, the corrupted class rises slowly and stabilizes at approximately 8% of the total population. Increasing Ω to 0.05 nearly doubles the equilibrium level to about 5%, with a transient peak at 17% before settling. At $\Omega = 0.10$, long-term level approaches 21%, while at $\Omega = 0.20$, it exceeds 25%, with a faster initial rise and no noticeable decline.

The presence of transient peaks in intermediate values of Ω(e.g., 0.05 and 0.10) suggests a short-term surge in risky behavior following relapse, possibly reflecting lag effects in awareness reinforcement programs. From an intervention perspective, these results indicate that even modest increases in relapse rates can significantly erode the long-term benefits of awareness campaigns. Sustained reductions in Ω-through continuous education, peer mentorship, and countering misinformation - are therefore crucial. Short-term intensification of interventions may also be warranted immediately after awareness programs to prevent early spikes in risky behavior.

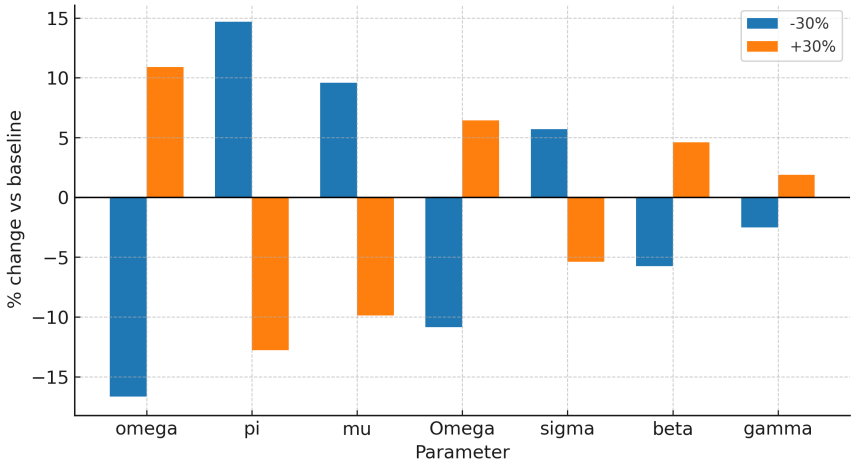

Figure 3. Local one-at-a-time sensitivity analysis results for the SIT-Ω model using baseline parameters from Table 2. Each bar represents the percent change in the long-run corrupted fraction C/N resulting from a $\pm 30\%$ perturbation of the parameter while keeping others fixed. Positive values indicate an increase in C/N, while negative values indicate a decrease. The most influential parameters were ω (recruitment rate), π (voluntary cessation rate), and μ (natural death rate).

4.2. Local sensitivity analysis

We performed a one-at-a-time (OAT) sensitivity analysis by varying each parameter (β, γ, σ, π, μ, Ω, ω) by $\pm 10\%$, $\pm 20\%$, and $\pm 30\%$ around their baseline values (Table 2), holding the others fixed. For each perturbation, the model was integrated to $t = 200$ and the long-run fraction of corrupted individuals C/N (mean over the last 10% of the simulation) was recorded. As shown in Figure 3, the parameters ω (recruitment rate), π (voluntary cessation rate), and μ (natural death rate) exert the greatest influence on the long-run C/N. Increasing ω increases C/N, while increasing π or μ decreases C/N. These findings suggest that interventions which reduce recruitment into risky behavior can substantially lower teenage pregnancy prevalence.

5. Conclusion

This study developed an SIT-based mathematical model incorporating a behavioral relapse pathway (Ω), capturing the tendency of informed adolescents to revert to risky behavior. Analytical results established stability conditions for both corruption-free and corruption-persistence equilibria, while numerical simulations demonstrated that higher relapse rates lead to substantial increases in the long-run prevalence of risky sexual behavior. Notably, even modest relapse rates produced short-term peaks of up to 22%, suggesting that awareness programs, when not reinforced, may only temporarily suppress risky behavior.

The local sensitivity analysis further revealed that the recruitment rate (ω), voluntary cessation rate (π), and natural death rate (μ) exert the greatest influence on long-term outcomes. This implies that reducing recruitment into risky behavior and enhancing voluntary or natural exit mechanisms are critical levers for mitigating teenage pregnancy prevalence.

Future research could extend this work in several directions. First, incorporating sociocultural heterogeneity (e.g., peer influence, family background, or regional disparities) would improve the model's realism. Second, integrating stochastic elements could capture the variability in adolescent decision-making and relapse rates. Third, global sensitivity analyses should be conducted once empirical parameter distributions are available, providing a more comprehensive assessment of parameter uncertainty. Finally, empirical validation of Ω using survey or longitudinal data would strengthen the practical applicability of the model and guide evidence-based interventions.

Author Contributions. **Apdo, R. B.**: Conceptualization, Methodology, Writing-original draft preparation, Writing-review and editing, Formal analysis. **Apdo, R. N.**: Software, Validation, Formal Analysis. **Gumombal, I. T.**: Conceptualization, Methodology, Formal analysis.

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Data availability. The data that support the findings of this study are available from the corresponding author upon reasonable request.

Abbreviations.

EOA	:	Example of Abbbreviation
AFW	:	Abbreviation of Words
ASO	:	and so on ...

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