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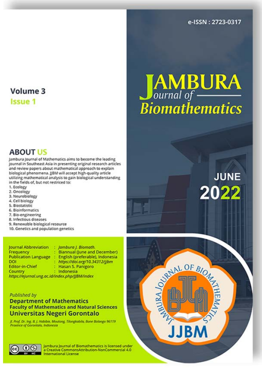

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Research Article

Analysis and Control of Chaotic Behaviour in a Plankton-Fish Interaction System with Fear and Refuge

Amit Sharma[1,*] **and Rajinder pal Kaur**[2]

[1] *School of Computer Science and Engineering, Lovely Professional University, Punjab-144811, India*
[2] *PG Department of Mathematics, Khalsa College, Punjab, India*

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ABSTRACT. *Controlling chaos in plankton-fish dynamics has been predominantly remained a rationale of many ecologists for managing and preserving ecosystem. In this paper, we have introduced a mathematical model consisting of phytoplankton, zooplankton, and fish population with a motive to study the simultaneous impact of prey refuge and fear. We have determined the existence of all feasible biological equilibria and proposed certain conditions of local stability of the given system around it. The Hopf-bifurcation analysis is carried out by considering phytoplankton refuge (n_1), zooplankton refuge (n_2), and fear effect (L) as significant bifurcation parameters. It is seen that fear of top predator mitigate unpredictable(chaotic) behavior of the plankton system and induce system stability for $L \geq 1.09$. Our investigations reveal that the defense mechanism developed by prey species due to the fear of predator population, namely n_1 and n_2 can also terminate chaos from the system. It is found that the given dynamical system remains stable in the intervals $n_1 \in [0.71, 0.73]$ and $n_2 \in [0.73, 0.75]$. We have applied feedback and non-feedback control mechanisms to stabilize the chaotic trajectories of the plankton-fish dynamics. All analytical findings are substantiated using numerical simulation.*

1. Introduction

Plankton-fish ecosystem is an important part of our biosphere and has potential influence on biological, social, and economical perspectives as well as on global climate. The sustainable life on earth depends critically on understanding the interactions among different aquatic species for their survival, coexistence, and extinction. These interactions can be better predicted through the prey-predator dynamical systems with direct and indirect approaches [1–5]. But, some experimental work presented in [6–10] demonstrated the significance of indirect approach over direct approach. It is noted from these studies that fear of predators in the prey population plays a very significant role in changing their behavior and physiology. Experimental study [11] shows that the predation rate of different species of crabs declines due to the fear of large carnivores and increases the density of prey population. [12] have analyzed plankton-fish dynamics and observed that the high fear effect in zooplankton species can affect the density of fish population but cannot induce its extinction. The fear of predator induces prey refuge, they hide physically to protect themselves from predation, as sea birds have nesting colonies on islands, and the semiaquatic animals, like mouse-deer, may use bodies of water as a refuge [13]. In the real world, predators are not always successful in catching and killing prey species due to their refuge. The effect of prey refuge has been extensively studied on various ecological systems [14–17], which are highly significant in ecology. The prey refuge is an important biological parameter for a pest, which increases its density and affects the predator population [18, 19]. In the aquatic ecosystem, both phytoplankton and zooplankton species can develop a defense mechanism to protect themselves from predation. In phytoplankton species, defense mechanisms are diverse and include physiological (toxicity, bioluminescence), morphological (silica shell, colony formation), and behavioral (escape response) traits [20]. Some mathematical studies [21, 22] claimed that benthic sediments and stratification of the water column can provide a temporal escape for phytoplankton from the predation of zooplankton. [14] have studied that both phytoplankton refuge and toxin significantly affect the occurrence and termination of harmful algal blooms. [15] has demonstrated that the zooplankton refuge reduces fish induced mortality and phytoplankton growth in lakes. Recently, Kaur et al. [12] have shown that zooplankton refuge can increase their biomass and induces extinction of its predators. The marine species are imbedded in periodically varying environments and so highly affected by seasons and noise. Many studies addressed the impact of seasonality, periodic fluctuations, and noise on marine ecosystem [23–26]. The chaotic behavior in aquatic ecosystems may result in unpredictable situations like the occurrence and termination of planktonic blooms, extinction of marine species, or any unexpected movement in oceans. Thus, it is necessary to study the chaos phenomenon and its control for sustainable life in oceans and ultimately in our biosphere. Various food chain ecological models [27–29] have been proposed and analyzed to study the complexity of these systems and their possible control. [30] have in-

*Corresponding Author.

Email : *amit.daviet@gmail.com* (A. Sharma)
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troduced nonlinear closure terms in plankton dynamics with different functional responses for eliminating chaos. Various other techniques have been developed for the control and tracking of chaotic systems such as active control [31], feedback control [32], adaptive control [33], back stepping control [34], sliding mode control [35], bounded feedback method [36], and time-delayed feedback control [37], etc. In literature, many researchers have proposed different techniques for controlling chaos in the plankton ecosystem. But, to the best of our knowledge, it is the first attempt wherein different chaos control mechanisms (feedback and non-feedback) are proposed along with phytoplankton(n_1), and zooplankton refuge(n_2), and fear effect (L). In feedback control mechanisms, noise is included in the given model system to stabilize the chaotic dynamics whereas in non-feedback techniques, a seasonal force is applied to given chaotic plankton-fish system for eliminating complexity. The present work shows the significance of internal (L, n_1, and n_2) and external factors (seasonality, noise, fluctuations, and time delay) for stabilising the chaotic behavior of the given plankton-fish dynamical system. The organization of this research article is as follows: In Section Section 2, a mathematical model of a given plankton system is proposed with certain assumptions. The boundedness and positivity are discussed in Section Section 3, followed by stability analysis of equilibria in Section Section 4. The feedback and non-feedback control schemes are discussed in Section Section 5 and Section Section 6, respectively. In Section Section 7, parameter range of internal factors (prey refuge and fear) are estimated numerically, primarily to control chaos by stabilizing a desired unstable periodic orbit embedded in a chaotic attractor followed by discussion and conclusion in section Section 8.

2. The Mathematical Model

The proposed ecological model consisting of the biomass of phytoplankton $P(t)$, zooplankton $Z(t)$, and fish $F(t)$ species at time t is given as follows:

$$\begin{aligned}\frac{dP}{dt} &= a_1P(1-\frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z,\\ \frac{dZ}{dt} &= \frac{b_1b_2(1-n_1)P}{(1+LF)(g_1+(1-n_1)P)}Z - d_1Z - \frac{c_1(1-n_2)ZF}{(g_2+(1-n_2)Z)},\\ \frac{dF}{dt} &= \frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2F.\end{aligned} \tag{1}$$

where, the biological interpretations of all parameters are provided in Table 1 such that $P(0) > 0$, $Z(0) > 0$, $F(0) > 0$.

The following assumptions have been made in the proposed model:

- The phytoplankton species grow logistically where the zooplankton and fish species depend on the phytoplankton and zooplankton population, respectively, for their growth and predate them with Holling-II type functional response.
- The zooplankton species extinct due to a high increase in phytoplankton refuge and in absence of a phytoplankton population. The zooplankton population rises due to increases in phytoplankton population and decline in phytoplankton refuge.The natural mortality rate of zooplankton is denoted d_1.
- The growth rate of fish species depends upon the availability of zooplankton species and their refuge where these species naturally die with rate d_2.

Table 1. Biological Interpretation of Parameters

Parameter	Biological Interpretation
a_1	Intrinsic growth rate of phytoplankton.
K	Carrying capacity.
b_1	Maximal ingestion rate of zooplankton.
g_1	Half saturation constant.
b_2	growth of zooplankton species due to phytoplankton.
L	Level of Fear factor in zooplankton due to fish predation.
d_1	Natural death rate of zooplankton.
c_1	Maximal ingestion rate of fish.
n_1	Rate of phytoplankton refuge.
n_2	Rate of zooplankton refuge.
g_2	Half saturation constant.
c_2	growth of fish species due to zooplankton.
d_2	Mortality rate of fish.

3. Positivity and Boundedness

Theorem 1. *All the solutions of the plankton fish system (1) lie in the positive octant* $\sum = \{(P,Z,F) \in \mathbb{R}^3_+, P(0) > 0, Z(0) > 0, F(0) > 0\}$ *and its non negative solutions are uniformly bounded in* $\Lambda = \{(P,Z,F) \in \mathbb{R}^3_+, 0 < P(t) < K, 0 < Z(t) < \alpha_2, 0 < F(t) < \alpha_1\}$.

Proof. The equations of system (1) can be written as:

$$\begin{aligned}P(t) &= P(0).e^{\int_0^t (a_1(1-\frac{P(s)}{K}) - \frac{b_1(1-n_1)}{(g_1+(1-n_1)P(s))}Z(s))ds},\\ Z(t) &= Z(0)e^{\int_0^t (\frac{b_2}{(1+LF(s))}\frac{b_1(1-n_1)P(s)}{(g_1+(1-n_1)P(s))} - d_1 - \frac{c_1(1-n_2)F(s)}{(g_2+(1-n_2)Z(s))})ds},\\ F(t) &= F(0).e^{\int_0^t (\frac{c_1c_2(1-n_2)Z(s)}{(g_2+(1-n_2)Z(s))} - d_2)ds}.\end{aligned}$$

It can be easily checked that, $P(t) > 0$, $Z(t) > 0$, and $F(t) > 0$ whenever $P(0) > 0$, $Z(0) > 0$, and $F(0) > 0$. Therefore, all the solutions of the plankton fish system (1) lie in the positive octant $\sum$. Further, we claim that all non negative solutions are uniformly bounded in the octant

$$\Lambda = \{(P,Z,F) \in \mathbb{R}^3_+, 0 < P(t) < K, 0 < Z(t) < \alpha_2,\\ 0 < F(t) < \alpha_1\}.$$

Now,

$$\frac{dP}{dt} \leq a_1P(t)(1-\frac{P(t)}{K})$$

it implies $0 < P \leq K$.

$$\frac{dF}{dt} \leq -(d_2 - c_1c_2)F(t)(\because \frac{(1-n_2)Z(t)}{(g_2+(1-n_2)Z(t))} < 1)$$

implies $F(t) \leq \alpha_1(e)^{-(d_2-c_1c_2)t}$. As $t \to \infty$, we have, $F(t) \leq \alpha_1$ whenever $d_2 > c_1c_2$. Further

$$\frac{dZ}{dt} \leq b_2b_1\frac{1}{(1+LF(t))}\frac{(1-n_1)P(t)}{(g_1+(1-n_1)P(t))}Z(t) - d_1Z(t),$$

implies $\frac{dZ}{dt} \leq b_2b_1(1)(1)Z(t) - d_1Z(t)$ $\Big(\because 0 < F(t) < \alpha_1$ implies $1 < 1 + LF(t) < 1 + L\alpha_1$, $1 > \frac{1}{(1+LF(t))} > \frac{1}{1+L\alpha_1}$ and $\frac{(1-n_1)P(t)}{(g_1+(1-n_1)P(t))} < 1\Big)$.

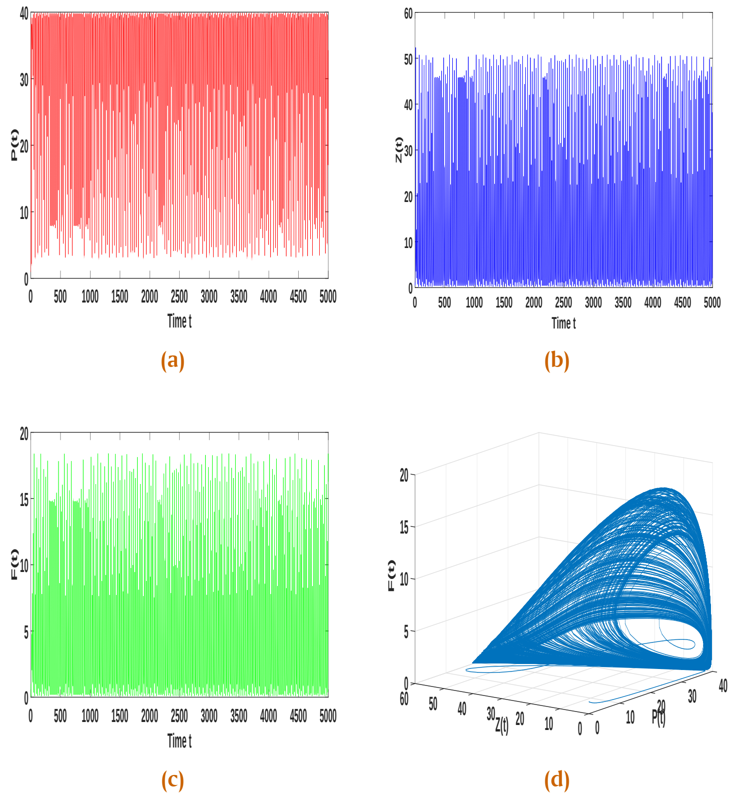


Figure 1. Occurrence of chaos using $[\ell_3]$

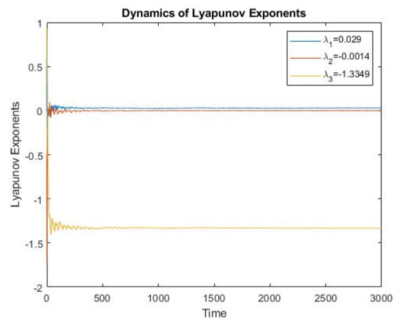


Figure 2. Lyapunov spectrum of system (1) using $[\ell_3]$

So, $\frac{dZ}{dt} \leq -(d_1 - b_1b_2)Z(t)$ implies $Z(t) \leq \alpha_2(e)^{-(d_1-b_1b_2)(t)}$ and as $t \to \infty$, we have, $Z(t) \leq \alpha_2$ whenever $d_1 > b_1b_2$.

Therefore, all solutions of the given plankton system lie in the octant, $\Lambda = \{(P, Z, F) \in \mathbb{R}^3_+, 0 < P(t) < K, 0 < Z(t) < \alpha_2, 0 < F(t) < \alpha_1\}$. □

4. Existence of equilibrium states

Lemma 1.
1. *The zero equilibrium $U_0(0, 0, 0)$ always exists.*
2. *The predator free equilibrium $U_1(K, 0, 0)$ always exists. Numerically, we obtain $U_1(40, 0, 0)$ for the set of parameters $[\ell_1]$:$a_1 = 2$, $n_1 = 0.8$, $b_1 = 1.0204$, $g_1 = 10$, $b_2 = 1.96$, $d_1 = 0.9$, $c_1 = 1.5$, $g_2 = 10$, $c_2 = 1.3333$, $d_2 = 0.7$, $K = 40$, $n_2 = 0.15$, $L = 0.05$.*

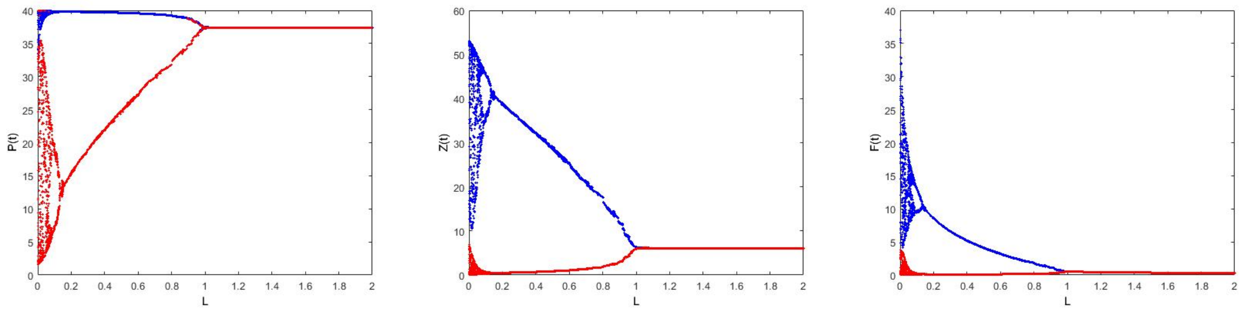
Lemma 2. *The fish free steady state*

$$U_2(\tilde{P}, \tilde{Z}, 0),$$
$$\tilde{P} = \frac{d_1g_1}{(b_1b_2 - d_1)(1 - n_1)},$$
$$\tilde{Z} = \frac{a_1(K(b_1b_2 - d_1)(1 - n_1) - d_1g_1)(d_1g_1 + g_1(b_1b_2 - d_1))}{b_1(1 - n_1)^2(b_1b_2 - d_1)^2}.$$

exists if $b_1b_2 > d_1$, $n_1 < 1$, and $K(b_1b_2 - d_1)(1 - n_1) > d_1g_1$. Numerically, taking $n_1 = 0.75$ in $[\ell_1]$, we obtain $U_2(38.4553, 5.7969, 0)$ and the conditions of existence i.e. $b_1b_2 > d_1$ $(2 > 0.9)$, $n_1 < 1 (0.75 < 1)$, and $K(b_1b_2 - d_1)(1 - n_1) > d_1g_1$ $(10.998 > 9)$ are clearly satisfied.

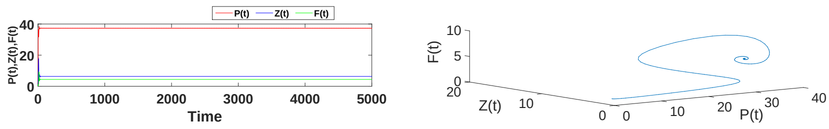
Theorem 2. *The positive interior equilibrium point $U_*(P_*, Z_*, F_*)$ exists under the following conditions*
1. *$n_2 < 1$ and $c_1c_2 > d_2$,*
2. *$0 < n_1 < 1 - \frac{g_1a_1}{b_1Z_*}$,*
3. *$d_1(g_1 + (1 - n_1)P_*) < b_1b_2P_*$.*

Proof. From 3rd equation of system (1), we obtain $Z_* = \frac{d_2g_2}{(1-n_2)(c_1c_2-d_2)}$, which exists if $n_2 < 1$, and $c_1c_2 >$



(a) (b) (c)

Figure 3. Bifurcation diagram with respect to fear factor L, where the blue lines and red lines represent local maxima and local minima, respectively



(a) (b)

Figure 4. Stable attractor of controlled dynamical system (4) at $h_1 = 0$, $h_2 = 0.5$ and $h_3 = 0$ (using $[\ell_3]$)

d_2. We determine a quadratic equation

$$I(P) = (1-n_1)P^2 + (g_1 - K(1-n_1))P - Kg_1 + \frac{b_1K(1-n_1)Z_*}{a_1}$$

from 1st equation of system (1), which has positive root P_* if $I(0) < 0$ i.e. $0 < n_1 < 1 - \frac{g_1a_1}{b_1Z_*}$.

From 2nd equation of dynamical system (1), we can find, $l(F) = F^2 + N_1F + N_2$, where

$$N_1 = \frac{c_1(1-n_2) + Ld_1(g_2 + (1-n_2)Z_*)}{c_1(1-n_2)L},$$
$$N_2 = \frac{(d_1(g_1 + (1-n_1)P_*) - b_1b_2P_*)(g_2 + (1-n_2)Z_*)}{c_1(1-n_2)(g_1 + (1-n_1)P_*)L}.$$

The equation $l(F)$ admits a $+ve$ zero say F_* if $l(0) = N_2 < 0$ i.e. $d_1(g_1 + (1-n_1)P_*) < b_1b_2P_*$. Numerically, for the set of hypothetical parameters $[\ell_2]$: $a_1 = 2, n_1 = 0.8, b_1 = 1, g_1 = 10, b_2 = 2, d_1 = 1, c_1 = 1.5, g_2 = 10, c_2 = 1.3333, d_2 = 0.7, K = 40, n_2 = 0.12, L = 0.1$, we obtain $U_*(38.0648, 4.5769, 0.5341)$. □

4.1. Stability of Equilibrium States

The main motive of this subsection is to determine the local stability of all feasible steady states under certain conditions.

Lemma 3. *The zero equilibrium state U_0 is unstable as a_1 is the positive eigenvalue of the corresponding variational matrix and the predator free steady state U_1 is locally asymptotically stable (LAS) if $0 < n_1 < 1 - \frac{b_1b_2K - d_1g_1}{d_1K}$.*

Theorem 3. *The fish free feasible point U_2 is LAS if $(\hat{H}_1)$: $\hat{A}_1\hat{A}_2 - \hat{A}_3 > 0$ and $\hat{A}_i > 0 \;\forall\; i = 1, 3$ holds true, where*

$$\hat{A}_1 = -(e_{100} + f_{010} + g_{001}),$$
$$\hat{A}_2 = -e_{010}f_{100} + f_{010}g_{001} + e_{100}f_{010} + e_{100}g_{001},$$
$$\hat{A}_3 = e_{010}f_{100}g_{001} - e_{100}f_{010}g_{001}.$$

Proof. The variational matrix $\hat{M}_*$ of the positive interior equilibrium U_2 is given by,

$$\hat{M}_* = \begin{bmatrix} e_{100} & e_{010} & 0 \\ f_{100} & f_{010} & f_{001} \\ 0 & 0 & g_{001} \end{bmatrix}$$

where

$$e_{100} = a_1 - \frac{2a_1P_2}{K} - \frac{g_1b_1(1-n_1)Z_2}{(g_1 + (1-n_1)P_2)^2},$$
$$e_{010} = -\frac{b_1(1-n_1)P_2}{(g_1 + (1-n_1)P_2)},$$
$$f_{100} = \frac{b_1b_2g_1(1-n_1)Z_2}{(g_1 + (1-n_1)P_2)^2},$$
$$f_{010} = \frac{b_1b_2(1-n_1)P_2}{(g_1 + (1-n_1)P_2)} - d_1,$$
$$f_{001} = -\frac{c_1(1-n_2)Z_2}{(g_2 + (1-n_2)Z_2)} - \frac{b_1b_2LP_2Z_2}{(g_1 + (1-n_1)P_2)},$$
$$g_{001} = \frac{(c_1c_2(1-n_2)Z_2)}{(g_2 + (1-n_2)Z_2)} - d_2.$$

The characteristic equation of the variational matrix $\hat{M}_*$ w.r.t. U_2 is

$$\lambda^3 + \hat{A}_1\lambda^2 + \hat{A}_2\lambda + \hat{A}_3 = 0, \quad (2)$$

where

$$\begin{aligned}\hat{A}_1 &= -(e_{100} + f_{010} + g_{001}),\\ \hat{A}_2 &= -e_{010}f_{100} + f_{010}g_{001} + e_{100}f_{010} + e_{100}g_{001},\\ \hat{A}_3 &= e_{010}f_{100}g_{001} - e_{100}f_{010}g_{001}.\end{aligned}$$

Further, using Routh-Hurwitz criterion, $\hat{M}_*$ has negative eigenvalues or eigenvalues with negative real parts if $(\hat{H}_1)$ holds true around U_2. □

Theorem 4. *The equilibrium point U_* is LAS if (H_1): $A_1A_2 - A_3 > 0$ and $A_i > 0\ \forall\ i = 1, 3$ holds true, where*

$$\begin{aligned}A_1 &= -(a_{100} + b_{010} + c_{001}),\\ A_2 &= -a_{010}b_{100} - c_{010}b_{001} + b_{010}c_{001} + a_{100}b_{010}\\ &\quad + a_{100}c_{001},\\ A_3 &= a_{100}c_{010}b_{001} + a_{010}b_{100}c_{001} - a_{100}b_{010}c_{001}.\end{aligned}$$

Proof. The variational matrix M_* of the positive interior equilibrium U_* is given by

$$M_* = \begin{bmatrix} a_{100} & a_{010} & 0\\ b_{100} & b_{010} & b_{001}\\ 0 & c_{010} & c_{001}\end{bmatrix},$$

where

$$\begin{aligned}a_{100} &= a_1 - \frac{2a_1P_*}{K} - \frac{g_1b_1(1-n_1)Z_*}{(g_1+(1-n_1)P_*)^2},\\ a_{010} &= -\frac{b_1(1-n_1)P_*}{(g_1+(1-n_1)P_*)},\\ b_{100} &= \frac{b_1b_2g_1(1-n_1)Z_*}{(1+LF_*)(g_1+(1-n_1)P_*)^2},\\ b_{010} &= \frac{b_1b_2(1-n_1)P_*}{(1+LF_*)(g_1+(1-n_1)P_*)} - d_1 - \frac{c_1g_2(1-n_2)F_*}{(g_2+(1-n_2)Z_*)^2},\\ b_{001} &= -\frac{c_1(1-n_2)Z_*}{(g_2+(1-n_2)Z_*)} - \frac{b_1b_2LP_*Z_*}{(1+LF_*)^2(g_1+(1-n_1)P_*)},\\ c_{010} &= \frac{(c_1c_2(1-n_2)g_2F_*)}{(g_2+(1-n_2)Z_*)^2},\\ c_{001} &= \frac{(c_1c_2(1-n_2)Z_*)}{(g_2+(1-n_2)Z_*)} - d_2.\end{aligned}$$

The characteristic equation of the variational matrix M_* w.r.t. U_* is

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0, \quad (3)$$

where

$$\begin{aligned}A_1 &= -(a_{100} + b_{010} + c_{001}),\\ A_2 &= -a_{010}b_{100} - c_{010}b_{001} + b_{010}c_{001} + a_{100}b_{010} + a_{100}c_{001},\\ A_3 &= a_{100}c_{010}b_{001} + a_{010}b_{100}c_{001} - a_{100}b_{010}c_{001}.\end{aligned}$$

Now, using Routh-Hurwitz criterion, M_* has negative eigenvalues or eigenvalues with negative real parts if (H_1) holds true around U_*. □

4.2. *Existence of chaotic behavior*

Consider the following set of parameters. $[\ell_3]$: $a_1 = 2$, $n_1 = 0.02$, $b_1 = 1.0204$, $g_1 = 10$, $b_2 = 1.96$, $d_1 = 1$, $c_1 = 1.5$, $g_2 = 10$, $c_2 = 1.3333$, $d_2 = 0.7$, $K = 40$, $n_2 = 0.12$, $L = 0.01$. Taking parameters as in $[\ell_3]$, the given dynamical system has chaotic behavior as shown in Figure 1. Lyapunov Exponents, one of the effective tools to explore chaotic dynamics, is shown in Figure 2. From the Lyapunov spectrum of the system, we observe that one of the Lyapunov exponents is positive that established the existence of chaotic dynamics. Bifurcation diagram with respect to the level of fear factor L is shown in Figure 3.

5. Role of noise in plankton dynamics

Number of analytical and experimental studies have been carried out to explore the impact of noise in prey-predator systems [16, 17, 38, 39]. The role of noise in the marine kingdom is also unavoidable as the aquatic ecosystems are highly affected by it. The noise-induced phenomenon known as stochastic resonance (SR) is detected when paddle fish search their prey [40]. In marine ecology, the impact of noise on aquatic ecosystems is extensively analyzed, but its role on plankton ecosystems through different feedback mechanisms is less known [24, 25]. Thus, in the present section, we study the impact of the noise on the plankton dynamics using feedback techniques (linear and bounded).

5.1. *Linear feedback control*

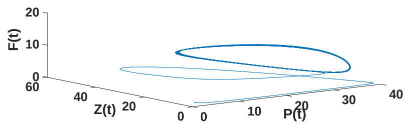


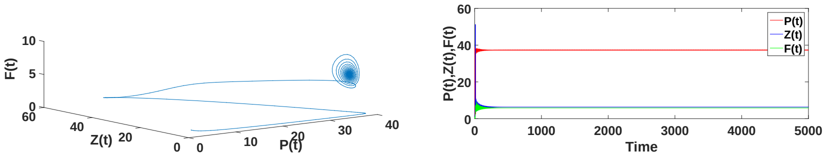
Figure 5. Stable limit cycle of controlled dynamical system (4) at $h_1 = 0$, $h_2 = 0$ and $h_3 = 0.5$ (using $[\ell_3]$)

In this subsection, we have introduced noise in terms of control parameters v_1, v_2, v_3 in the given plankton system (1). The main objective of this subsection is to guide the chaotic trajectories to the equilibrium point of the system. In the feedback control approach, the structure of equations of the controlled dynamics is given by

$$\begin{aligned}\frac{dP}{dt} &= a_1P(1-\frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z + v_1,\\ \frac{dZ}{dt} &= \frac{b_2}{(1+LF)}\frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z - d_1Z + v_2\\ &\quad - \frac{c_1(1-n_2)ZF}{(g_2+(1-n_2)Z)},\\ \frac{dF}{dt} &= \frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2F + v_3.\end{aligned} \quad (4)$$

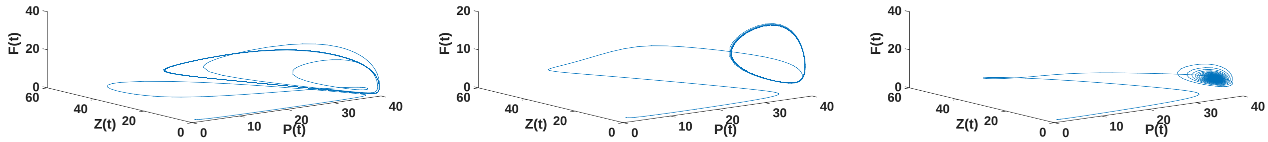
Let $U_*(P_*, Z_*, F_*)$ be the unstable equilibrium of the unpredictable chaotic dynamics (4). A simple feedback controller is required for real world application, which is given below as:

$$\begin{pmatrix} v_1\\ v_2\\ v_3\end{pmatrix} = \begin{pmatrix} -h_1 & 0 & 0\\ 0 & -h_2 & 0\\ 0 & 0 & -h_3\end{pmatrix}\begin{pmatrix} P-P_*\\ Z-Z_*\\ F-F_*\end{pmatrix}$$



(a) (b)

Figure 6. Stable behavior of controlled dynamical system (8) at $L = 0.4$ (using $[\ell_3]$)



(a) quasi periodic behavior at $J_1 = 0.2$, $J_2 = 0.1$, $\alpha = 22/7$

(b) Stable limit cycle at $J_1 = 0.9$, $J_2 = 0.1$, $\alpha = 22/7$

(c) converges at $J_1 = 1.4$, $J_2 = 2.5$, $\alpha = 22/7$

Figure 7. Dynamics of controlled dynamical system (10) (using $[\ell_3]$).

Now, we choose the positive constants in such a way that the orbits of the controlled system become stable at a feasible state U_*. Substituting the values of control inputs v_1, v_2, v_3 in (4), we get

$$\begin{aligned}
\frac{dP}{dt} &= a_1 P(1 - \frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1 + (1-n_1)P)} Z - h_1(P - P_*),\\
\frac{dZ}{dt} &= \frac{b_2}{(1+LF)} \frac{b_1(1-n_1)P}{(g_1 + (1-n_1)P)} Z - d_1 Z - h_2(Z - Z_*)\\
&\quad - \frac{c_1(1-n_2)ZF}{(g_2 + (1-n_2)Z)},\\
\frac{dF}{dt} &= \frac{c_1 c_2 (1-n_2)ZF}{(g_2 + (1-n_2)Z)} - d_2 F - h_3(F - F_*).
\end{aligned} \tag{5}$$

Theorem 5. *The equilibrium point U_* is locally asymptotically stable if $(G_1): B_1 > 0,\ B_3 > 0,$ and $B_1 B_2 > B_3$ holds true. Here*

$$\begin{aligned}
B_1 &= -(a_{11} + a_{22} + a_{33}),\\
B_2 &= -a_{12}a_{21} - a_{32}a_{33} + a_{22}a_{33} + a_{11}a_{22} + a_{11}a_{33},\\
B_3 &= a_{11}a_{32}a_{23} + a_{12}a_{21}a_{33} - a_{11}a_{22}a_{33}.
\end{aligned}$$

Proof. To discuss the local stability of the controlled system (5) about U_*, we choose $h_1 = 0$, $h_2 > 0$ and $h_3 = 0$. The Jacobian matrix of the system (5) around U_* is given by

$$M_1^* = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & c_{32} & c_{33} \end{bmatrix},$$

where

$$\begin{aligned}
a_{11} &= a_1 - \frac{2a_1 P_*}{K} - \frac{g_1 b_1 (1-n_1) Z_*}{(g_1 + (1-n_1)P_*)^2} - h_1,\\
a_{12} &= -\frac{b_1(1-n_1)P_*}{(g_1 + (1-n_1)P_*)},\\
a_{21} &= \frac{b_1 b_2 g_1 (1-n_1) Z_*}{(1+LF_*)(g_1 + (1-n_1)P_*)^2},\\
a_{22} &= \frac{b_1 b_2 (1-n_1) P_*}{(1+LF_*)(g_1 + (1-n_1)P_*)} - \frac{c_1 g_2 (1-n_2) F_*}{(g_2 + (1-n_2)Z_*)^2}\\
&\quad - d_1 - h_2,\\
a_{23} &= -\frac{c_1(1-n_2)Z_*}{(g_2 + (1-n_2)Z_*)} - \frac{b_1 b_2 L P_* Z_*}{(1+LF_*)^2 (g_1 + (1-n_1)P_*)},\\
a_{32} &= \frac{(c_1 c_2 (1-n_2) g_2 F_*)}{(g_2 + (1-n_2)Z_*)^2},\\
a_{33} &= \frac{(c_1 c_2 (1-n_2) Z_*)}{(g_2 + (1-n_2)Z_*)} - d_2 - h_3.
\end{aligned}$$

The characteristic equation of the variational matrix M_1^* w.r.t. U_* is

$$\lambda^3 + B_1 \lambda^2 + B_2 \lambda + B_3 = 0, \tag{6}$$

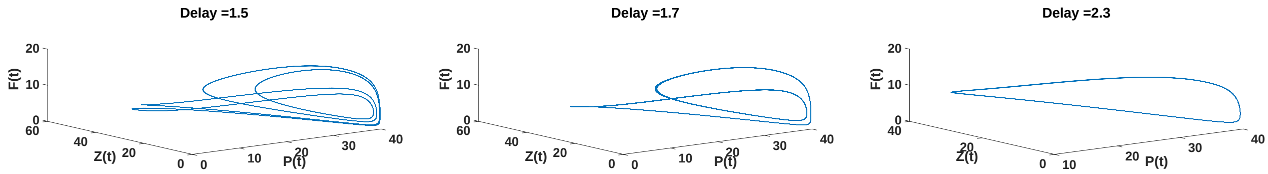
where

$$\begin{aligned}
B_1 &= -(a_{11} + a_{22} + a_{33}),\\
B_2 &= -a_{12}a_{21} - a_{32}a_{33} + a_{22}a_{33} + a_{11}a_{22} + a_{11}a_{33},\\
B_3 &= a_{11}a_{32}a_{23} + a_{12}a_{21}a_{33} - a_{11}a_{22}a_{33}.
\end{aligned}$$

Now, using Routh-Hurwitz criterion, (6) has negative eigenvalues or eigenvalues with negative real parts if (G_1) holds true around U_*, where $(G_1): B_1 > 0, B_3 > 0,$ and $B_1 B_2 > B_3$. The constants h_i, $i = 1, 2, 3$ are taken so as to satisfy condition (G_1). Such a choice of $h_i' s$ exclude the chaotic nature of the system and make it stable. □

5.2. Numerical validation

The chaotic system (1) and controlled dynamics (5) are simulated for the same choice of parameters $[\ell_3]$. Here, we choose the gain parameters as $h_1 = 0$, $h_2 = 0.5$, $h_3 = 0$. Figure 4



(a) Quasi-periodic trajectory at $\tau = 1.5$ (b) Double periodic at $\tau = 1.7$ (c) A stable limit cycle at $\tau = 2.3$

Figure 8. Dynamics of controlled dynamical systems (11) for different values of τ

shows the stable behavior of the system (4), and the trajectory converges to the same steady state $U_*(38.06489, 4.5769, 0.5341)$ of (1).

Remark 1. In the same way, if we again set constants as $h_1 = 0$, $h_2 = 0$, and $h_3 = 0.5$, the controlled dynamical system (5) shows stable limit cycle (Figure 5). Thus, the unpredictable and complex plankton dynamics (1) can be stabilize through the controller system (5) to equilibrium point or to a stable limit cycle.

5.3. Bounded feedback control scheme

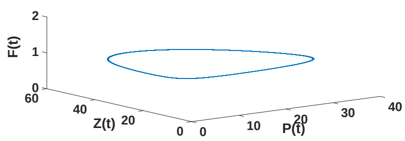


Figure 9. Stable limit cycles of controlled dynamical systems (12) at $\tau = 0.001$

In the previous subsection, we have perturbed the given system by including noise in phytoplankton, zooplankton, and fish dynamics to suppress chaos. Here, we will analyze the impact of noise on plankton-fish dynamics by applying noise only in zooplankton dynamics in a bounded region. We include the control parameter w(t) as noise in the given chaotic dynamics (1). To ensure small values of the noise perturbations, we restrict the controller w(t) in the following way,

$$w(t) = \begin{cases} w_0 & w_0 > w(t) \\ w(t) & -w_0 < w(t) < w_0 \\ -w_0 & w(t) < -w_0 \end{cases} \tag{7}$$

where w_0 is a small saturating positive constant. To stabilize the chaotic dynamics (1), we propose a control design of system (1) using bounded feedback control mechanism as,

$$\begin{aligned} \frac{dP}{dt} &= a_1 P(1 - \frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1 + (1-n_1)P)} Z \\ \frac{dZ}{dt} &= \frac{b_2}{(1+LF)} \frac{b_1(1-n_1)P}{(g_1 + (1-n_1)P)} Z - \frac{c_1(1-n_2)ZF}{(g_2 + (1-n_2)Z)} \\ &\quad - d_1 Z - w(t) \\ \frac{dF}{dt} &= \frac{c_1 c_2 (1-n_2) ZF}{(g_2 + (1-n_2)Z)} - d_2 F \end{aligned} \tag{8}$$

where $w(t) = L_1 * (\frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2 F)$.

Theorem 6. *The system (8) is locally asymptotically stable around its equilibrium point U_* if (G_{11}): $C_1 > 0$, $C_3 > 0$, $C_1C_2 > C_3$ holds true. Here*

$$\begin{aligned} C_1 &= -(b_{11} + b_{22} + b_{33}), \\ C_2 &= -b_{12}b_{21} - b_{32}b_{33} + b_{22}b_{33} + b_{11}b_{22} + b_{11}b_{33}, \\ C_3 &= b_{11}b_{32}b_{23} + b_{12}b_{21}b_{33} - b_{11}b_{22}b_{33}. \end{aligned}$$

Proof. The jacobian matrix M_2^* of the controlled system (8) is

$$M_2^* = \begin{bmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{bmatrix},$$

where

$$\begin{aligned} b_{11} &= a_1 - \frac{2a_1 P_*}{K} - \frac{g_1 b_1 (1-n_1) Z_*}{(g_1 + (1-n_1)P_*)^2}, \\ b_{12} &= -\frac{b_1(1-n_1)P_*}{(g_1 + (1-n_1)P_*)}, \\ b_{21} &= \frac{b_1 b_2 g_1 (1-n_1) Z_*}{(1+LF_*)(g_1 + (1-n_1)P_*)^2}, \\ b_{22} &= \frac{b_1 b_2 (1-n_1) P_*}{(1+LF_*)(g_1 + (1-n_1)P_*)} - \frac{c_1 g_2 (1-n_2) F_*}{(g_2 + (1-n_2)Z_*)^2} \\ &\quad - d_1 - L_1 (\frac{(c_1 c_2 (1-n_2) g_2 F_*)}{(g_2 + (1-n_2)Z_*)^2}), \\ b_{23} &= -\frac{c_1(1-n_2)Z_*}{(g_2 + (1-n_2)Z_*)} - \frac{b_1 b_2 L P_* Z_*}{(1+LF_*)^2 (g_1 + (1-n_1)P_*)} \\ &\quad - L_1 (\frac{(c_1 c_2 (1-n_2) Z_*)}{(g_2 + (1-n_2)Z_*)} - d_2), \\ b_{32} &= \frac{(c_1 c_2 (1-n_2) g_2 F_*)}{(g_2 + (1-n_2)Z_*)^2}, \\ b_{33} &= \frac{(c_1 c_2 (1-n_2) Z_*)}{(g_2 + (1-n_2)Z_*)} - d_2. \end{aligned}$$

The characteristic equation of the variational matrix M_2^* w.r.t. U_* is

$$\lambda^3 + C_1 \lambda^2 + C_2 \lambda + C_3 = 0, \tag{9}$$

where,

$$C_1 = -(b_{11} + b_{22} + b_{33}),$$

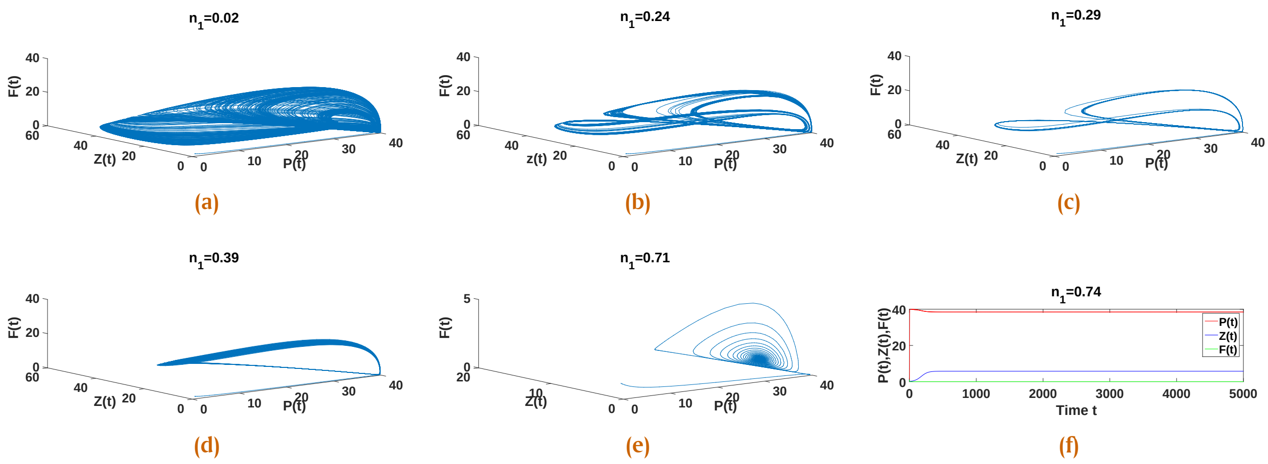


Figure 10. Controlling of chaos w.r.t. phytoplankton refuge n_1

$$C_2 = -b_{12}b_{21} - b_{32}b_{33} + b_{22}b_{33} + b_{11}b_{22} + b_{11}b_{33},$$
$$C_3 = b_{11}b_{32}b_{23} + b_{12}b_{21}b_{33} - b_{11}b_{22}b_{33}.$$

Now, using the Routh-Hurwitz criterion, M_2^* has negative eigenvalues or eigenvalues with negative real parts if (G_{11}) holds around U_*. Here (G_{11}): $C_1 > 0$, $C_3 > 0$, and $C_1C_2 > C_3$. It is observed that the chaotic system (1) can be controlled through bounded feedback scheme (8) using set of parameters $[\ell_3]$, $L_1 = 0.4$, and $w_0 = 1.4$. Figure 6 shows that the controlled system converges to $U_*(37.2847, 6.3346, 5.8353)$. □

6. Stability analysis of seasonally perturbed system

Aquatic species survive in the periodically varying environment and so severally affected by seasonal fluctuations [23, 41]. In this section, we include seasonal forces in the dynamical system (1), which exhibited unpredictable behavior through an infinite number of unstable periodic orbits embedded within the attractor (using $[\ell_3]$). To suppress the chaotic behavior of the dynamics (1), we apply two control mechanisms, namely, non-feedback control scheme and delayed feedback control scheme. These two schemes convert the chaotic trajectory of system (1) to a stable limit cycle.

6.1. *Non-feedback control scheme using sinusoidal force*

In [39] a plankton dynamical system is analysed by applying sinusoidal force on the growth rate of phytoplankton and death rate of zooplankton as a diagnostic tool to detect chaos. Here, we have included the sinusoidal force in zooplankton dynamics to stabilize the given chaotic system. Thus, in this mechanism, we have proposed a non-feedback controller g(t) using a weak seasonal periodic sinusoidal force in zooplankton population. Here $g(t) = J_1 + J_2 \sin(\alpha t)$, in which J_1 is a constant bias. The parameters J_2 and α are the amplitude and frequency of sinusoidal periodic force function, respectively. We insert this controller in the second equation of (1) and the following controlled system is obtained.

$$\frac{dP}{dt} = a_1P(1-\frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z,$$
$$\frac{dZ}{dt} = \frac{b_2}{(1+LF)}\frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z - \frac{c_1(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_1Z + g(t),$$
$$\frac{dF}{dt} = \frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2F. \quad (10)$$

We have carried out numerical simulation of the controlled system (10) using set of parameters $[O_1]$: $[\ell_3]$, $J_1 = 0.2$, $J_2 = 0.1$, and $\alpha = 22/7$. Figure 7 shows that for the set of parametric values $[O_1]$, the controlled system (10) converts the unpredictable trajectories of given model (1) to quasi periodic orbit and as J_1 increases from 0.2 to 0.9 in $[O_1]$, the chaotic attractors suppressed to stable limit cycle.

> **Remark 2.** It is notable here that when we take $J_1 = 1.4$ and $J_2 = 2.5$ in $[O_1]$, the system (10) becomes stable and converges to $U_*(37.2754, 6.3349, 8.1180)$ (Figure 7c).

6.2. *Delayed feedback control scheme*

In this control scheme, we do not require external forces and internal parameters to suppress chaos. In this mechanism, we will again control the chaos by perturbing the system in a more realistic way using delay. These perturbations are the difference between the current and delayed state of the plankton-fish system. We have applied this delayed feedback scheme to the given dynamical system (1) to eliminate its chaotic behavior and will stabilize the unstable periodic orbits of period τ. The set of differential equations of the controlled system is as given below:

$$\frac{dP}{dt} = a_1P(1-\frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z - J_3(P(t)-P(t-\tau)),$$
$$\frac{dZ}{dt} = \frac{b_2}{(1+LF)}\frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z - \frac{c_1(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_1Z - J_4(Z(t)-Z(t-\tau)),$$
$$\frac{dF}{dt} = \frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2F - J_5(F(t)-F(t-\tau)) \quad (11)$$

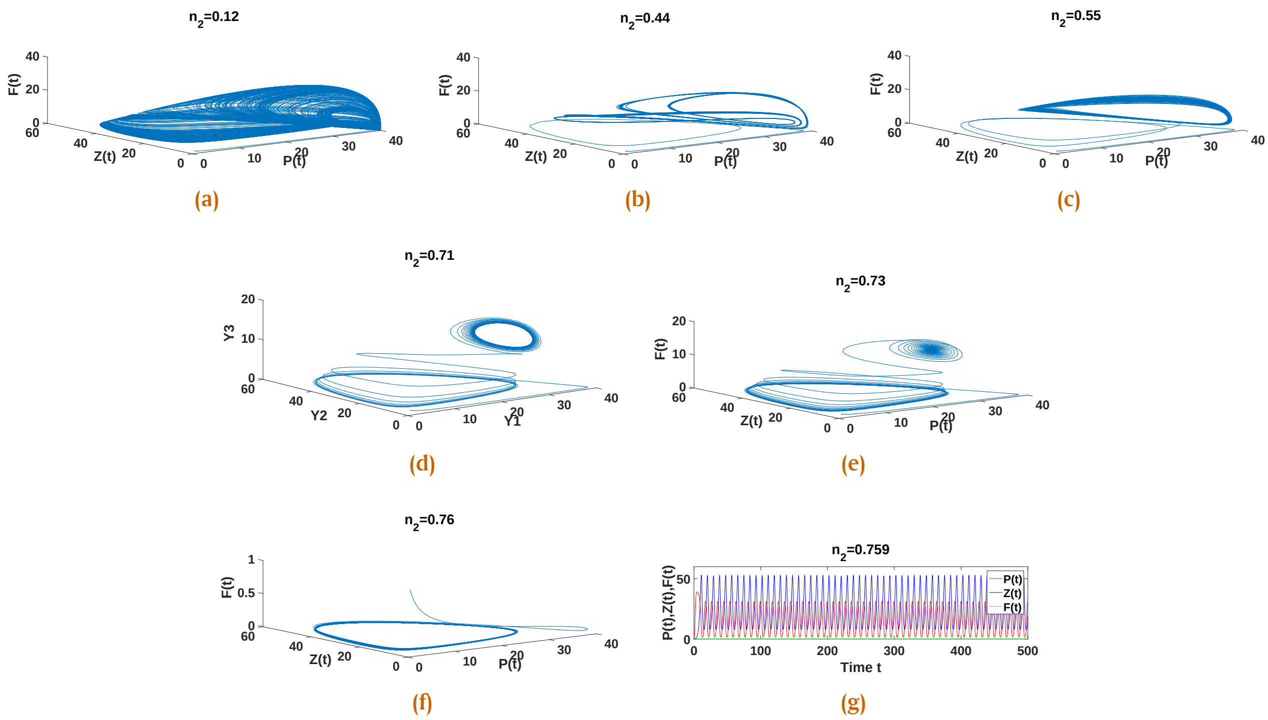


Figure 11. Controlling of chaos w.r.t. zooplankton refuge n_2

Here, diag(J_3, J_4, J_5) is the feedback gain matrix, and τ is taken to be the same as the period of the target unstable periodic orbits. The controller in the dynamics (11) tends to zero as trajectories converge to periodic orbit with a period of τ.

Firstly, we consider set of parameters $[O_2]$: $[\ell_3]$, $J_3 = 0.1$, $J_4 = 0.1$, $J_5 = 1$, and $\tau = 1.5$. Now, simulating the dynamical system (11) using set of parameters $[O_2]$, the unpredictable chaotic orbits controlled by quasi periodic orbits (Figure 8a), if we gradually increase time delay τ from 1.5 to 1.7 in $[O_2]$, the system converts into double periodic (Figure 8b) and becomes stable around a limit cycle for $\tau = 2.3$ in $[O_2]$ (Figure 8c).

6.3. *Approximated delay feedback scheme (ADFS)*

In this subsection, we apply the ADFS (for detailed explanation of this method reader can refer to [42]) to terminate the chaotic nature of the given system (1) and convert it into a stable limit cycle with a small period. The chaotic trajectory (P(t),Z(t),F(t)) is very close to periodic trajectory with period τ. Now, we apply Taylor's theorem for small τ and get,

$$
\begin{aligned}
P(t-\tau) &= P(t) - \tau\dot{P}(t) + O(\tau^2),\\
P(t) - P(t-\tau) &\cong \tau\dot{P}(t)\\
&= \tau(a_1P(1-\frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z)\\
Z(t) - Z(t-\tau) &\cong \tau\dot{Z}(t)\\
&= \tau(\frac{b_2}{(1+LF)}\frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z - d_1Z\\
&\quad - \frac{c_1(1-n_2)ZF}{(g_2+(1-n_2)Z)})\\
F(t) - F(t-\tau) &\cong \tau\dot{F}(t)\\
&= \tau(\frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2F) \qquad (12)
\end{aligned}
$$

From (11) and (12), we get

$$
\begin{aligned}
\dot{P}(t) &= (1-J_3\tau)(a_1P(1-\frac{P}{K}) - \frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z)\\
\dot{Z}(t) &= (1-J_4\tau)(\frac{b_2}{(1+LF)}\frac{b_1(1-n_1)P}{(g_1+(1-n_1)P)}Z - d_1Z\\
&\quad - \frac{c_1(1-n_2)ZF}{(g_2+(1-n_2)Z)})\\
\dot{F}(t) &= (1-J_5\tau)(\frac{c_1c_2(1-n_2)ZF}{(g_2+(1-n_2)Z)} - d_2F) \qquad (13)
\end{aligned}
$$

where $0 < J_3\tau < 1$, $0 < J_4\tau < 1$, and $0 < J_5\tau < 1$. For the set of parameters $[\ell_3]$, $J_3 = 0.01$, $J_4 = 0.01$, $J_5 = 1$, and $\tau = 0.001$, we integrate (13) numerically and observed that the chaotic orbit is controlled by stable limit cycle (Figure 9).

7. Controlling of chaos using internal parameters n_1, n_2, and L.

In this section, we will study the role of phytoplankton refuge (n_1), zooplankton refuge (n_2), and fear factor (L) in controlling the unpredictable behavior of the chaotic planktonic dynamical system (1).

- We simulate the given system using $[\ell_3]$ and determined that when $n_1 \in (0, 0.23)$ it shows chaotic behavior as the value of n_1 gradually increased to 0.24, we found that the system shows quasi-periodic trajectories and turns to double periodic at $n_1 = 0.29$. The dynamical system enters into Hopf-bifurcation at $n_1 = 0.39$, becomes stable for $n_1 \in [0.71, 0.73]$ but as n_1 crosses 0.73, the top predator

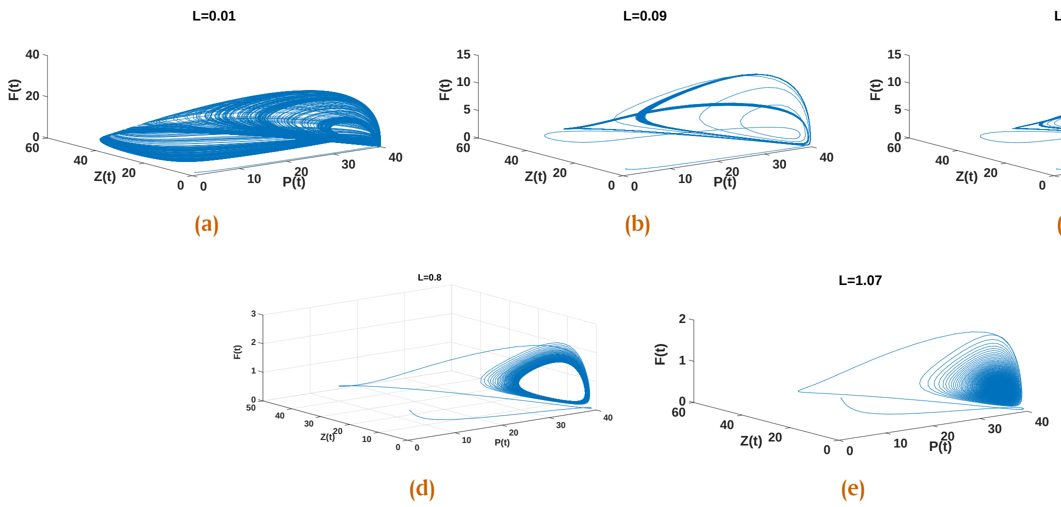


Figure 12. Controlling of chaos w.r.t. fear effect L

extinct. Therefore the anti predator behavior of the phytoplankton should not exceed 0.73 (Figure 10).

- It is observed from Figure 11 that the model system (1) is chaotic for $0 < n_2 \leq 0.43$, quasi-periodic for $0.44 \leq n_2 < 0.54$, double periodic for $0.55 \leq n_2 < 0.67$, Hopf-bifurcation occur in $0.68 \leq n_2 \leq 0.72$, complexity disappear with existence of stability in $0.73 \leq n_2 \leq 0.75$, and stable limit cycle exists for $n_2 \geq 0.758$ with the extinction of top predator. These findings reveal that increase in prey refuges makes the plankton dynamics stable from chaotic, but for the co-existence of all plankton species n_2 should not exceed 0.758.
- We have studied the impact of fear effect (L) on the given plankton dynamics and observed that the model system is chaotic for $0 < L \leq 0.07$, quasi-periodic in $0.08 \leq L \leq 0.1$, double periodic in $0.1 < L \leq 0.13$, bifurcates in $0.14 \leq L < 1.06$, and becomes stable beyond $L = 1.09$ (Figure 12).

8. Discussion and Conclusion

In this paper, we have proposed and investigated the dynamical behaviour of a plankton-fish interaction model system with prey refuge and fear effect. We have discussed the impact of internal parameters (prey refuge and fear) on the given plankton-fish dynamical system. It is observed that the system exhibits chaotic dynamic with respect to fear parameter L and existence of chaotic dynamics is confirmed by a positive Lyapunov exponents coefficient (Figure 3). Further, it is found in Section 7 that the chaotic behavior of the given system (1) can also be controlled through different control parameters viz., phytoplankton refuge (n_1), zooplankton refuge (n_2), and fear effect (L). The results of our study reveal that the given chaotic plankton system remains stable for $n_1 \in [0.71, 0.73]$, $n_2 \in [0.73, 0.75]$, and $L \geq 1.09$ (see Figure 10 to Figure 12). After that, in order to determine the effect of external factors (seasonality, noise, fluctuations, and time delay) on the given system, we have applied two chaos control schemes, viz., feedback and non-feedback. These schemes have the following ecological implications;

- In linear feedback control scheme (subsection 5.1), a new controlled dynamical system (4) has formulated with the help of control agents $v_i's$, $i = 1, 2, 3$, which represent noise in the system. Our numerical simulations have shown that noise transfer the chaotic trajectories of the given plankton system into a stable orbit around U_* (see Figure 4).
- In bounded feedback control scheme (subsection 5.3), we have included a controller $w(t)$ as a noise parameter in the zooplankton dynamics to construct a controlled dynamics (8). It is numerically observed that noise converts the chaotic system to a stable one around U_* (see Figure 6).
- In the non-feedback control scheme (subsection 6.1), a week periodic sinusoidal force as a non-feedback controller g(t) is applied to the given chaotic plankton dynamics (1). Numerically, it is seen that the sinusoidal force suppress the chaotic behavior of the system to a great extent (see Figure 7).

Thus, it can be concluded that under certain parametric conditions, both external and internal factors (n_1, n_2 and L) have a stabilizing effect on the given plankton-fish interactive system and reshape irregular dynamic of the system into a balancing state which is necessary for the sustenance and co-existence of the marine ecosystem.

Author Contributions. **Sharma, A.**: Conceptualization, writing—review and editing, data curation, visualization, supervision. **Kaur, R. P.**: methodology, software, validation, formal analysis, resources, writing—original draft preparation.

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Data availability. Not applicable.

References

[1] S. L. Lima, "Nonlethal effects in the ecology of predator-prey interactions," *Bioscience*, vol. 48, no. 1, pp. 25–34, 1998.

[2] S. Creel and D. Christianson, "Relationships between direct predation and risk effects," *Trends in Ecology and Evolution*, vol. 23, no. 4, pp. 194–201, 2008. DOI:10.1016/j.tree.2007.12.004

[3] S. L. Lima, "Predators and the breeding bird: behavioral and reproductive flexibility under the risk of predation," *Biological Reviews*, vol. 84, no. 3, pp. 485–513, 2009. DOI:10.1111/j.1469-185X.2009.00085.x

[4] J. Bhattacharyya and S. Pal, "Stage-structured cannibalism with delay in maturation and harvesting of an adult predator," *Journal of Biological Physics*, vol. 39, no. 1, pp. 37–65, 2013. DOI:10.1007/s10867-012-9284-6

[5] J. Bhattacharyya and S. Pal, "Hysteresis in coral reefs under macroalgal toxicity and overfishing," *Journal of Biological Physics*, vol. 41, no. 2, pp. 151–172, 2015. DOI:10.1007/s10867-014-9371-y

[6] S. Eggers *et al.*, "Predation risk induces changes in nest-site selection and clutch size in the siberian jay," *Proceedings of the Royal Society B*, vol. 273, pp. 701–706, 2006. DOI:10.1098/rspb.2005.3373

[7] J. Fontaine and T. Martin, "Parent birds assess nest predation risk and adjust their reproductive strategies," *Ecology Letters*, vol. 9, no. 4, pp. 428–434, 2006. DOI:10.1111/j.1461-0248.2006.00892.x

[8] J. D. Ibáñez-Álamo and M. Soler, "Predator-induced female behavior in the absence of male incubation feeding: an experimental study," *Behavioral Ecology and Sociobiology*, vol. 66, no. 7, pp. 1067–1073, 2012. DOI:10.1007/s00265-012-1357-9

[9] S. Creel *et al.*, "Predation risk affects reproductive physiology and demography of elk," *Science*, vol. 315, no. 5814, pp. 960–960, 2007. DOI:10.1126/science.1135918

[10] A. J. Wirsing and W. J. Ripple, "A comparison of shark and wolf research reveals similar behavioral responses by prey," *Frontiers in Ecology and the Environment*, vol. 9, pp. 335–341, 2011. DOI:10.1890/090226

[11] J. P. Suraci *et al.*, "Fear of large carnivores causes a trophic cascade," *Nature Communications*, vol. 7, p. 10698, 2016. DOI:10.1038/ncomms10698

[12] R. P. Kaur, A. Sharma, and A. K. Sharma, "Impact of fear effect on plankton-fish system dynamics incorporating zooplankton refuge," *Chaos, Solitons and Fractals*, vol. 143, p. 110563, 2021. DOI:10.1016/j.chaos.2020.110563

[13] W. Sun *et al.*, "Effects of zooplankton refuge on the growth of tilapia (oreochromis niloticus) and plankton dynamics in pond," *Aquaculture International*, vol. 18, no. 4, pp. 647–655, 2010. DOI:10.1007/s10499-009-9286-y

[14] J. Li *et al.*, "Dynamical analysis of a toxin-producing phytoplankton-zooplankton model with refuge," *Mathematical Biosciences snd Engineering*, vol. 14, no. 2, pp. 529–552, 2017. DOI:Doi: 10.3934/mbe.2017032

[15] A. Bertolo *et al.*, "Effects of physical refuges on fish–plankton interactions," *Freshwater Biology*, vol. 41, no. 4, pp. 795–808, 1999. DOI:10.1046/j.1365-2427.1999.00424.x

[16] G. C. W. Sabin and D. Summers, "Chaos in a periodically forced predator□prey ecosystem model," *Mathematical Biosciences*, vol. 113, no. 1, pp. 91–113, 1993. DOI:10.1016/0025-5564(93)90010-8

[17] S. Rinaldi, S. Muratori, and Y. Kuznetsov, "Multiple attractors, catastrophes and chaos in seasonally perturbed predator-prey communities," *Bulletin of Mathematical Biology*, vol. 55, no. 1, pp. 15–35, 1993. DOI:10.1007/BF02460293

[18] T. K. Kar, "Stability analysis of a prey–predator model incorporating a prey refuge," *Communications in Nonlinear Science and Numerical Simulation*, vol. 10, no. 6, pp. 681–691, 2005. DOI:10.1016/j.cnsns.2003.08.006

[19] Y. Huang, F. Chen, and L. Zhong, "Stability analysis of a prey–predator model with holling type iii response function incorporating a prey refuge," *Applied Mathematics and Computation*, vol. 182, no. 1, pp. 672–683, 2006. DOI:10.1016/j.amc.2006.04.030

[20] M. Pancic and T. Kiorboe, "Phytoplankton defence mechanisms: traits and trade-offs," *Biological Reviews*, vol. 93, no. 2, pp. 1269–1303, 2018. DOI:10.1111/brv.12395

[21] D. E. Schindler and M. D. Scheuerell, "Habitat coupling in lake ecosystems," *Oikos*, vol. 98, no. 2, pp. 177–189, 2002. DOI:10.1034/j.1600-0706.2002.980201.x

[22] P. J. Wiles *et al.*, "Stratification and mixing in the limfjorden in relation to mussel culture," *Journal of Marine Systems*, vol. 60, no. 1–2, pp. 129–143, 2006. DOI:0.1016/j.jmarsys.2005.09.009

[23] L. Zhang *et al.*, "Periodic fluctuations of marine oxygen content during the latest permian," *Global and Planetary Change*, vol. 195, p. 103326, 2020. DOI:10.1016/j.gloplacha.2020.103326

[24] C. Peng, X. Zhao, and G. Liu, "Noise in the sea and its impacts on marine organisms," *International Journal of Environmental Research and Public Health*, vol. 12, no. 10, pp. 12304–12323, 2015. DOI:10.3390/ijerph121012304

[25] S. S. Sabet, Y. Y. Neo, and H. Slabbekoorn, "Impact of anthropogenic noise on aquatic animals: from single species to community-level effects," in *The effects of noise on aquatic life II*, pp. 957–961, 2016. DOI:10.1007/978-1-4939-2981-8_118

[26] D. P. Häder *et al.*, "Effects of uv radiation on aquatic ecosystems and interactions with climate change," *Photochemical and Photobiological Sciences*, vol. 10, no. 2, pp. 242–260, 2011. DOI:10.1039/c0pp90036b

[27] R. K. Naji and A. T. Balasim, "Dynamical behavior of a three species food chain model with beddington–deangelis functional response," *Chaos, Solitons and Fractals*, vol. 32, no. 5, pp. 1853–1866, 2007. DOI:10.1016/j.chaos.2005.12.019

[28] R. K. Upadhyay and R. K. Naji, "Dynamics of a three species food chain model with crowley–martin type functional response," *Chaos, Solitons and Fractals*, vol. 42, no. 3, pp. 1337–1346, 2009. DOI:10.1016/j.chaos.2009.03.020

[29] M. Zhao and S. Lv, "Chaos in a three-species food chain model with a beddington–deangelis functional response," *Chaos, Solitons and Fractals*, vol. 40, no. 5, pp. 2305–2316, 2009. DOI:10.1016/j.chaos.2007.10.025

[30] A. Singh and S. Gakkhar, "Controlling chaos in a food chain model," *Mathematics and Computers in Simulation*, vol. 115, pp. 24–36, 2015. DOI:10.1016/j.matcom.2015.04.001

[31] V. Sundarapandian, "Output regulation of the liu chaotic system," *Applied Mechanics and Materials*, vol. 110, pp. 3982–3989, 2011. DOI:10.4028/www.scientific.net/AMM.110-116.3982

[32] G. Chen, "A simple adaptive feedback control method for chaos and hyper□chaos control," *Applied Mathematics and Computation*, vol. 217, no. 17, pp. 7258–7264, 2011. DOI:10.1016/j.amc.2011.02.017

[33] D. Yang and J. Zhou, "Connections among several chaos feedback control approaches and chaotic vibration control of mechanical systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 11, pp. 3954–3968, 2014. DOI:10.1016/j.cnsns.2014.04.001

[34] J. A. Laoye, U. E. Vincent, and S. O. Kareem, "Chaos control of 4d chaotic systems using recursive backstepping nonlinear controller," *Chaos, Solitons and Fractals*, vol. 39, no. 1, pp. 356–362, 2009. DOI:10.1016/j.chaos.2007.04.020

[35] S. Vaidyanathan, "Sliding mode control based global chaos control of liu-liu-liu-su chaotic system," *International Journal of Control Theory and Applications*, vol. 5, no. 1, pp. 15–20, 2012.

[36] P. A. Cook, "*Nonlinear Dynamical Systems.*" USA: Prentice-Hall International, 1986

[37] T. Botmart, P. Niamsup, and X. Liu, "Synchronization of non-autonomous chaotic systems with time-varying delay via delayed feedback control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 4, pp. 1894–1907, 2012. DOI:10.1016/j.cnsns.2011.07.038

[38] T. S. Parker and L. O. Chua, "*Practical Numerical Algorithms for Chaotic Systems.*" Springer-Verlag, 1989. DOI:0.1007/978-1-4612-3486-9_7

[39] M. Gao, H. Shi, and Z. Li, "Chaos in a seasonally and periodically forced phytoplankton–zooplankton system," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 3, pp. 1643–1650, 2009. DOI:10.1016/j.nonrwa.2008.02.005

[40] Y. Pilpel, "Noise in biological systems: pros, cons, and mechanisms of control," *Yeast Systems Biology*, pp. 407–425, 2011. DOI:10.1007/978-1-61779-173-4_23

[41] I. M. Moroz, R. Cropp, and J. Norbury, "Chaos in plankton models: Foraging strategy and seasonal forcing," *Ecological Modelling*, vol. 332, pp. 103–111, 2016. DOI:10.1016/j.ecolmodel.2016.04.011

[42] T. Insperger, "On the approximation of delayed systems by taylor series expansion," *Journal of computational and non linear dynamics*, vol. 10, no. 2, p. 024503, 2015. DOI:10.1115/1.4027180