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An Exact Solution for a Single Machine Scheduling Under Uncertainty

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ABSTRACT

Here we have n jobs on one machine where the processing times are triangular fuzzy numbers. The jobs are available to process without interruption. The purpose is to find a best sequence of the jobs that minimizes total fuzzy completion times and maximum fuzzy tardiness. In this paper a new definition is presented called D-strongly positive fuzzy number, then an exact solution of the problem through this definition is found. This definition opens new ideas about converting scheduling problems into fuzzy cases.

Keywords:
Scheduling; Fuzzy Processing Times; Fuzzy Tardiness; Efficient Solutions

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1. Introduction

A theory of fuzzy set was developed by Lotfi Zadeh in 1965 [1], after that many research papers have been published [2]. Nowadays fuzzy set theory has many applications in engineering and business. In the past, job processing time was usually treated as a crisp value. In reality, the processing time of a job can vary, due to some factors. The first study in fuzzy scheduling problems was by Prade [3], and some of articles were edited by Hapke andJaszkiewicz [4]. Fuzzy scheduling problems occur when processing times, due dates or both of them are in fuzzy environment. Han et al. [5] studied scheduling problem with fuzzy due dates. Ishii et al. [6] focused on two different problems under fuzzy due dates. Lam and Cai [7] studied the earliness and tardiness where due dates were fuzzy numbers. In the field of fuzzy environment, few researches treated the problems with fuzzy processing time. Ishibuchi et al. [8] formulated flow shop scheduling problem nuder fuzzy processing time. Tsujimura et al. [9] used genetic algorithm to solve scheduling problem with fuzzy processing times.

In addition that, the problems of job scheduling could be in the form that both of the processing times and due dates are fuzzy numbers. Chanas and Kasperski [10] considered two different problems where both of them were fuzzy numbers. Wu [11] minimized the fuzzy earliness and fuzzy tardiness using a new definition of fuzzy subtraction and fuzzy maximum of any given two fuzzy numbers. Nowadays, more

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than one objective has a great role in applications, so the idea of efficient solutions is one of the methods that used to understand the structure of this type of problem. An efficient solution defined as a solution that it cannot be dominated by any other solutions [12]. Recognizing all efficient solutions is done in [13]. Few studies treat with this way, because no enough information exists regarding the objectives. Multi-objective scheduling problems in fuzzy environment are more interesting in real life situations. Khalifa [14] proposed a method to minimize the total penalty cost due to earliness or lateness of job in fuzzy environment. Ramadan [15] introduced a new technique to find a relation between some important factors in fuzzy environment

In this paper we deal with a single machine under the fuzzy processing times and fuzzy due date. The criteria function is, total fuzzy completion times and maximum fuzzy tardiness $\sum_{j=1}^{n} \widetilde{c}_j$ and $\tilde{T}_{max}$.

2. Method and Preliminaries

In this work we restrict ourselves to the most popular type of fuzzy number which is Triangular Fuzzy Number (TFN).

Definition 1. Triangular Fuzzy Number (TFN) is a fuzzy number represented by three points $\tilde{A} = (a^L, a, a^U)$. The membership function of a triangular fuzzy number $\widetilde{A}$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad x < a^L, \\ \frac{x-a^L}{a-a^L} & , \quad a^L \le x \le a\,, \\ \frac{a^U-x}{a^U-a} & , \quad a < x \le a^U. \end{cases}$$

Suppose that $\tilde{A}$ and $\tilde{B}$ are two (TFN^s). Where $\tilde{A} = (a^L, a, a^U)$ and $\tilde{B} = (b^L, b, b^U)$, then *i)* $\tilde{A} \oplus \tilde{B} = (a^L + b^L, a + b, a^U + b^U)$, which is also triangular fuzzy number,*ii)* $\tilde{A} \ominus \tilde{B} = (a^L - b^U, a - b, a^U - b^L)$, which is also triangular fuzzy number [11].

2.1. Defuzzification Method

It is not possible to find the order of fuzzy numbers, so it is not possible to compare them directly. There are many proposed methods to do this and each method computes a crisp value, then used this value for comparison. This procedure is called defuzzification. In this work we used the centroid point method of fuzzy numbers.

Applying it to triangular fuzzy number $\tilde{p} = (p_1, p_2, p_3)$, we have $D = \frac{p_1+p_2+p_3}{3}$ [17]. Let $\tilde{p}_1$ and $\tilde{p}_2$ be two triangular fuzzy processing times where $\tilde{p}_1 = (p_1^L, p_1, p_1^U)$ and $\tilde{p}_2 = (p_2^L, p_2, p_2^U)$. Using this method we say that $\tilde{p}_1 < \tilde{p}_2$ if $D(\tilde{p}_1) < D(\tilde{p}_2)$. A special case will occur when $p_1^U < p_2^L$, in this case the two fuzzy numbers are comparable and there is no need to use ranking methods to map them to crisp values.

2.2. Notations and Problem Description

A set N, $N = 1, 2, ..., n$ of n independent jobs to be processed on a single machine. Each job j has a fuzzy processing time $\widetilde{\mathrm{p}}_\mathrm{j}$ which is a (TFN) $\tilde{p}_j = (p_j^L, p_j, p_j^U)$, and a fuzzy due date $\tilde{d}_j = (d_j^L, d_j, d_j^U)$. The jobs are available at time zero and the machine can only process one job at a time without interrupted. For each order the fuzzy completion time $\tilde{c}_j$ of each job j can be computed. In applications, all the data are uncertain. As well as the completion time. Let

Π	:	set of permutation schedules,
π	:	a permutation schedule,
$\widetilde{p}_j$	:	the fuzzy processing time of job j,
SPT-rule	:	order the jobs in non-decreasing order of $\tilde{p}_j$,
p_j^c	:	the crisp value of the fuzzy processing time $\tilde{p}_j$,
$\widetilde{d}_j$	:	the fuzzy due date of job j,
d_j^c	:	the crisp value of the fuzzy due date $\tilde{d}_j$,
EDD-rule	:	order the jobs in non-decreasing order of d_j,
$\widetilde{c}_j$	:	the fuzzy completion time of job j,
$\widetilde{L}_j = \widetilde{c}_j \ominus d_j$	:	the fuzzy lateness of job j,
$\tilde{T}_j = \widetilde{max}\ \ \tilde{c}_j \ominus \tilde{d}_j, \tilde{0}$	:	fuzzy tardiness of job j,
$\tilde{T}_{max} = \widetilde{max}\ \ \tilde{T}_j$	:	maximum fuzzy tardiness,
A^c	:	crisp value of the fuzzy number $\tilde{A}$,
m_1	:	the denominator of the ranking method.

Then, we have

$$\sum_{j=1}^{n} \widetilde{c}_j = G(\pi) \ \ and \ \ \tilde{T}_{max} = H(\pi) \tag{1}$$

A solution of these objectives is difficult and sometimes is not possible, this means, there is in general no π which minimizes G and H. So to find this fair solution we have the following concept [13].

Definition 2. A sequence $\sigma^* \in \Pi$ is efficient to problem (1) if $G(\sigma) \geq G(\sigma^*)$ and $H(\sigma) \geq H(\sigma^*)$ for any $\sigma \in \Pi$, where at least one relation holds with strict inequality.

It is possible to find all efficient solutions for this problem, then through these solutions one can find an optimal solution of the sum of the two objective functions $(\sum_{j=1}^{n} \widetilde{c}_j + \tilde{T}_{max})$. Van Wassenhove and Gelders solved this problem in the crisp case by finding all efficient solutions.

2.3. Calculations for Fuzzy Completion Times

Consider n-job on single machine scheduling problem with processing times that are assumed as fuzzy numbers. Let $\tilde{p}_j$ be the fuzzy processing time for job. Then for this job, the fuzzy completion times can be calculated us:

$$\begin{aligned} &\tilde{c}_1 = \tilde{p}_1, \\ &\tilde{c}_2 = \tilde{c}_1 \oplus \tilde{p}_2, \\ &\ \vdots \\ &\widetilde{c}_j = \tilde{c}_{j-1} \oplus \tilde{p}_j, \text{for} j = 1, ..., n. \end{aligned} \tag{2}$$

If for a job j the completion time is after its due date, the penalties are incurred [11].

2.4. Fuzzy Tardiness and Maximum Tardiness

For a given sequence Chanas and Kasperski [10] we defined the fuzzy tardiness of a job as a fuzzy maximum of zero and the difference between the fuzzy completion time and the fuzzy due date of this job, this means

$$\tilde{T}_j = \widetilde{max}\ \tilde{c}_j \ominus \tilde{d}_j, \tilde{0}\ , \tag{3}$$

where the fuzzy completion time $\tilde{c}_j = (c_j^L, c_j, c_j^U)\ \forall_j$ is found using (1), and maximum fuzzy tardiness is

$$\tilde{T}_{max} = \widetilde{max}\ \widetilde{max}\ \tilde{c}_j \ominus d_j, \tilde{0}\ . \tag{4}$$

3. Results and Discussions

3.1. New Structure of Fuzzy Numbers

To find the Pareto set for the problem (1) under fuzzy environment we will give some new ideas and definitions.

Definition 3. For any D positive number greater than or equal 1, $\tilde{A} = a^L, a, a^U$ and $\tilde{B} = b^L, b, b^U$ are said to be D-strongly positive iff $(a^L + a + a^U)$-$(b^L + b + b^L) \geq D$.

Now, let S be the set of maximum fuzzy tardiness $S = \tilde{T}_{max}\ (\sigma_i)$, S_1 the set of total fuzzy completion times „ where each two elements in both sets are D-strongly positive number, this means that $\tilde{T}_{max}(\sigma_i)$ and $\tilde{T}_{max}(\sigma_{i+1})$ are D-strongly positive number , and $\sum_{j=1}^{n} \tilde{c_j}(\sigma_{i+1})$ and $\sum_{j=1}^{n} \tilde{c}_j(\sigma_i)$ are D-strongly positive number too, $i = 1, ..., k$, where k is the number of efficient solutions.

Theorem 1. *The SPT sequence is one of the efficient solutions.*

Proof. If all p_j^c, $j = 1, 2, ..., n$ are different, so we have unique SPT sequence which gives the absolute minimum of G. If not, let SPT^* be a sequence arranged by SPT-rule and for equal $\mathrm{p}_\mathrm{j}^\mathrm{c}$ use EDD-rule. So the SPT is one of the efficient solution. □

In general this is not work with EDD-rule. Using the above defuzzification method,

$$D(\widetilde{p}_j) = p_j^c = \frac{p_j^{c*}}{3} = \frac{p_j^L + p_j + p_j^U}{3}$$

and for the fuzzy tardiness the crisp value of it is T_j^c and $T_{max}^c = max\ T_j^c$.

Van Wassenhove and Gelders (VG) algorithm [13] found all efficient solutions for the problem (1), his algorithm does not work in the fuzzy case. In this work we will find the efficient solutions for the problem (2) in particular case by modifying the (VG) algorithm.

3.2. Modified VG algorithm

We have 5 steps for this algorithm:

- Step(0): Compute $p_j^c\ \int_j \in N$, $N = 1, 2, ..., n$ according to a given method, then rank them. Put $\tilde{R} = \sum_{j=1}^{n} \widetilde{p}_j$, $\widetilde{\Delta} = \tilde{R}$. Compute R^c.

- Step (1): Let $\widetilde{D_i} = d_i \oplus \widetilde{\Delta}$, $K = n$, $N = 1, 2, ..., n$.
- Step (2): Find job $j^* \ni p_j^{c*} \geq p_j^c$; $D_j^{c*}, D_j^c \geq R^c$, Assign job j^* in position K.
- Step (3): Set $\tilde{R} = \sum_{j=1}^{n} \widetilde{p}_j$, $j \neq j^*$, $N = N - j^*$, $K = K - 1$. If $K = 0$, go to step (4). Else, go to step (2).
- Step (4): A solution α is exists. Compute $\tilde{T}_{max}(\alpha)$. If $T_{max}^c(\alpha) = T_{max}^c(EDD)$ go to step (5). Else, compute $\widetilde{\Delta} = \tilde{T}_{max}(\alpha) \ominus m_1$. Go to step (1).
- Step (5): Stop.

Theorem 2. *If the efficient solutions are D-strongly positive fuzzy number, then the modified VG algorithm finds all the efficient solutions where $D \geq m_1$.*

Proof. Clearly $< n!$, and $(\alpha_1, \alpha_2, ..., \alpha_k)$ be the efficient solutions, so $T_{max}^c(\alpha_i)$ -$T_{max}^c(\alpha_{i+1})$ *is* $\geq D$, $1 \leq i \leq k-1$. Let,

$$T_{max}^c(\alpha_i) = \frac{T_{max}^{c*}(\alpha_i)}{m_1}$$

and

$$T_{max}^c(\alpha_{i+1}) = \frac{T_{max}^{c*}(\alpha_{i+1})}{m_1},\ 1 \leq i \leq k-1.$$

Therefore, $\frac{T_{max}^{c*}(\alpha_i)}{m_1}$ - $\frac{T_{max}^{c*}(\alpha_{i+1})}{m_1}$ *is* $\frac{D}{m_1}$, $1 \leq i \leq k-1$. In the step (4) of the algorithm, we have

$$\frac{T_{max}^{c*}(\alpha_i)}{m_1} - m_1.$$

The next solution may be infeasible if $D < m_1$. The condition is that $D \geq m_1$. □

An important special case will be when m_1= 1. In this case the problem (1) reduces to the case that is solved by *VG* algorithm.

4. Conclusion

In the fuzzy environment the algorithms cannot apply, it is necessary to developt ideas. In all efficient solutions are D-strongly positive, a new algorithm is presented to recognize the efficient solutions for the criteria under fuzzy environment. A new ideas for further research is to find the efficient solutions for other scheduling problems.

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