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Mathematical Model of Coastline Changes in the Ujung Pangkah Gresik Using the Polynomial Lagrange Approach

Hendrata Wibisana[1,*], Primasari Cahya Wardhani[2], Novie Handajani[1]

[1]*Civil Engineering Department, Faculty of Engineering, Universitas Pembangunan Nasional Veteran Jawa Timur, Surabaya 60294, Indonesia*
[2]*Physics Department, Faculty of Engineering, Universitas Pembangunan Nasional Veteran Jawa Timur, Surabaya 60294, Indonesia*
[*]*Corresponding author. Email: hendrata.ts@upnjatim.ac.id*

ABSTRACT

Ujung Pangkah Gresik is an area where there is an estuary of a vast river, the Bengawan Solo river. This area annually sends sediment loads transported by the Bengawan Solo river to the Ujung Pangkah estuary so that gradually accumulation occurs, which results in changes in the morphology of the coast on the edge of the Ujung Pangkah coast, Gresik. This study seeks to see the side of shoreline changes on the Ujung Pangkah coast for five years. Then take the change model as a mathematical model that can describe the speed of shoreline change per year using the Lagrange polynomial interpolation method for degrees 1, degrees 2, and degrees 3. The results of shoreline changes that occur are obtained by the Lagrangge polynomial algorithm of degree 3 with R2 of 0.3747 which has a better correlation than degree 1 (0.0313) and degree 2 (0.3741). The results of this study obtained a mathematical model with a Lagrange polynomial approach where degree 3 has the best correlation among other models. This study concludes that by using a mathematical model, an overview of the process of change or natural phenomena can be obtained, where the existing model can predict future changes in the existing coastline.

Keywords:
Change of Coastline; Mathematical model; Lagrangge Polynomial; Ujung Pangkah Gresik

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1. Introduction

Change in the coastline is a fascinating natural phenomenon to study because changes in the coastline have enormous implications for the people who inhabit the area around the coast. For this reason, coastline changes need to be monitored and analyzed at any time to get a complete picture of these changes, which will result in forecasting the causes and effects affecting the ecosystem and local people's lives [1].

Researchers from various countries have researched coastline changes. Both manually and by using satellite imagery, the satellite imagery technology will provide more efficient and accurate results [2–4]. This is due to the ability of multi-temporal satellite

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imagery and has an excellent spatial recording process compared to making direct measurements in the field per measurement point, which takes a very long time and cannot avoid measurement errors due to observations from humans who have limited time and energy [3, 5, 6].

Satellite imagery plays a very important role in the process of changing coastlines. By doing a temporal analysis, it will be possible to know these changes over a certain period of time. These changes will impact sedimentation or erosion [5, 7, 8]. Thereby, policymakers and the local community will be able to know about future events so that it will be easier to handle what can be done [4, 9, 10]. Satellite imagery is also possible to predict floods and natural disasters in the future using the representatives of coastline changes image results [11–13]. Manual measurement of shoreline changes requires a long time and inaccurate results, so a settlement method is needed that can facilitate the efficient measurement of shoreline changes at a relatively low cost. For this reason, the existence of satellite image data in the form of coordinates of changes in the coastline is solved by a mathematical method, which in this case is represented by a polynomial lagrange.

The purpose of this study is to try to calculate the change in coastline on the coast of Ujung Pangkah Gresik on the side adjacent to the island of Madura with the Lagrange polynomial interpolation approach. By applying the Lagrange approach, a mathematical model will be obtained to describe the rate of change per sample data point [13–15]. Many changes to the coastline itself have been done manually by applying basic formulas in civil engineering [16–19]. However, the use of satellite imagery data has so far not been widely implemented for shoreline changes. So, it is necessary to process the data using accurate mathematical methods. The selection of the lagrange polynomial used is limited to the 3rd degree polynomial, this is because for the 4th degree and the nth degree, although they give the same good results, they have low effectiveness when applied to satellite data calculations [20].

In addition, to find out how much influence the Bengawan Solo River flows, which empties into the Ujung Pangkah Gresik area with several tributaries spread around Ujung Pangkah, this is because the influence of a river will make a significant contribution to sediment transport flowing from the river. Towards the end point of disposal and Ujung Pangkah is the estuary point of the Bengawan Solo River.

2. Method

2.1. Research Location

The research location is on the coast of Ujung Pangkah Gresik with coordinates -6.792o to -6.998o south latitude and 112.383o to 112.70o east longitude. The Ujung Pangkah coast of Gresik is bordered by the Java Sea in the north, Bangkalan Madura district in the east, Lamongan district in the west and Gresik City in the south, for the appearance of the area with satellite imagery can be seen in Figure 1 which shows the whole Ujung Pangkah area before image cropping.

On the Figure 2 is a view from satellite image from cropped of satellite image of Landsar 8 which is shown in Figure 1 previously, this method aims to further clarify the area from the data collection point to measure shoreline changes.

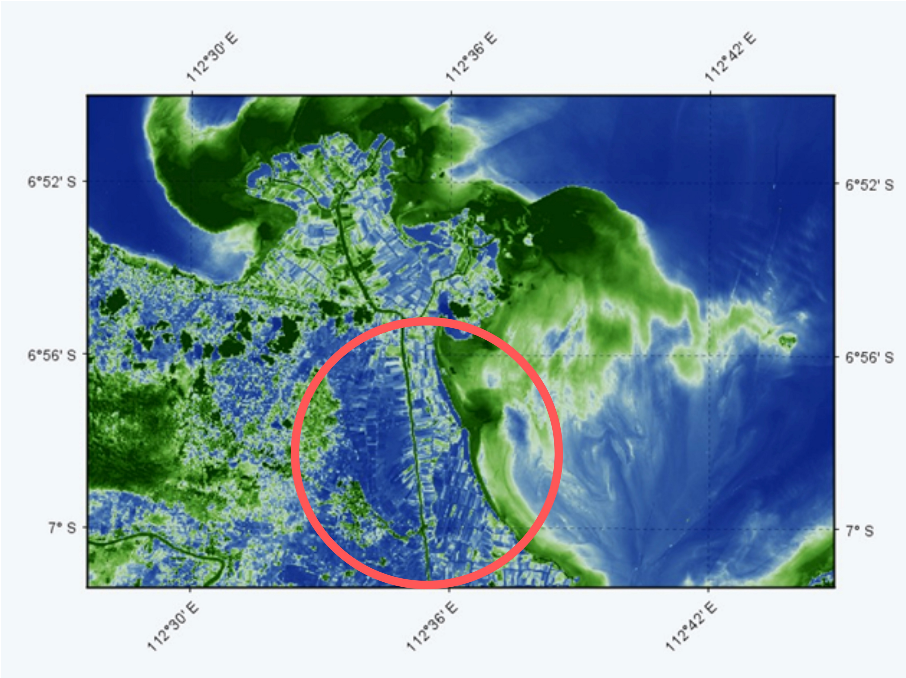

Figure 1. Research location on Ujung Pangkah Gresik area

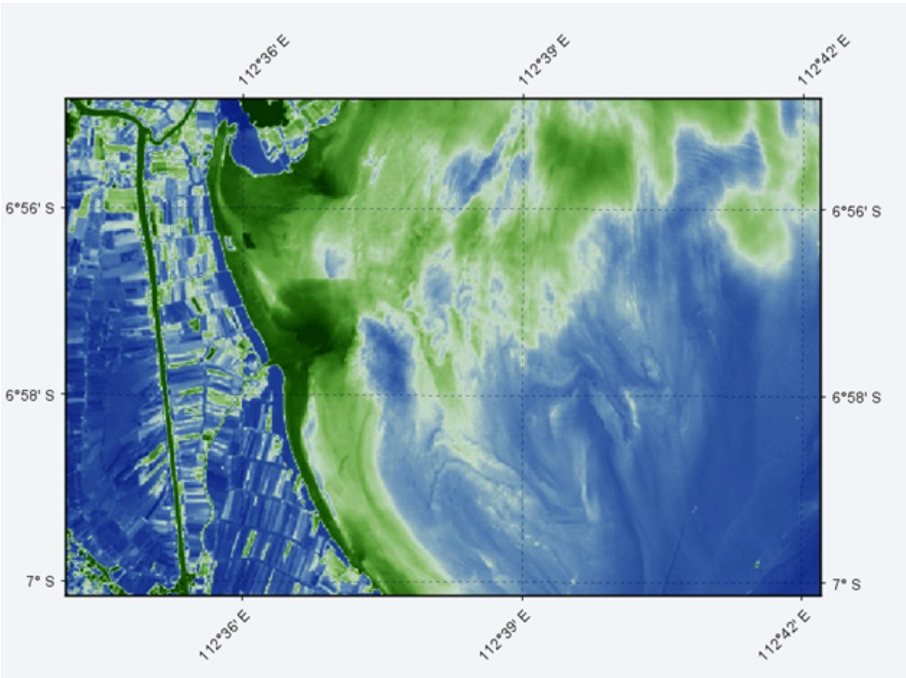

Figure 2. Overview of the research area from cropped of landsat 8 OLI satellite image

2.2. Satellite Image Acquisition

The satellite imagery used in this study is the Landsat 8 OLI (object land imagery) satellite image in 2020. This Landsat 8 satellite image can be downloaded for free via the https://www.earthexplorer.usgs.gov website, and images that are downloaded have a file name according to the date of the month and year of acquisition, namely

LC08_L1TP_118065_20200312_20200317_01_T1_B4.data. LC08 is an initial characteristic of Landsat 8 satellite imagery that distinguishes it from previous Landsat satellite images, namely Landsat 7, 5, and earlier, which are no longer operating other than Landsat 7 and 5.

2.3. Measuring Shoreline Change

Measuring shoreline change is carried out by placing ten (10) points along the elongated coastline of Ujung Pangkah with a predetermined distance of about 200 meters shown in Figure 3.

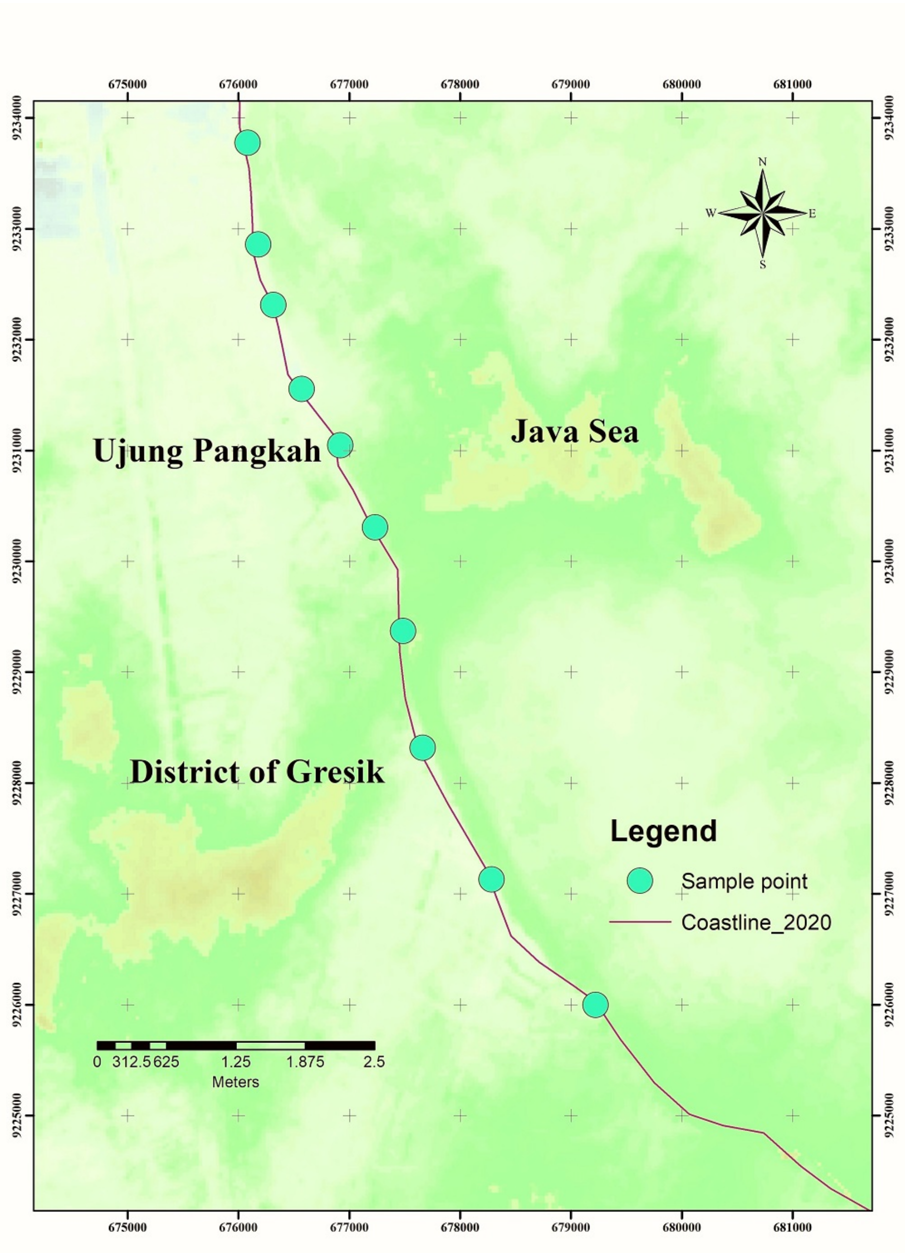

Figure 3. Placement of observation points for coastline changes with the grid system

The coordinate measured by GPS navigation is recorded and tabulated, shown in Table 1.

Table 1. The results of measuring the UTM coordinates of the Landsat image in 2020 for the Ujung Pangkah Gresik area

Castline In 2020	
Latitude	Longitude
9230122	677487
9229443	677566
9228789	677637
9228163	677829
9227522	678091
9227038	678352
9226444	678782
9225791	679288
9225206	679744
9224778	680441

In this study a mathematical approach was used using the Lagrange model of degrees 1 to 3 as follows (1) to (3).

1 Degree Model:

$$f_1(x) = \frac{(x-x_1)}{(x_0-x_1)} f(x_0) + \frac{(x-x_0)}{(x_1-x_0)} f(x_1) \tag{1}$$

2 Degree Model:

$$\begin{aligned} f_1(x) =& \frac{(x-x_1)}{(x_0-x_1)} \frac{(x-x_2)}{(x_0-x_2)} f(x_0) + \frac{(x-x_0)}{(x_1-x_0)} \frac{(x-x_2)}{(x_1-x_2)} f(x_1) \\ &+ \frac{(x-x_0)}{(x_2-x_0)} \frac{(x-x_1)}{(x_2-x_1)} f(x_2) \end{aligned} \tag{2}$$

3 Degree Model:

$$\begin{aligned} f_1(x) =& \frac{(x-x_1)}{(x_0-x_1)} \frac{(x-x_2)}{(x_0-x_2)} \frac{(x-x_3)}{(x_0-x_3)} f(x_0) + \frac{(x-x_0)}{(x_1-x_0)} \frac{(x-x_2)}{(x_1-x_2)} \frac{(x-x_3)}{(x_1-x_3)} f(x_1) \\ &+ \frac{(x-x_0)}{(x_2-x_0)} \frac{(x-x_1)}{(x_2-x_1)} \frac{(x-x_3)}{(x_2-x_3)} f(x_2) + \frac{(x-x_0)}{(x_3-x_0)} \frac{(x-x_1)}{(x_3-x_1)} \frac{(x-x_2)}{(x_3-x_2)} f(x_3) \end{aligned} \tag{3}$$

A coastline digitization process is carried out to measure the coastline coordinates by taking Landsat 8 satellite imagery in 2020 as a measurement basis. The digitization process is carried out using satellite image processing software. After the coastline in each year is formed, at the predetermined sample point, a straight line is drawn, and the UTM (Universal Transfer Mercator) coordinates are measured in 2020 and then continued with measurement in 2019, 2018, and 2017, the result can be seen in Table 1 and Table 2.

For measuring coastline changes, UTM coordinates are used as calculation data. The annual data are compared, and calculating the difference in the coordinates of both Latitude and Longitude is carried out. The formula is used to measure the distance to the shoreline change in (4). The data obtained will be used to construct a mathematical model using a lagrange polynomial. The formula of Euclidean distance used is the basic formula as follows:

$$d_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4}$$

where

D	=	distance between year 1 line and year 2 line
x	=	longitude
y	=	latitude
1,2	=	year 1 and year 2 coastlines

3. Results and Discussions

3.1. Coastline Change Results from 2016 to 2020

The digitized coastline length is a Landsat image from 2020 to 2016. The calculation result of the difference in coastline length with formula one is shown in Table 2.

Table 2. Data on annual changes in coastline differences 2020-2016

Observation Points	2020-2019	2019-2018	2018-2017	2017-2016
1	78.7464	138.1050	27.5862	108.5219
2	114.2016	94.7048	70.4911	76.5768
3	48.7647	51.9230	101.6366	81.3941
4	118.5327	83.4865	68.0955	32.0156
5	138.4052	53.8516	136.6931	164.9394
6	164.9242	91.0220	90.6091	61.0574
7	106.5082	39.6232	36.4005	31.0161
8	78.3901	50.2494	58.5235	55.4707
9	46.9574	27.7308	140.4172	43.9318
10	74.6726	54.7814	104.3456	67.3573

The results of the map of changes each year are shown in Figure 4, measured from 2020 to 2016 with Landsat 8 OLI imagery. The point of change of the coastline each year can be depicted with a graph to see the characteristics of each current year in Ujung Pangkah Gresik. The mapping characteristic of coastline changes from 2016 to 2020 showed in Figure 4.

In Figure 4, it can be seen that the change in the coastline that dominates in the magnitude of the line difference is between 2020 and 2019, shown by a blue line, followed by changes in 2018 and 2017, shown by a gray line, while for changes in 2017 and 2016. Although some points look low at observation point 5, there is a very significant rate of change, which is more than 160 meters in a year, so it can be said that at point 5, there has been a sedimentation process that has resulted in the coastline moving forward for approximately 160 meters. Figure 5 presents the comparison graph of each difference per year.

3.2. Mathematical Models with Lagrange Polynomials

To calculate shoreline changes with Lagrange polynomial interpolation, the 1st year changes are given the notation number 1 is 2020-2019. The 2nd year changes are 2019-2018, and so on. Thereby, the 4th year changes are obtained.

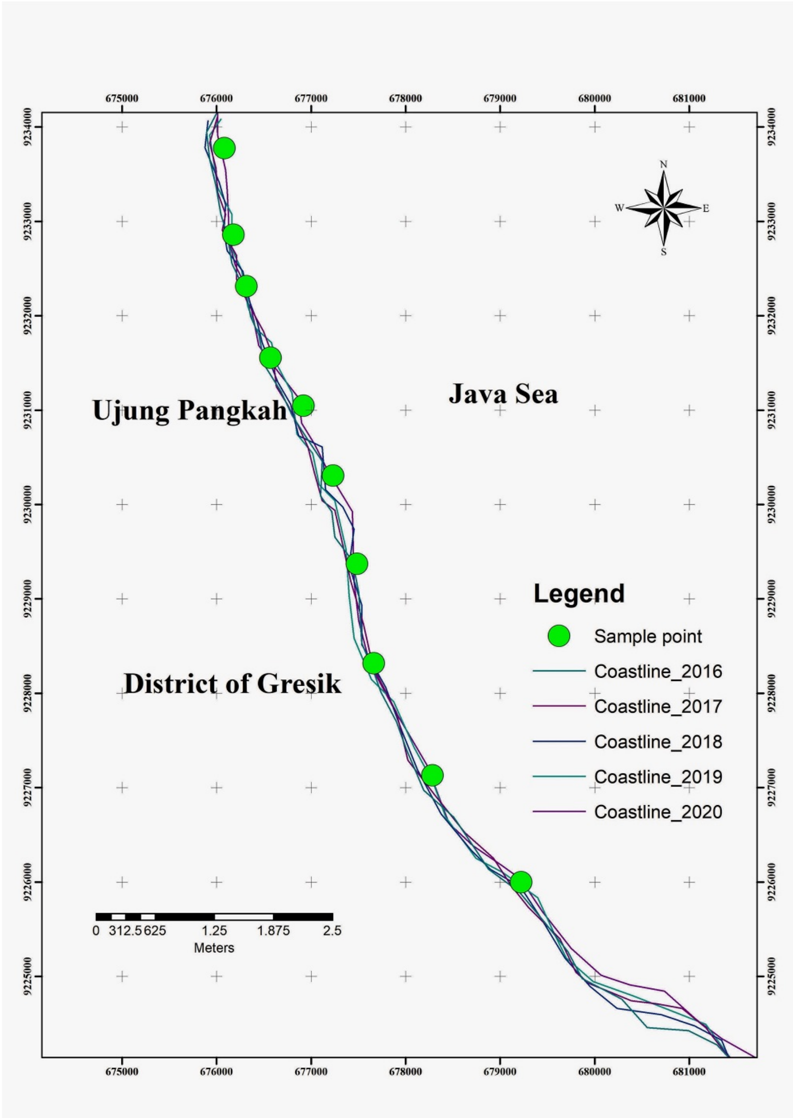

Figure 4. Map characteristic of each coastline change from 2016 to 2020

1 Degree Model

$$
\begin{aligned}
f(x) =& \frac{(x-2)}{(1-2)} 138.40 + \frac{(x-1)}{(2-1)} 53.85 f(x) \\
=& (x-2)(-138.40) + (x-1)\, 53.85 f(x) \\
=& -84.55x + 222.95.
\end{aligned} \tag{5}
$$

where x is the current year's coastline difference and $f(x)$ is the coastline change that occurred. By using a degree 1 mathematical model, it is found that the R square value is 0.0313 so that the use of a degree 1 mathematical model is less able to represent the value of shoreline changes.

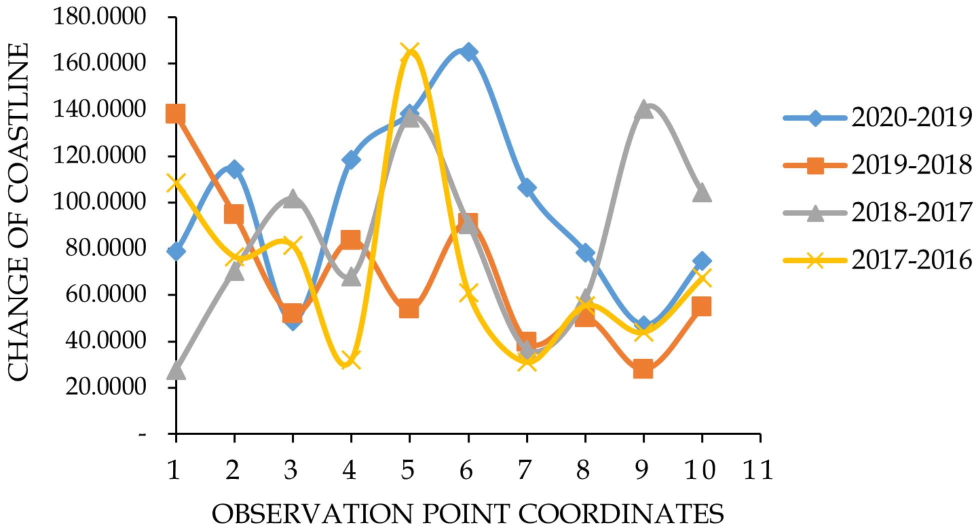

Figure 5. Characteristics of each coastline change from 2016 to 2020

2 Degree Model

$$\begin{aligned}f(x) =&\frac{(x-2)(x-3)}{(1-2)(1-3)}138.40+\frac{(x-1)(x-3)}{(2-1)(2-3)}53.83+\frac{(x-1)(x-2)}{(3-1)(3-2)}136.70\\ =&\left(x^2-5x+6\right)69.20+\left(x^2-4x+3\right)(-53.83)+\left(x^2-3x+2\right)68.35\\ =&83.72x^2-335.73x+390.41.\end{aligned} \tag{6}$$

where x is the current year's coastline difference and $f(x)$ is the coastline change that occurred. Using the mathematics model of Lagrange Polynomial for 2 degree model was obtained that the R square value is 0.3741.

3 Degree Model

$$\begin{aligned}f(x) =&\frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)}138.40+\frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)}53.85\\ &+\frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)}136.70+\frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}164.90\\ =&\left(x^3-9x^2+26x-24\right)-23.07+\left(x^3-8x^2+19x-12\right)26.92\\ &+\left(x^3-7x^2+14x-8\right)-68.35+(x^3-6x^2+11x-6)27.48\\ =&-37.02x^3+305.84x^2-742.96x+612.56.\end{aligned} \tag{7}$$

In this study, the use of a mathematical model using a degree 3 lagrange polynomial experienced an increase in the value of R square ($R = 0.3743$) compared to the degree 1 and degree 2 lagrange polynomial mathematical models. For the 3-degree model, it is known that the third power coefficient is negative, so the difference in coastline changes will experience erosion in the next ten years. However, for that, it still needs to be studied further what will happen in 10 years with other parameters such as wave motion, wind direction, and the suspended cargo carried by the Bengawan Solo River because all of these contribute to the morphology of the Ujung Pangkah coast in the next 10 to 20 years. This research only intends to describe a change model with numerical applications using

Lagrange polynomial interpolation to add insight into the numerical analysis of natural phenomena that occur mainly in dynamic coastal areas such as Ujung Pangkah Gresik.

4. Conclusion

Changes that occur on the coast of Ujung Pangkah Gresik are quite dynamic, and the value of these changes is quite significant annually; the lowest value is 27.58 meters, and the highest is 164.93 meters per year. Landsat 8 OLI satellite imagery can be appropriately used and accurately describes the process of changing the coastline, which is calculated annually from 2016 to 2020. The shoreline change model with the Lagrange polynomial interpolation model is sufficient to provide variations in accuracy, whereas the most optimal model to describe the change is a 3-degree model. The more data changes a study has; the more Lagrange polynomial models can be made with varying degrees.

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Reference

[1] J. Liu, L. Zhou, Y. Tan, Q. Wang, Z. Hu, J. Li, X. Jiang, and Z. Ke, "Distribution of reactive aluminum under the influence of mesoscale eddies in the western South China Sea," *Acta Oceanologica Sinica*, vol. 36, no. 6, pp. 95–103, jun 2017, doi: 10.1007/s13131-017-1046-7.

[2] H. Phan, A. Reniers, T. Ye, and M. Stive, "Response in the mekong deltaic coast to its changing sediment sources and sinks," in *Proceedings of Coastal Dynamics*, 2017, pp. 311–322.

[3] C. Wang, Y. Zhao, X. Zou, X. Xu, and C. Ge, "Recent changing patterns of the Changjiang (Yangtze River) Estuary caused by human activities," *Acta Oceanologica Sinica*, vol. 36, no. 4, pp. 87–96, apr 2017, doi: 10.1007/s13131-017-1017-z.

[4] U. U. Henry, J. Igbokwe, and A. C. Chukwocha, "Assessment of coastaline vulnerability for sustainable coastal development and planning in Nigeria using medium resolution satellite images and GIS," *International Journal of Engineering and Management Sciences*, vol. 4, no. 4, pp. 408–416, 2013.

[5] L. O. Dragomir, P. D. E. H. Petrosani, and S. Oncia, "Using satellite images LANDSAT TM for calculating normalized difference indexes for the landscape of Parang Mountains," *GeoCAD*, 2012.

[6] S. Zeinali, M. Dehghani, M. Rastegar, and M. Mojarrad, "Detecting shoreline changes in Chabahar Bay by processing satellite images," *Scientia Iranica*, vol. 24, no. 4, pp. 1802–1809, aug 2017, doi: 10.24200/sci.2017.4271.

[7] X. Li and M. C. Damen, "Coastline change detection with satellite remote sensing for environmental management of the Pearl River Estuary, China," *Journal of Marine Systems*, vol. 82, pp. S54–S61, aug 2010, doi: 10.1016/j.jmarsys.2010.02.005.

[8] A. Paytan, K. R. M. Mackey, Y. Chen, I. D. Lima, S. C. Doney, N. Mahowald, R. Labiosa, and A. F. Post, "Toxicity of atmospheric aerosols on marine phytoplankton," *Proceedings of the National Academy of Sciences*, vol. 106, no. 12, pp. 4601–4605, mar 2009, doi: 10.1073/pnas.0811486106.

[9] C. Zhong, W. P. Xiao, and B. Huang, "The response of phytoplankton to mesoscale eddies in western South China Sea," *Adv. Mar. Sci*, vol. 31, no. 2, pp. 213–220, 2013.

[10] T. R. Parsons, Y. Maita, and C. M. Lalli, *A Manual of Chemical & Biological Methods for Seawater Analysis*. Elsevier, 1984. ISBN 9780080302874 Doi: 10.1016/C2009-0-07774-5.

[11] M. V. Khamutova, V. A. Kushnikov, and O. I. Dranko, "A mathematical model for choosing an action plan for the prevention and elimination of flood effects," *IFAC-PapersOnLine*, vol. 55, no. 3, pp. 113–118, 2022, doi: 10.1016/j.ifacol.2022.05.020.

[12] J. G. Wallwork, *Mesh Adaptation and Adjoint Methods for Finite Element Coastal Ocean Modelling*. PhD Thesis: Imperial College London, 2021.

[13] T. Nguyen, J. Peterson, L. Gordon-Brown, and P. Wheeler, "Coastal changes predictive modelling: a fuzzy set approach," *World Acad Sci Eng Technol*, vol. 48, pp. 468–473, 2008.

[14] C. Chalons and A. Del Grosso, "Exploring different possibilities for second-order well-balanced Lagrange-projection numerical schemes applied to shallow water Exner equations," *International Journal for Numerical Methods in Fluids*, vol. 94, no. 6, pp. 505–535, jun 2022, doi: 10.1002/fld.5064.

[15] P. Othata and N. Pochai, "Irrigation Water Management Strategies for Salinity Control in the Chao Phraya River Using Sualyev Finite Difference Method with Lagrange Interpolation Technique," *Engineering Letters*, vol. 29, no. 2, pp. 332–338, 2021.

[16] R. Gens, "Remote sensing of coastlines: detection, extraction and monitoring," *International Journal of Remote Sensing*, vol. 31, no. 7, pp. 1819–1836, apr 2010, doi: 10.1080/01431160902926673.

[17] R. Bynoe, "Trekking the Shore: Changing Coastlines and the Antiquity of Coastal Settlement," *Quaternary Science Reviews*, vol. 34, pp. 146–147, feb 2012, doi: 10.1016/j.quascirev.2011.12.016.

[18] H. Wibisana, N. Handajani, and P. C. Wardhani, "The Mathematics Model Analysis of Distribution Concentration of Total Suspended Solid in Gresik Coastal Coast with Landsat 8 Satellite Imagery," *Nusantara Science and Technology Proceedings*, pp. 288–294, 2021.

[19] H. Wibisana and S. Zainab, "Time Series Analysis of Sea Surface Temperature With Aqua MODIS from 2011 to 2016. Case Studi: North Coast of Gresik and Madura," *IPTEK Journal of Science and Technology*, vol. 28, no. 1, pp. 15–19, 2017.

[20] P. Soille and M. Pesaresi, "Advances in mathematical morphology applied to geoscience and remote sensing," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 40, no. 9, pp. 2042–2055, sep 2002, doi: 10.1109/TGRS.2002.804618.