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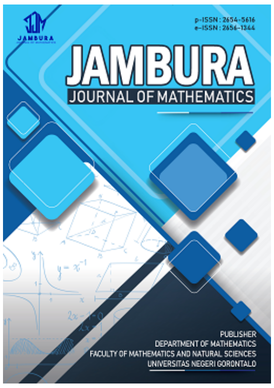

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Research Article

On The Hidden Structure of Odd Numbers and Its Consequences for the Riemann Hypothesis

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ABSTRACT. *The Riemann hypothesis remains unconfirmed or invalidated to this day, although local verifications on the calculation of its zeros have never found it faulty. Mertens reformulated the problem to make it much more accessible and surely more easily solvable. Unfortunately his conjecture, also called a strong conjecture, turned out to be incorrect. There remain 2 other conjectures, the weak and the general, which do not yet have fixed status. Would it then be possible that the resolution of the Riemann hypothesis arises through one of these 2 conjectures? We answer yes and we turn our attention to the Mertens weak conjecture. Equipped with a new equation to date and a methodical approach which uses a bounded description of numbers, we solve the conjecture by placing ourselves under the criteria of Hausdorff's theorem concerning the evolution of the sum, by showing first of all that odd numbers have a structure similar to that of triangular numbers, and then the randomness arises from their intrinsic regularity; which does not contradict the Martin-Löf definition of random sequences despite everything. We therefore resolve the Riemann hypothesis and we provide an equation which will certainly make it possible to resolve other types of problems, and thus to extend the means made available to mathematicians to examine various types of questions whether in number theory or in other fields of mathematics, or even in physics, cryptography and computer science.*

1. Introduction

Attacking the Riemann hypothesis [1, 2] head-on is quite difficult and this difficulty comes from the function $\zeta(s)$ itself [3, 4], whose domain is $s \in \mathbb{C}|Re(s) > 1$ and analytically extended to $s \in \mathbb{C}|Re(s) > 0,\ s \neq 1$. It is still not very well known and her zeros continue to raise questions, even within the narrow scope of the actual plan. Thus, we still do not know if the Apéry's constant [5] (the value of $\zeta(3)$) is a transcendent number or not, nor if it even has a closed form. The entire approach put in place by Mertens starts from the Möbius function [6]. Let μ be the Möbius function, a particular multiplicative function, defined on all strictly positive integers and with range in the set $-1, 0, 1$ such that:

$$\mu(N) = \begin{cases} 1, & \text{if } N = 1 \\ 0, & \text{if } N \text{ is divisible by a perfect square} \\ (-1)^t, & \text{if } N \text{ has a number t of factors.} \end{cases}$$

This function is also involved in combinatorics and in Dirichlet series [7].

From the Möbius function, we define the Mertens function [8], for any given integer N, as follows:

$$M(N) = \sum_{1 \leq k \leq N} \mu(k). \tag{1}$$

As the Möbius function μ only takes unit values in absolute value or zero, we always have:

$$|M(N)| < N. \tag{2}$$

After the application of Abel's summation formula [9, 10], we have:

$$\sum_{1}^{\infty} \frac{\mu(N)}{N^s} = s \int_1^{\infty} \frac{M(u)}{u^{1+s}} du = \frac{1}{\zeta(s)} \tag{3}$$

$\forall N \in \mathbb{N}, \forall s \in \mathbb{C}$, $Re(s) > 1$ and $M(u) = \sum_{1 \leq N \leq u} \mu(N)$.

Merten's function theory is very obscure. But we know how to prove the following estimate by taking into account the largest known region in the critical band that does not contain zero of the zeta function [11]:

$$M(u) = O\left(ue^{-a(ln u)^{3/5}(lnln u)^{-1/5}}\right) \tag{4}$$

where a is a constant, $u > 0$, and based on $Re(s) > 1 - \frac{a}{(ln\ u)^{2/3}(ln\ u)^{1/3}}$, and O represents the asymptotic comparison of 2 functions in Landau notation [12].

There are 3 Mertens conjectures and all involve the Riemann hypothesis:

1. Mertens weak conjecture:

$$\forall \varepsilon > 0, M(N) = O\left(N^{\frac{1}{2}+\varepsilon}\right) \tag{5}$$

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2. Mertens strong conjecture:
$$|M(N)| < \sqrt{N}. \tag{6}$$
Jànos Pintz showed shortly that there exists at least one integer less than $exp\,(3.21\;10^{64})$ refuting the conjecture [13].
3. Mertens generalized conjecture:
$$|M(N)| < A\sqrt{N} \tag{7}$$
where $A \in \mathbb{R}$ is a positive constant. It is still unknown whether $M(N)/\sqrt{N}$ is bounded, but te Riele and Odlyzko consider it probable that it is not [14, 15].

Our article attempts to demonstrate Merten's weak conjecture (6), because it seems a natural and more accessible way to lead to the Riemann hypothesis from the elements in our possession. This will open the way to a new approach to several mathematical problems.

Theorem 1 (Hausdorff's theorem). *[16] With a large quantity of numbers taken at random, the sum does not grow faster than $C^{ste}N^{\frac{1}{2}+\varepsilon}$, $\forall\varepsilon > 0$, when N tends towards infinity, and this with a probability of 1.*

We will show that taking numbers from 1 to N is the same as drawing N numbers at random.

2. Method

We will base ourselves on a bounded description of numbers, adequate in our opinion, to lead directly to the expected result, namely the demonstration that Hausdorff's theorem remains valid even for an arithmetic sequence. To do this, we will study the characteristic equation that describes the odd numbers in $\mathbb{Z}$ and arrive at the hidden structure of these numbers. This structure presents a distribution similar to that of triangular numbers. We will then show that from the regular distribution of odd numbers emerges random behavior, which does not contradict the definition of Martin-Löf on random sequences [17]. This fact will allow us to place ourselves within the criterion of Hausdorff's theorem and show that Mertens' weak conjecture is correct and merges with Hausdorff's theorem. This will lead to the confirmation of Riemann's hypothesis. This will also give suggestions for a new way of generating random numbers.

3. Results and Discussion

Let B be a positive square integer. Let N be any number for which we can write:
$$N = p \times q \tag{8}$$
with $p \leq q$. In particular $p = 1$, if N is a prime number.

Let $T(N)$ be a number called the witness of N for the factorization, such that we can write:
$$N + T(N) = B. \tag{9}$$
B is then a bound for N and N is said to be bounded by B, so that we have:
$$N \leq B. \tag{10}$$

Proposition 1. *Any number N bounded by B can be written as the product of 2 factors defined by the 3 integers numbers b, a, x as follows:*
$$\left(\frac{b-1}{2} - x - a\right)\left(\frac{b-1}{2} - x + a\right) = N. \tag{11}$$

Proof. Let us define from B, b, which we will call the base, such that:
$$b = 2\sqrt{B} - 1.$$
We can show that $B - b$ is still a square. Indeed,
$$B - b = B - 2\sqrt{B} + 1 \Rightarrow (B - b) = \left(\sqrt{B} - 1\right)^2.$$
Let x such that:
$$\frac{b-1}{2} - x = \frac{S}{2}$$
with S the sum of the factors of N:
$$S = p + q.$$
Let a be half the difference of the factors of the same number N:
$$a = \frac{q-p}{2}.$$
Therefore $\forall N \leq B$, then we have:
$$\left(\frac{b-1}{2} - x - a\right)\left(\frac{b-1}{2} - x + a\right) = N.$$
□

Corollary 1. *The witness of N, $T(N)$, is then given by the following expression:*
$$T(N) = (b - x)(x + 1) + a^2. \tag{12}$$

Proof. Indeed, the expansion of the expression (11) in proposition 1 leads to:
$$\left(\frac{b-1}{2} - x\right)^2 - a^2 = N$$
$$\left(\frac{b-1}{2}\right)^2 - (b-1)x + x^2 - a^2 = N$$
$$\left(\frac{2\sqrt{B}-2}{2}\right)^2 - (b-1)x + x^2 - a^2 = N$$
$$\left(\sqrt{B} - 1\right)^2 - (b-1)x + x^2 - a^2 = N.$$
It is known that $B - b = \left(\sqrt{B} - 1\right)^2$, so we have:

$$\begin{aligned} B-b-(b-1)\,x+x^2-a^2 &= N\\ B-bx-b+x+x^2-a^2 &= N\\ B-b\,(x+1)+x\,(x+1)-a^2 &= N\\ B-(b-x)\,(x+1)-a^2 &= N\\ (b-x)\,(x+1)+a^2 &= B-N. \end{aligned}$$

Therefore $B-N=T(N)$, then we have:

$$T\,(N)=(b-x)\,(x+1)+a^2.$$

□

Corollary 2. *For the numbers x and a to always be integers, the number N must not be even.*

Proof. For any even number N such that, for 2 numbers n and $m\in\mathbb{Z}$:

$$N=2n\times(2m+1).$$

We know how to find a and x such that:

$$a=\frac{2\,(m-n)+1}{2}\ \text{and}\ \frac{s}{2}=\frac{2\,(m+n)+1}{2}$$

are not integers and therefore x neither.

Therefore, equation (12) in $\mathbb{Z}$ is the characteristic equation of odd numbers when described in a bounded way. In the following, we will subsequently reserve the notation "N" for odd integers. □

Corollary 3. *As N can only be odd, $T(N)$ must also be odd.*

Proof. This goes without saying since every $T(N)$ also obeys equation (11) at preposition 1 because every witness is also a number. This implies that B can only be even due to equation (9). □

Corollary 4. *The numbers x and a obey the same restriction on parity as N and $T(N)$.*

Proof. This is obvious by considering equation (12) at corollary 1 and knowing that b and $T(N)$ are always odd. □

Proposition 2. *If we consider b, x and a modulo 8, then there are 4 kinds of odd numbers.*

Proof. The fact that x and a must always have the same parity (corollary 4) means that they do not contribute simultaneously to the modular value of (12):

- If x and a are even, then x does not influence $T(N)$, and therefore the 2 values of $T(N)$ will depend only on those of a. Indeed, in $(b-x)\,(x+1)$ we have the following 4 cases:
  1. Let $x\equiv 0\ mod8 \Rightarrow (b-x)\,(x+1)\equiv b\ mod8$
  2. Let $x\equiv 2\ mod8 \Rightarrow (b-x)\,(x+1)\equiv (3b-6)\ \ mod8$
  3. Let $x\equiv 4\ mod8 \Rightarrow (b-x)\,(x+1)\equiv (5b-4)\ \ mod8$
  4. Let $x\equiv 6\ mod8 \Rightarrow (b-x)(x+1)\equiv (7b-2)\ \ mod8$
     b having 2 possible values $3\ mod8$ and $7\ mod8$, when we replace these 2 values in each of the 4 cases we always find the same value of b. x therefore has no influence.

  Only the 2 values of a^2 (0 and $4\ mod8$) therefore influence the expression of $T(N)$.
- If x and a are odd, then it is around a not to influence $T(N)$ since $a^2\equiv 1\ mod4$. Indeed, in $(b-x)\,(x+1)$ we have the following 4 cases:
  1. Let $x\equiv 1\ mod8 \Rightarrow (b-x)\,(x+1)\equiv 2(b-1)\ mod8$
  2. Let $x\equiv 3\ mod8 \Rightarrow (b-x)\,(x+1)\equiv 4\,(b-3)\ \ mod8$
  3. Let $x\equiv 5\ mod8 \Rightarrow (b-x)\,(x+1)\equiv 6\,(b-5)\ \ mod8$
  4. Let $x\equiv 7\ mod8 \Rightarrow (b-x)(x+1)\equiv 0\,(b-7)\ \ mod8$
     b having 2 possible values $3\ mod8$ and $7\ mod8$, when we replace these 2 values in each of the 4 cases we find either $0\ mod8$ and $4\ mod8$.

  As previously with a and x even, only these 2 values (0 and $4\ mod8$) therefore influence the expression of $T(N)$.

We, therefore, end up with 4 types of numbers, those that depend on a and those which depend on x. And, following the influential modular values, we have: T_{a0}, T_{a4}, T_{x0}, and T_{x4}. □

Proposition 3. *If we consider the respective influences of x (respectively of a) in the types induced by a (resp. by x), the odd numbers are distributed in each of these 4 types as are the triangular numbers.*

Proof. 3 successive triangular numbers present by successive difference from the largest to the smallest the following property which defines their distribution:

$$t_n = \frac{n\,(n+1)}{2}, \tag{13}$$

$$t_{n+1} = \frac{(n+1)\,(n+2)}{2}, \tag{14}$$

$$t_{n+2} = \frac{(n+2)\,(n+3)}{2}. \tag{15}$$

By doing (15) − (14), we have:

$$\begin{aligned} t_{n+2}-t_{n+1} &= \frac{n^2+5n+6-n^2-3n-2}{2}\\ &= \frac{2n+4}{2}\\ &= n+2. \end{aligned}$$

By doing (14) − (13), we have:

$$\begin{aligned} t_{n+1}-t_n &= \frac{n^2+3n+2-n^2-n}{2}\\ \frac{2n+2}{2} &= n+1. \end{aligned}$$

Therefore we have that:

$$t_{n+2}-t_{n+1}=(t_{n+1}-t_n)+1. \tag{16}$$

Now that we have established what we mean by the distribution of triangular numbers, let's apply the same thing to odd numbers.

1. Influence of x in $T_{a0}(N)$ and $T_{a4}(N)$
We will be interested in the $T_a(N)$ which share the same values of a. Using the fact that $x < \frac{b-1}{2}$ by definition (cfr. (16)) and the fact that x is even, we can then introduce 3 numbers taking the first $x = \frac{b-1}{2} - 2u - 1,\ \forall u \in \mathbb{N}$:

$$T_{1x}(N) = \left(b - \frac{b-3}{2} + 2u\right) \times \left(\frac{b-3}{2} - 2u + 1\right), \tag{17}$$

$$T_{2x}(N) = \left(b - \frac{b-3}{2} + 2u + 2\right) \times \left(\frac{b-3}{2} - 2u - 2 + 1\right), \tag{18}$$

$$T_{3x}(N) = \left(b - \frac{b-3}{2} + 2u + 4\right) \times \left(\frac{b-3}{2} - 2u - 4 + 1\right). \tag{19}$$

- Let's first rewrite $T_{2x}(N)$ in order to highlight the elements that resemble those of $T_{1x}(N)$:

$$\begin{aligned} T_{2x}(N) = &\left(b - \frac{b-3}{2} + 2u\right)\left(\frac{b-3}{2} - 2u + 1\right) \\ &- 2\left(b - \frac{b-3}{2} + 2u\right) \\ &+ 2\left(\frac{b-3}{2} - 2u - 1\right) \end{aligned} \tag{20}$$

We recognize in the first term $T_{1x}(N)$, and so:

$$\begin{aligned} T_{2x}(N) &= T_{1x}(N) - 2\left(b - \frac{b-3}{2} + 2u\right) \\ &\quad + 2\left(\frac{b-3}{2} - 2u - 1\right) \\ &= T_{1x}(N) - 2\left(b - \frac{b-3}{2} + 2u\right) \\ &\quad + 2\left(\frac{b-3}{2} - 2u\right) - 2 \\ &= T_{1x}(N) + 2\left(\frac{b-3}{2} - 2u\right) \\ &\quad - 2b + 2\left(\frac{b-3}{2} - 2u\right) - 2 \\ &= T_{1x}(N) + b - 3 - 4u - 2b + b \\ &\quad - 3 - 4u - 2 \\ &= T_{1x}(N) - 8 - 8u. \end{aligned} \tag{21}$$

so that:

$$\frac{T_{1x}(N) - T_{2x}(N)}{8} = u + 1. \tag{22}$$

- Let's arrange equation (19) in the same way as a function of $T_{1x}(N)$:

$$\begin{aligned} T_{3x}(N) = &\left(b - \frac{b-3}{2} + 2u\right)\left(\frac{b-3}{2} - 2u + 1\right) \\ &- 4\left(b - \frac{b-3}{2} + 2u\right) \\ &+ 4\left(\frac{b-3}{2} - 2u - 3\right) \\ = &\, T_{1x}(N) - 4\left(b - \frac{b-3}{2} + 2u\right) \\ &+ 4\left(\frac{b-3}{2} - 2u - 3\right) \\ = &\, T_{1x}(N) - 4\left(b - \frac{b-3}{2} + 2u\right) \\ &+ 4\left(\frac{b-3}{2} - 2u\right) - 12 \\ = &\, T_{1x}(N) + 4\left(\frac{b-3}{2} - 2u\right) - 4b \\ &+ 4\left(\frac{b-3}{2} - 2u\right) - 12 \\ = &\, T_{1x}(N) + 2b - 6 - 8u - 4b + 2b \\ &- 6 - 8u - 12 \\ = &\, T_{1x}(N) - 24 - 16u. \end{aligned} \tag{23}$$

so that:

$$\begin{aligned} T_{2x}(N) - T_{3x}(N) &= T_{1x}(N) - 8 - 8u - T_{1x}(N) \\ &\quad + 24 + 16u \\ &= 8u + 16 \end{aligned} \tag{24}$$

or

$$\frac{T_{2x}(N) - T_{3x}(N)}{8} = u + 2. \tag{25}$$

Furthermore, it will be obtained that:

$$\text{eq.}(25) = \text{eq.}(22) + 1. \tag{26}$$

2. Influence of a in $T_{x0}(N)$ and $T_{x4}(N)$
We will be interested in the $T_x(N)$ which share the same values of x. We therefore introduce 3 numbers as previously, taking the first $a = 2u + 1,\ \forall u \in \mathbb{N}$:

$$T_{1a}(N) = (2u+1)^2, \tag{27}$$
$$T_{2a}(N) = (2u+1+2)^2, \tag{28}$$
$$T_{3a}(N) = (2u+1+4)^2. \tag{29}$$

By doing (28) − (27), we have:

$$\begin{aligned} T_{2a}(N) - T_{1a}(N) &= 4u^2 + 12u + 9 - 4u^2 - 4u - 1 \\ \frac{T_{2a}(N) - T_{1a}(N)}{8} &= u + 1. \end{aligned} \tag{30}$$

By doing (29) − (28), we have:

$$\begin{aligned} T_{3a}(N) - T_{2a}(N) &= 4u^2 + 20u + 25 - 4u^2 - 12u - 9 \\ \frac{T_{3a}(N) - T_{2a}(N)}{8} &= u + 2. \end{aligned} \tag{31}$$

We also know that:

$$\text{eq.}(31) = \text{eq.}(30) + 1. \tag{32}$$

Using the multiplicative number of 8, the variation is the same in the 2 cases (eq. (26) et eq. (32)) as in eq. (16). Odd numbers, therefore, have a structure similar to that of triangular numbers. This also shows that numbers sharing a common property (such as having the same sum or the difference of factors) will be aligned on the same line. This structure is just the same as the one we want to prove for the zeros of the function $\zeta(s)$. □

To do without the factor 8, we thus introduce the parameter kikua k such that, for an element $e = \{0, 2, 4, 6\}$, we have:

$$k = \frac{T(N) - b - e}{8}.$$

This is not the only way to define k. k is quite malleable and is not fixed; it is therefore not a number in the sense in which we understand this word. This aspect is likely to be exploited in cryptography. In general we therefore have:

$$k = \frac{T(N) - b - e}{8} \pm C^{ste}.$$

We then name "ben", denoted $b|e$, the type to which k belongs. The set of 4 $b|e$ is called "ben ben". Here is a visual example for $b|0$: with $B = 256$. We have the figure 1:

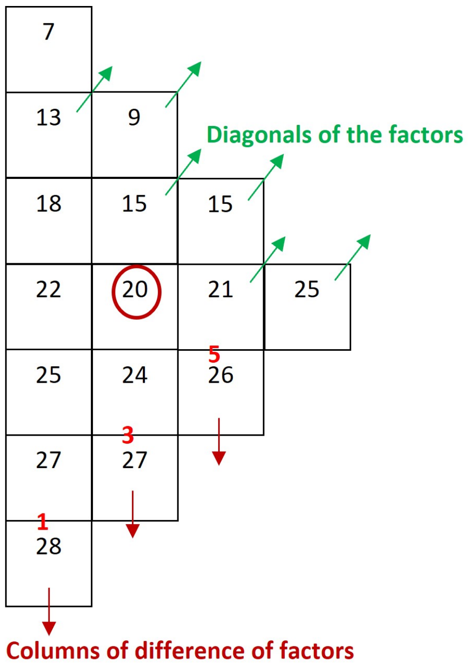

Figure 1. $b|0$ represented at the bound $B = 256$

Consider $N = 65$. b will therefore be worth: $b = 2\sqrt{256} - 1 = 31$. k will therefore be worth: $k = \frac{256-65-31}{8} = \frac{160}{8} = 20$. We see in Figure 1 that we have $k = 20$ on the diagonal of factor 5. We also have $\frac{D}{4} = \frac{13-5}{4} = 2$, i.e. the distance between the last 2 in the column (24 and 27) reduced by 1. We can make the same observations with any k of $b|0$.

An interesting aspect of the construction is the appearance of the following property when k is 0, that is to say when k is subtracted from all the other k_i of $b|0$, as we can see in figure 2:

$$\frac{65-1}{4} = 2 \times 7 + 3 - 1.$$

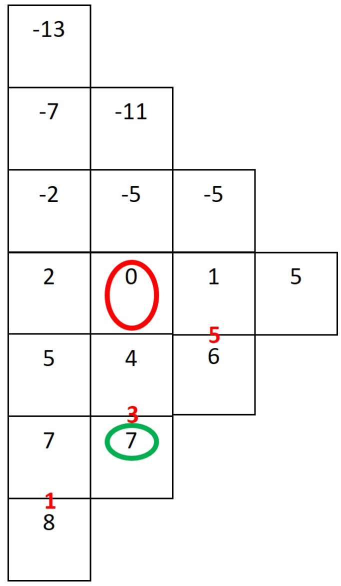

Figure 2. $b|0$ and k subtracted from other k_i

This value of 7 is given by the factors of 65:

$$7 = \frac{(5-1) \times (13+1)}{8}$$

i.e.:

$$\frac{65-1}{4} = \frac{(5-1) \times (13+1)}{4} + \frac{13-5}{4}.$$

What can still be written in general form:

$$\frac{N-1}{4} = \frac{(p-1) \times (q+1)}{4} + \frac{D}{4}$$

and by setting $\frac{(p-1)\times(q+1)}{4} = 2x'_0$, we have the following more condensed form:

$$\frac{N-1}{4} = 2x'_0 + \frac{a}{2}.$$

However we find the same value by changing the terms of the right hand side, by taking the elements of the right column:

$$\frac{65-1}{4} = 2 \times 6 + 5 - 1$$

or even those of the left column:

$$\frac{65-1}{4} = 2 \times 8 + 1 - 1.$$

By introducing an integer r, we have for the right columns:

$$\frac{N-1}{4} = 2x_0' - 2r + \frac{a}{2} + 2r$$

and for the left columns:

$$\frac{N-1}{4} = 2x_0' + 2r + \frac{a}{2} - 2r.$$

Therefore, in all generality, we cannot determine with precision the exact column of the right-hand side, there exists a probability attached to the exact position of a number in $b|0$. This is the source of the difficulty we encounter when trying to factor integers. In doing so, we also cannot distinguish the columns on the left from those on the right. The position of the numbers is completely random. This is starting to sound like quantum mechanics. This conclusion applies to all numbers of $b|0$ and of $b|4$.

The relation for $b|2$ and $b|6$ is slightly different:

$$\frac{N+1}{4} = 2x_0 \pm 2r + \frac{S}{4} \mp 2r$$

with $2x_0 = \frac{(p-1)(q-1)}{4}$ and S the sum of the factors. All the elements which are attached to a or to S are random and this leads us to state the following proposition.

Proposition 4. *Let 1 to N numbers such that we associate with each the sum S of its factors, then since the S are random, the N numbers are also necessarily random.*

Proof. This goes without saying, since the N determine the S and reciprocally the S determine the N, if the seconds are random then the first are necessarily random too, otherwise we could, from the N found, have a way of predicting the seconds, which would enter in conflict with the randomness of seconds. Understanding that to calculate $M(N)$ we must be able to factor all the integers from 1 to N, which is similar, as shown, to finding their random distribution. Manipulating $M(N)$ cannot do without dealing with randomness.

Taking numbers from 1 to N is therefore the same as N random numbers. Hausdorff's theorem then applies to N numbers and therefore Mertens weak conjecture is confirmed. So, we have:

$$M(N) = o\left(N^{\frac{1}{2}+\varepsilon}\right).$$

Therefore, the Riemann hypothesis is demonstrated. □

4. Conclusion

We addressed the problem of the 3 conjectures of Mertens, which will likely lead us to the demonstration of the Riemann hypothesis. In view of the new elements in our possession, namely a new way of approaching the factorization of numbers coupled with the hidden structure of odd numbers, we judged that Merten's weak conjecture could lead us to this demonstration. We then established a model to achieve this demonstration based on the hidden structure of odd numbers. We discovered that random behavior emerged under the regularity of these numbers, which allows us to place ourselves under the criterion of Hausdorff's theorem. This, therefore, allowed us to arrive at the result that:

$$M(N) = o\left(N^{\frac{1}{2}+\varepsilon}\right).$$

So, based on equation (3), since the integral converges for $Re(s) > \frac{1}{2}$ this implies that $\frac{1}{\zeta}$ is defined for $Re(s) > \frac{1}{2}$ and therefore by symmetry for $Re(s) < \frac{1}{2}$. Thus the only non-trivial zeros of ζ satisfy $Re\,(s) = \frac{1}{2}$, which is the statement of the Riemann hypothesis. The Riemann hypothesis is over. It is over to say that Riemann was right. It is also over to say that generations of mathematicians, each more talented than the other, have not been able to solve it come from the randomness attached to the numbers themselves.

This demonstration has implications for the distribution of prime numbers, in particular on the estimation of the error in the prime counting function $\pi(n)$:

$$\pi\,(n) = Li\,(n) + O\left(\sqrt{n}\ln n\right).$$

where n is the number for which we calculate the numbers of prime numbers less than or equal to n, Li is the offset logarithmic integral function defined by $Li\,(n) = \int_2^n \frac{dt}{\mathrm{log}t}$.

The Riemann hypothesis was the most important unsolved problem in number theory. It aroused the greatest hopes, given the innumerable consequences that would result from it if it proved correct. We, in turn, bring consequences that will change mathematics in depth. So, following this article, here are several applications and uses that we will develop as a direct consequence of this result:

1. In mathematics, there is a new way to solve differential equations, Hilbert space, ABC conjecture, Navier-Stokes equations, random matrices, etc.
2. In cryptography, information-theoretically secure, post-quantum security systems are developed and deployed.
3. In computer science, it is solving the halting problem probabilistically and designing a quantum computer free from issues related to decoherence.
4. Development of random number generators.
5. In quantum mechanics, the problem of measurement is solved based on the analogy between integers and subatomic particles.
6. Establishment of ways to solve the shortest vector problem (SVP) and thus lead to the resolution of the problem $P = NP$?, etc.

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