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Gustina Saputri, Netti Herawati, Tiryono Ruby, and Khoirin Nisa

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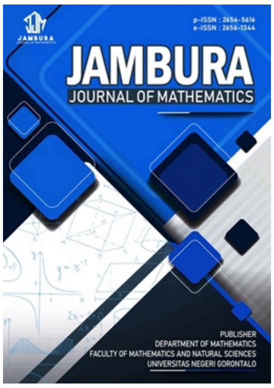

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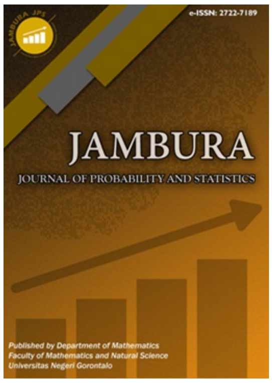

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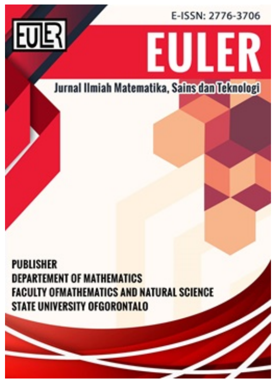

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Research Article

Comparative Study in Controlling Outliers and Multicollinearity Using Robust Performance Jackknife Ridge Regression Estimator Based on Generalized-M and Least Trimmed Square Estimator

Gustina Saputri[1], **Netti Herawati**[1,*], **Tiryono Ruby**[1], **and Khoirin Nisa**[1]

[1]*Dapartemen of Mathematics, University of Lampung, Bandar Lampung, Indonesia*

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ABSTRACT. *Regression analysis is one of the statistical methods used to determine the causal relationship between one or more explanatory variables to the affected variable. The problem that often occurs in regression analysis is that there are multicollonity and outliers. To deal with such problems can be solved using ridge regression analysis and robust regression. Ridge regression can solve the problem of multicollinearas by assigning a constant k to the matrix $\boldsymbol{Z}'\boldsymbol{Z}$. But in this method the resulting bias value is still high, so to overcome this problem, the jackknife ridge regression method is used. Meanwhile, to overcome outliers in the data using robust regression methods which have several estimation methods, two of which are the Generalized-M (GM) estimator and the Least Trimmed Square (LTS) estimator. The aim of the study is to solve the problem of multicollinearity and outliers simultaneously using robust jackknife ridge regression method with GM estimators and LTS estimators. The results showed that the robust ridge jackknife regression method with LTS estimator can control multicollinearity and outliers simultaneously better based on MSE, AIC and BIC values compared to the robust ridge jackknife regression method with GM estimators. This is indicated by the value MSE = -6.60371, AIC = 75.823 and BIC = 81.642 on LTS estimators that are of lower value than GM estimators.*

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1. Introduction

To look for relationships between one or more explanatory variables to response variables, regression analysis is one of the most frequently used statistical method. This method is very susceptible to non-fulfillment of the assumption of multicollinearity and the existence of outliers. The presence of outliers and multicollinearity can cause the confident interval value to be less precise [1]. One method that can overcome outliers is robust regression [2]. In robust regression there are several estimators that can be used in overcoming outliers. Generalized M- estimator that bounds outliers in variable xi using a weight function that depends on xi was studied by [3].

In addition, research on overcoming multicollinearity in regression models has been carried out by many researchers using various methods. For example, the study of PCR, LASSO and RR which has been done by [4] shows that RR estimation is better than OLS in overcoming multicollinearity. Another recommended method to overcome multicollinearity is jackknife ridge regression which is a development of the ridge regression method. This method has been shown to reduce the bias that may arise from the resulting model to overcome multicollinearity [5].

When outliers and multicollinearity occur simultaneously in data, robust regression or jackknife ridge regression cannot be used separately [6]. Both methods must be combined to handle the problem simultaneously [7–10]. Furthermore, the research of Robust MM- estimators is more significant that the Robust M-estimators and Robust S-estimators which was shown by [11]. However, comprehensive research in controlling outliers and multicollinearity has not been carried out thoroughly, especially using the Robust Jackknife Ridge regression method. Therefore, in this study the Robust Jackknife Ridge GM regression estimator method and the Robust Jackknife Ridge LTS Regression estimator method were investigated and compared using low birth weight data in South Sulawesi Province based on MSE, AIC and BIC values.

2. Methods

To compare both the GM estimator and the LTS estimator on the robust jackknife ridge regression estimator used low birth weight data (Y) in South Sulawesi Province using 10 independent variables with 24 observations from dinkes-PROFIL_2020_FINISH1.pdf (sulselprov.go.id). The variables consist of pregnant women with K4 immunization (X_1), pregnant women with K1 immunization (X_2), pregnant women who get blood-added tablets (X_3), pregnant women with complications (X_4), proper sanitation (X_5), human development index (X_6), percentage of poor people (X_7), average length of schooling (X_8), population density (X_9) and expected length of schooling (X_{10}).

*Corresponding Author.

Email : *netti.herawati@fmipa.unila.ac.id* (N. Herawati)
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Table 1. Descriptive Analysis

Variable	Y	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Mean	264.70	83.946	93.967	87.507	79.229	97.736	72.206	9.4183	8.1479	640.95	13.251
Min	65	53.67	69.38	45.90	30.60	71.28	67.22	4.54	6.59	43	11.98
Max	888	98.43	107.22	98.80	131.4	100	83.58	14.58	11.21	8101	15.57
SD	200.11	11.18	9.174	13.97	20.39	5.994	3.805	2.918	1.176	1618.4	0.8539

First, conducted a descriptive analysis on each variable used in the study. Then, evaluated the VIF value to determine the presence or absence of multicollinearity and identify outliers with box plot. Next was to conduct analysis with GM estimators, transformation centering and re scaling are carried out, calculate the estimated value of the GM estimator regression coefficient, calculate value k_{GM}, calculate regression coefficient estimator values using ridge jackknife robust regression method $(\widehat{\alpha}_{RJRR})$ for GM estimators. In order to perform LTS estimator analysis, transformation centering and re scaling are carried out, calculate the estimated value of the LTS estimator regression coefficient, calculate value k_{LTS}, calculate regression coefficient estimator values using ridge jackknife robust regression method $(\widehat{\alpha}_{RJRR})$ for LTS estimators. To see the best model by comparing the results of the robust jackknife ridge regression coefficient estimator using the GM estimator and the robust jackknife ridge regression using the LTS estimator by looking at the MSE, AIC, and BIC values.

2.1. Generalized-M Estimator

GM estimators can overcome outlier and multicollinearity problems in regression data both in Variable X and in Variable Y [12, 13]. GM estimators are generally solutions of normal equations:

$$\sum_{i=1}^{n} x_{ij}\pi_i\psi\left(\frac{y_i-\sum_{j=1}^{p}x_{ij}\alpha_j}{\sigma}\right)=0 \quad ,j=1,\ 2\ ,\ldots,\ k. \quad (1)$$

Exist π_i which aims to determine the weight of observations that have a high leverage point value using Schweppe weighting with $\pi_i = \sqrt{1-h_{ii}}$. If the weighting function of GM estimates $w_i = \dfrac{\psi(u_i/\pi_i)}{u_i/\pi_i}$ which has a value between 0 and 1 with $u_i = \dfrac{\varepsilon_i}{\sigma}$, then obtained:

$$w_i = \frac{\psi\left(\frac{u_i}{\pi_i}\right)}{u_i/\pi_i}$$

$$w_i = \frac{\psi\left(\frac{y_i-\sum_{j=1}^{p}x_{ij}\alpha_j}{\sigma\pi_i}\right)}{\frac{y_i-\sum_{j=1}^{p}x_{ij}\alpha_j}{\sigma\pi_i}}$$

$$w_i\left(\frac{y_i-\sum_{j=1}^{p}x_{ij}\alpha_j}{\sigma}\right)=\pi_i\psi\left(\frac{y_i-\sum_{j=1}^{p}x_{ij}\alpha_j}{\sigma\pi_i}\right)$$

with $\sigma = 1,4825\ MAD$, $\psi(x) = \max\{-K, \min(K,x)\}$ is a function of Huber's influence with $K = 2\sqrt{(k+1)/n}$, so that the equation will be:

$$\sum_{i=1}^{n} x_{ij}w_i\left(y_i-\sum_{j=1}^{p}x_{ij}\alpha_j\right)=0\ ,\ j=0,\ 1,\ 2,\ \ldots,\ k. \quad (2)$$

Furthermore, this equation can be solved by an iterate-weighted least squares method called Iteratively Reweighted Least Square (IRLS) until obtaining $\widehat{\alpha}$ which converges.

2.2. Least Trimmed Square (LTS) Estimator

Least Trimmed Square (LTS) is one method of estimating the parameters of the robust regression model against outliers in data [14]. LTS is a method of estimating robust regression parameters that have a high breakdown point value by minimizing the number of residual h squares [15]. The value of h is at the interval $\dfrac{n}{2}+1 \le h \le \dfrac{3n+p+1}{4}$. The h value will result in a breakdown point of 50%. A breakdown point is the smallest fraction or percentage of outliers that causes the estimator value to be large [11, 15]. If the breakdown point value is more than 50%, then the estimation from the regression model cannot provide information from the majority of the data. The estimation procedure with the Least Trimmed Square (LTS) method is as follows:

1. Calculate the estimated coefficient of regression parameters $\widehat{\alpha}_0$ using the least squares method.
2. Calculate the residual square with $\varepsilon_i^2 = (y_i-\hat{y}_i)^2$.
3. Sort ε_i^2 from smallest value to largest.
4. Calculate values $h_0 = \dfrac{n+p+1}{2}$.
5. Count ε_i^2 that has been sorted as many as h_0 or $\sum_{i=1}^{h_0}\varepsilon_i^2$.
6. Calculate the estimated parameter coefficient $\widehat{\alpha}_1$ from h_0 observation using the least squares method.
7. Calculate residual squares $\varepsilon_i^2 = (y_i-\hat{y}_i)^2$ corresponding to $\widehat{\alpha}_1$, next sort from smallest value to largest.
8. Counting amounts h_1 observation with value ε_i^2 smallest.
9. Counting $\sum_{i=1}^{h_1}\varepsilon_i^2$.
10. Iterate from stages 6 to 9 until obtaining a convergent parameter coefficient estimation value.

3. Results and Discussion

The first step in applying the Jackknife Ridge regression method with the Generic M-estimator and LTS estimator is to carry out descriptive analysis of the data. This is done to get a clearer picture of the form of data used. The following are the results of a descriptive analysis of the low birth weight data shown in Table 1.

Table 1 shows that low birth weight (Y) has an average of 264.70 with a standard deviation of 200.11, a minimum value of 65 in Tana Toraja Regency, and a maximum value of 888 in Bulukumba Regency. This indicates that low birth weight cases in East Java Province in 2022 are quite high in Bulukumba Regency, with 888 affected infants. Next, multicollinearity in the data is evaluated by calculating the VIF for each independent variable. The results of this analysis are presented in Table 2.

Table 2. VIF value

Variable	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
VIF	7.0815	6.2104	2.1038	1.3005	1.4182	12.5951	2.63333	16.0598	2.1924	7.0059

Table 3. GM estimator iteration

Iteration	α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}
MKT	386.793	-4.819	-5.085	5.170	-0.096	-4.497	17.77	-23.07	-189.13	0.07428	91.1673
1	-554.739	0.541	-5.255	2.523	0.054	-1.952	27.92	-12.09	-179.15	0.07157	56.1434
2	-800.092	2.767	-4.835	0.759	0.303	-1.052	30.86	-8.202	-168.53	0.07051	34.9883
3	-792.192	3.732	-4.850	0.150	0.447	-0.723	31.07	-6.624	-156.32	0.06946	19.0762
4	-784.500	3.9420	-4.811	-0.004	0.544	-0.564	31.32	-5.921	-150.20	0.06736	10.5400
5	-735.496	4.0469	-4.838	-0.026	0.584	-0.489	30.96	-5.659	-143.85	0.06430	3.70438
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
78	-458.41875	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7652
79	-458.41874	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654
80	-458.41872	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654
81	-458.41871	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654
82	-458.41871	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654
83	-458.41871	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654
84	-458.41871	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654
85	-458.41871	4.3399	-4.938	-0.062	0.664	-0.813	29.09	-5.331	-116.93	0.0193	-21.7654

From Table 2, it can be seen that the value of VIF in variables X_6 and X_8 is more than 10, meaning that it shows that there is multicollinearity. The following step is to identify whether there are outliers in the data by looking at the leverage value used. If the leverage value is greater than $\frac{2m}{n}$ where m is the number of independent variables plus a constant and n is the number of observations, then it can be said that there are outliers. The results are shown in Figure 1.

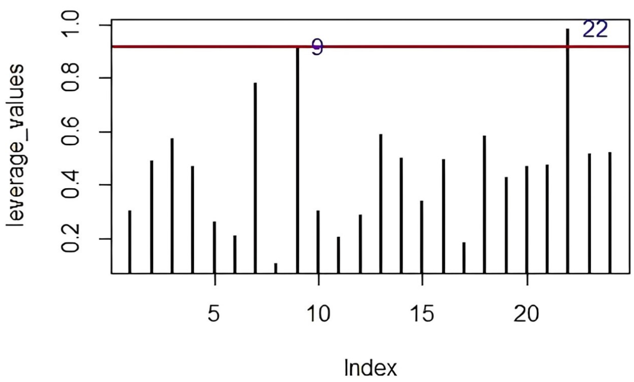

Figure 1. Leverage value chart

In Figure 1, it is observed that the 9th and 22nd observations have values greater than $\frac{2m}{n}$, indicating outliers. Additionally, the box plot method is employed to identify outliers, and the analysis results are shown in Figure 2.

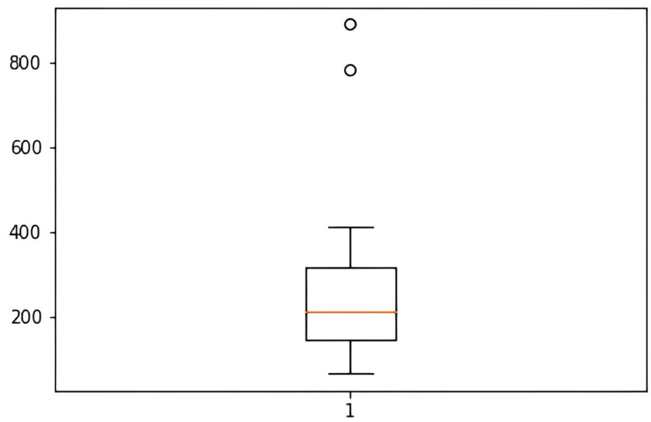

Figure 2. Low Birth Weight data box plot in South Sulawesi Province in 2020

In Figure 2, it is evident that the 2nd and 22nd data points exceed the maximum limit, indicating the presence of outliers. Therefore, both Figure 1 and Figure 2 confirm that the low birth weight data in South Sulawesi Province in 2020 contains outliers. To perform robust jackknife ridge regression using the GM estimator begins with centering and calling the data. This process obtained a new value used for the analysis with the GM estimator. The next step was to iterate until a convergent value was obtained. The results of the iteration can be seen in Table 3.

Table 3 shows that Iteration 82 is the last iteration, because the value of iteration 82 is the same as the next iteration which has converged. There fore the final estimate of the strong regression with the GM estimator is:

$$\hat{Y} = -458.41871 + 4.3399X_1 - 4.938X_2 - 0.062X_3 + 0.664X_4 - 0.813X_5 + 29.09X_6 - 5.331X_7 - 116.93X_8 + 0.0193\ X_9 - 21.7654X_{10}.$$

Then, robust jackknife ridge regression analysis with the LTS estimator was performed. Just like estimating using robust jackknife ridge regression with the GM estimator, the first thing to do was to centering and recalling the data. Until a new value is obtained to used for the analysis process with the LTS estimator. The iteration process was carried out until the α value converges. The iteration results are in Table 4.

Table 4. LTS estimator iteration

Iteration	MKT	1	2	3	4	5	6	7	8
h_i	24	18	15	13	12	12	12	12	12
α_0	386.793	-625.350	-287.024	-614.585	-812.300	-812.300	-812.300	-812.300	-812.300
α_1	-4.819	8.574	7.561	8.019	8.703	8.703	8.703	8.703	8.703
α_2	-5.085	-3.960	-3.572	-3.480	-4.293	-4.293	-4.293	-4.293	-4.293
α_3	5.170	-4.297	-1.243	-1.994	-1.663	-1.663	-1.663	-1.663	-1.663
α_4	-0.096	0.654	0.679	0.955	1.146	1.146	1.146	1.146	1.146
α_5	-4.497	-0.351	0.150	-0.135	0.424	0.424	0.424	0.424	0.424
α_6	17.775	28.767	22.133	26.249	28.420	28.420	28.420	28.420	28.420
α_7	-23.071	-5.663	-6.211	-3.956	-2.000	-2.000	-2.000	-2.000	-2.000
α_8	-189.130	-134.029	-98.816	-110.321	-111.900	-111.900	-111.900	-111.900	-111.900
α_9	0.074	0.076	0.0784	0.076	0.074	0.074	0.074	0.074	0.074
α_{10}	91.167	-8.690	-38.793	-29.864	-32.840	-32.840	-32.840	-32.840	-32.840
ε^2_{LTS}	0.410000	0.039000	0.005480	0.001510	0.000108	0.000108	0.000108	0.000108	0.000108

The results in Table 4 show that the 5th iteration with $hi = 12$ is the last iteration because the value of the iteration is the same as the value of the next iteration which has converged. It can be said that the 5^{th} iteration is the last iteration and the final estimate of robust regression with the LTS estimator is:

$$\begin{aligned}\hat{Y} = & -812.300 + 8.7030X_1 - 4.2930X_2 - 1.6630X_3 \\ & + 1.1460X_4 + 0.4249X_5 + 28.420X_6 - 2.0000X_7 \\ & - 111.900X_8 0.0744 + X_9 - 32.8400X_{10}.\end{aligned}$$

Next, we will compare which estimator is good for overcoming multicollinearity and outliers based on MSE, AIC, and BIC values. The results can be seen in Table 5.

Table 5. Comparison of model feasibility values

Statistics	MSE	AIC	BIC
Robust Jackknife Ridge Regression GM estimator	50810.11	337.45	351.59
Robust Jackknife Ridge Regression LTS estimator	-6.60371	75.823	81.642

Table 5 shows that the robust jackknife ridge regression model with LTS estimator produces smaller MSE, AIC, and BIC values compared to the robust jackknife ridge regression model with GM estimator. This indicates that robust jackknife ridge regression with LTS estimator can solve multicollinearity and outlier problems better than robust jackknife ridge regression with GM estimator.

4. Conclusion

Based on the results comparing the robust jackknife ridge regression with estimator GM and with LTS estimators, it can be concluded that the regression of the robust ridge ridge using both of estimators can overcome the problem of multicollinearity and outliers simultaneously. In addition, the comparison results of the two estimators show that the LTS estimator is better than the GM estimator because it has $MSE = -6.60371$, $AIC = 75.823$ and $BIC = 81.642$ where these values are smaller than the MSE, AIC and BIC values of the GM estimator.

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