RESEARCH ARTICLE • OPEN ACCESS

Comparison of Fuzzy Grey Markov Model (1,1) and Fuzzy Grey Markov Model (2,1) in Forecasting Gold Prices in Indonesia

Arthamevia Najwa Soraya, Firdaniza, and Kankan Parmikanti

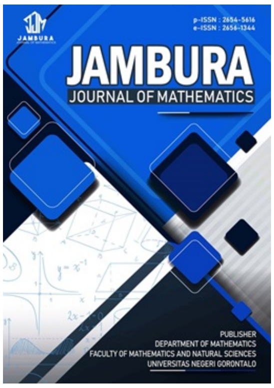
JAMBURA
JOURNAL OF MATHEMATICS

Volume 6, Issue 2, Pages 236–242, August 2024

Received 18 June 2024, Revised 30 July 2024, Accepted 3 August 2024, Published 6 August 2024

To Cite this Article : A. N. Soraya, F. Firdaniza, and K. Parmikanti,“Comparison of Fuzzy Grey Markov Model (1,1) and Fuzzy Grey Markov Model (2,1) in Forecasting Gold Prices in Indonesia”, *Jambura J. Math*, vol. 6, no. 2, pp. 236–242, 2024, *https://doi.org/10.37905/jjom.v6i2.26679*

JOURNAL INFO • JAMBURA JOURNAL OF MATHEMATICS



Homepage	:	http://ejurnal.ung.ac.id/index.php/jjom/index
Journal Abbreviation	:	Jambura J. Math.
Frequency	:	Biannual (February and August)
Publication Language	:	English (preferable), Indonesia
DOI	:	https://doi.org/10.37905/jjom
Online ISSN	:	2656-1344
Editor-in-Chief	:	Hasan S. Panigoro
Publisher	:	Department of Mathematics, Universitas Negeri Gorontalo
Country	:	Indonesia
OAI Address	:	http://ejurnal.ung.ac.id/index.php/jjom/oai
Google Scholar ID	:	iWLjgaUAAAAJ
Email	:	info.jjom@ung.ac.id

Jambura Journal of Mathematics, Volume 6, Issue 2, Pages 236–242, August 2024 https://doi.org/10.37905/jjom.v6i2.26679

Research Article

Check for updates

Comparison of Fuzzy Grey Markov Model (1,1) and Fuzzy Grey Markov Model (2,1) in Forecasting Gold Prices in Indonesia

Arthamevia Najwa Soraya[1,*], **Firdaniza**[1], and **Kankan Parmikanti**[1]

[1]*Department of Mathematics, Universitas Padjadjaran, Bandung, Indonesia*

ARTICLE HISTORY
Received 18 June 2024
Revised 30 July 2024
Accepted 3 August 2024
Published 6 August 2024

KEYWORDS
Forecasting
Gold Price
Fuzzy Grey Markov Model (1,1)
Fuzzy Grey Markov Model (2,1)

ABSTRACT. *Currently, gold investment is considered promising despite the ever-changing price of gold. However, obtaining optimal profits is a challenge for investors. Therefore, a proper forecasting method is needed to forecast the gold price so investors can know the best transaction time. This study used two forecasting methods: the Fuzzy Grey Markov Model (1,1) and a new, never-before-used approach, the Fuzzy Grey Markov Model (2,1). The Fuzzy Grey Markov Model (2,1) approach is interesting because it can be considered for forecast data that shows varying increases and decreases, such as the gold price data used in this study. Both methods are combined models that utilize fuzzy logic to handle uncertainty in data; the Grey model forms a forecasting model, and the Markov chain determines the state transition probability matrix. Next, the error rates of the two methods are compared based on the Mean Absolute Percentage Error (MAPE) value to obtain the best forecasting method. As a result of this study, the Fuzzy Grey Markov Model (1,1) was chosen as the best forecasting method with a MAPE value of 0.28%.*

1. Introduction

One activity that illustrates the importance of the economy in a country is investment [1]. Currently, gold investment is considered attractive because gold prices tend to experience gradual increases in the long term [2], especially during a severe financial crisis when the value of many assets fell sharply, but the price of gold increased [3]. This condition encourages many individuals to choose gold as an investment vehicle. Also, easy access to gold shops across various regions facilitates gold investment. These gold investors want optimal profits by getting a low price when buying and a high price when selling. However, in reality, the constantly changing price of gold brings light to investment activities [4], so forecasting is needed to know when an event will occur so that appropriate action can be taken to overcome it [5]. That way, investors can determine the right time to invest in gold based on future prices [6]. Gold price forecasting can be calculated using various forecasting methods, such as using double exponential smoothing [4], Conditional Heteroscedasticity-Mixed Data Sampling (GARCH-MIDAS) [7], as well as Autoregressive Integrated Moving Average (ARIMA) and Support Vector Machine (SVM) [8], and others. However, this method requires assumptions, such as following particular data patterns, which may only sometimes be suitable for all data types.

The Fuzzy Grey Markov Model (FGMM) is an alternative method that can be used to forecast gold prices without requiring assumptions about data patterns. This model is obtained by combining the Grey model, Markov chains, and fuzzy logic. Ju-Long introduced the Grey model in 1982 [9] as a forecasting model that has comprehensive coverage and achieves good prediction accuracy in dealing with small data samples and poor information, as well as handling the uncertainty that arises [10, 11]. The Grey model (MG(1,1)) with a first-order differential equation and one research variable is the simplest form of the Grey model for use in forecasting various fields, such as forecasting demand for electricity use [12], as well as forecasting the number of tourists [13]. In addition, Grey models (MG(2,1)) with second-order differential equations are used to handle more complex and dynamic data [14]. Markov chains are used to predict random data with large fluctuations and improve the forecasting accuracy of the Grey model [15, 16]. The combination of Grey models, both GM(1,1) and GM(2,1), with Markov Chains, is known as the Grey Markov Model (1,1) (GMM(1,1)) and Grey Markov Model (2,1) (GMM(2,1)). Then, fuzzy logic, first developed by Zadeh in 1965 [17] from fuzzy set theory, was used to handle the influence of random fluctuations and the weak anti-interference ability of Markov chains [18].

In FGMM, forecasting is done by adapting to the information in the data so that FGMM can be used in a broader variety of data. FGMM's integration of the Grey model, Markov chains, and fuzzy logic makes this model a forecasting model that can be considered for gold prices compared to traditional methods due to its ability to handle volatile and uncertain financial markets. This aspect is essential in forecasting gold prices, which are influenced by economic, social, political, and market sentiment. Therefore, FGMM was chosen with the hope that it can provide more accurate forecasting results and be more effective so that it can help gold investors obtain optimal profits from their investments

Some researchers have used the Fuzzy Grey Markov Model

*Corresponding Author.

Email : arthamevia20001@mail.unpad.ac.id (A. N. Soraya)
Homepage : http://ejurnal.ung.ac.id/index.php/jjom/index / E-ISSN : 2656-1344

based on GM(1,1) or FGMM(1,1) to help predict traffic volumes [19] and the real-time COVID-19 disease [20]. These studies show the superiority of using FGMM(1,1) in monotonic data compared to GM(1,1) and GMM(1,1), but when dealing with non-monotonic data, other approaches can be considered. Based on these studies, it is possible to carry out forecasting using a new approach that has never been used before, namely the Fuzzy Grey Markov Model based on GM(2,1) or FGMM(2,1). GM(2,1) makes FGMM(2,1) a forecasting method that can be used to handle data that shows varying increases and decreases so that it can be considered in describing non-monotonic data. Therefore, in this research, gold price forecasting in Indonesia was carried out using FGMM(1,1) and a new forecasting method that has never been used before, namely FGMM(2,1). By introducing FGMM(2,1), this research fills the gap by offering a method that is better suited for non-monotonic data, specifically used on gold price forecasting in Indonesia, with the expectation that FGMM(2,1) can provide better forecasting values than FGMM(1,1), considering the data used is non-monotonic. Next, the performance of the two methods is compared using Mean Absolute Percentage Error (MAPE) to choose the best forecasting method.

2. Methods

The objects of this study are FGMM(1,1) and FGMM(2,1), which are extensions of GM(1,1) and GM(2,1) by combining Markov chains and fuzzy logic into the forecasting process. Furthermore, these two methods will forecast the daily gold price data weighing one gram in Indonesia from February 1, 2024, to March 1, 2024. The data obtained from the official website of PT ANTAM Tbk Precious Metal Processing and Refining Business Unit (https://logammulia.com).

2.1. Fuzzy Grey Markov Model (1,1)

FGMM(1,1) is a forecasting model that combines the theory of GM(1,1) with fuzzy logic and Markov chains. FGMM(1,1) concept utilizes relative error values obtained from GM(1,1) forecasting results into interval classes. Next, a fuzzification process uses Markov chains to form a state transition probability matrix. It ends with a defuzzification process using the FGMM(1,1) method to obtain forecasting results. In FGMM(1,1), the forecast initiation uses the GM(1,1), which uses the coefficient of time variable to update the model to new data, known as Accumulated Generating Operation (AGO) . Theoretically, GM(1,1) helps describe data by a monotonic change process [21]. After obtaining the predicted value's relative error from GM(1,1), the next step is to apply the Markov chain and fuzzy logic concept into the calculation to obtain the forecast value of FGMM(1,1).

To construct the FGMM(1,1), we adhere to the following steps:

1. The first step is to perform GM(1,1) forecasting by forming a non-negative sequence of actual gold prices data, $X^{(0)}$, as follows:
$$X^{(0)} = \left(x^{(0)}(1),\ x^{(0)}(2),\ \cdots,\ x^{(0)}(n)\right), \tag{1}$$
where $x^{(0)}(k) \geq 0,\ k = 1, 2, \ldots, n$ and n specifies the amount of data used.
2. To avoid the vibration in gold prices data, form $X^{(1)}$ as One-time Accumulated Generating Operation (1-AGO), given by
$$X^{(1)} = \left(x^{(1)}(1),\ x^{(1)}(2),\ \cdots,\ x^{(1)}(n)\right), \tag{2}$$
where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)\,, k = 1, 2, \ldots, n.$
3. Let $Z^{(1)}$ be a Mean Generating Operation (MGO) sequence of the average value of two consecutive $X^{(1)}$ data,
$$Z^{(1)} = \left(z^{(1)}(2),\ z^{(1)}(3),\ \cdots,\ z^{(1)}(k)\right), \tag{3}$$
where $z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}$ and $k = 2, 3, \ \ldots, n.$
4. The GM(1,1) difference equation is
$$x^{(0)}(k) + az^{(1)}(k) = b, \tag{4}$$
where parameters $'a'$ is the developing coefficient and $'b'$ is the Grey input value. The values of parameters $'a'$ and $'b'$ can be calculated by the least square method, resulting in Equation (5) and Equation (6),
$$a = \frac{\frac{1}{n-1} S_0 \cdot S_1 - \sum_{k=2}^{n} x^{(0)}(k) \cdot z^{(1)}(k)}{\sum_{k=2}^{n} \left[z^{(1)}(k)\right]^2 - \frac{1}{n-1}\left[\sum_{k=2}^{n} z^{(1)}(k)\right]^2}, \tag{5}$$
$$b = \frac{1}{n-1}\left[S_0 + aS_1\right], \tag{6}$$
with
$$S_0 = \sum_{k=2}^{n} x^{(0)}(k) \text{ and } S_1 = \sum_{k=2}^{n} z^{(1)}(k).$$
5. The whitening differential equation of GM(1,1) is
$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b, \tag{7}$$
with the general solution of Equation (7) given by
$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a}\,,\ k = 1, 2, \ldots, n. \tag{8}$$
6. The actual data's forecasting gold prices can be calculated by applying an Inverse Accumulating Generator Operator (IAGO) from Equation (2), obtaining
$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)\,, \tag{9}$$
where $\hat{x}^{(0)}(1) = x^{(0)}(1)\,.$
7. The forecast value of GM(1,1) can be calculated by substituting Equation (8) into (9), obtaining
$$\hat{x}^{(0)}(k+1) = (1 - e^{a}) \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak}. \tag{10}$$
8. Calculate the relative error between the predicted value from GM(1,1) and the actual data by,
$$\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100. \tag{11}$$
9. Determine the Universe of Discourse of the predicted value's relative error obtained from Equation (11), by taking advantage of the minimum and maximum data values [22] becomes
$$U = \left[D_{min} - D_1,\ D_{max} + D_2\right], \tag{12}$$
where D_{min} is the minimum value data, D_{max} is the maximum, and D_1 & D_2 are arbitrary constants.

10. Determine the boundary of each state description by defining many classes of intervals (K) adjusted for the needs of observation data and determining the length of intervals (l) calculated by

$$l = \frac{[(D_{max} + D_2) - (D_{min} - D_1)]}{K}. \tag{13}$$

Then, the interval of each existing class stands by

$$u_i = [D_{min} - D_1 + (i-1)\, l\ ,\ D_{max} + D_2 + il]\,, \tag{14}$$

where $i = 1, 2, \dots, K$. The interval of each class can be written as $u_1 = [d_0, d_1]$, $u_2 = [d_1, d_2]$,..., $u_i = [d_{(i-1)}, d_i]$, where each interval represents a particular state description.

11. Construct the membership function for each fuzzy state. The membership function is a curve that maps data input points into fuzzy membership degree [23]. Utilizing the triangular method, the membership function is defined by

$$\mu_1(x) = \begin{cases} 1 & ;\quad d_0 \le x \le \frac{d_0+d_1}{2} \\ \frac{d_1+d_2-2x}{d_2-d_0} & ;\quad \frac{d_0+d_1}{2} \le x \le \frac{d_1+d_2}{2} \\ 0 & ;\quad \text{otherwise}, \end{cases} \tag{15}$$

$$\mu_i(x) = \begin{cases} \frac{2x-d_{i-2}-d_{i-1}}{d_i-d_{i-2}} & ;\quad \frac{d_{i-2}+d_{i-1}}{2} \le x \le \frac{d_{i-1}+d_i}{2} \\ \frac{d_{i+1}-d_{i-1}-2x}{d_{i+1}-d_{i-1}} & ;\quad \frac{d_{i-1}+d_i}{2} \le x \le \frac{d_i+d_{i+1}}{2} \\ 0 & ;\quad \text{otherwise}, \end{cases} \tag{16}$$

and

$$\mu_n\,(x) = \begin{cases} \frac{2x-d_{n-2}-d_{n-1}}{d_n-d_{n-2}} & ;\quad \frac{d_{n-2}+d_{n-1}}{2} \le x \le \frac{d_{n-1}+d_n}{2} \\ 1 & ;\quad \frac{d_{n-1}+d_n}{2} \le x \le d_n \\ 0 & ;\quad \text{otherwise}. \end{cases} \tag{17}$$

12. To define a fuzzy vector, calculate each fuzzy state's membership degree by substituting each state's relative error value to the corresponding member function. The fuzzy vector result for each data value is defined by Govindan et al. [19]:

$$F\,(\varepsilon(k)) = \mu_{A_1}\,(\varepsilon(k))\,, \mu_{A_2}\,(\varepsilon(k))\,, \dots, \mu_{A_n}\,(\varepsilon(k))\ , \tag{18}$$

where $\mu_{A_i}\,(\varepsilon(k))$ is the membership function of the relative error $(\varepsilon(k))$ of the fuzzy set A_i.

13. Use Markov chains to create a one-step state transition probability matrix for each fuzzy state transition. If $p_{ij}$is the transition probability from state i to state j [24], then the one-step transition probability matrix from state i to state j is defined as

$$\mathbf{P} = [p_{ij}] = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots & p_{0k} \\ p_{10} & p_{11} & p_{12} & \cdots & p_{1k} \\ p_{20} & p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k0} & p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}, \tag{19}$$

where $p_{ij} \ge 0$ dan $\sum_{j=0}^{\infty} p_{ij} = 1\ (i, j = 0, 1, 2, \dots)$.

14. Perform fuzzification, which maps crisp values to fuzzy value [25]. The fuzzified form for the next period is calculated by multiplying the fuzzy vector of the previous period with the one-step transition probability matrix in Equation (19), expressed as

$$\begin{aligned} F\,(\varepsilon\,(k+1)) &= F\,(\varepsilon\,(k)) \times \mathbf{P} \\ &= \{\mu_{A_1}\,(\varepsilon(k+1))\,, \mu_{A_2}\,(\varepsilon(k+1)\,, \dots, \\ &\quad \mu_{A_n}\,(\varepsilon(k+1))\}\,, \end{aligned} \tag{20}$$

where $\mu_{A_i}\,(\varepsilon\,(k+1))$ is the membership function of the relative error $(k+1)\ (\varepsilon(k+1))$ of the fuzzy set A_i.

15. Perform defuzzification to change fuzzy values back into crisp values [26] by calculating the crisp value as the sum of the average membership functions, with the weight being the membership degree of each existing fuzzy set [17],

$$\varepsilon\,(k+1) = \frac{1}{2}\left[\sum_{i=1}^{n} \mu_{A_i}\,(\varepsilon\,(k+1))\,(d_{i-1} + d_i)\right], \tag{21}$$

where d_{i-1} and d_i are the lower and upper limits of the interval class.

16. Lastly, the FGMM(1,1) forecast value of gold prices is given by Equation (22),

$$\hat{y}\,(k+1) = \frac{\hat{x}^{(0)}\,(k+1)}{1 - \varepsilon(k+1)}\ ,\ k = 1, 2, \dots, n. \tag{22}$$

2.2. Fuzzy Grey Markov Model (2,1)

FGMM(2,1) has almost the same concept as FGMM(1,1), but it utilizes relative error values obtained from GM(2,1). In FGMM(2,1), the forecast initiation uses the GM(2,1), which theoretically a derivative of GM(1,1). In the previous GM(1,1), 1-AGO was used to form new data in forecasting. Meanwhile, GM(2,1), a One-time Inverse Accumulated Generating Operation (1-IAGO), is used as input in the calculation. Theoretically, GM(2,1) helps describe data by a non-monotonic change process [21].

To construct the FGMM(2,1), we adhere to the following steps:

1. The first step is to perform GM(2,1) forecasting by using the actual gold price data sequence in Equation (1), the 1-AGO sequence in Equation (2), and the MGO sequence in Equation (3). Then, create a new data sequence from actual data, the 1-IAGO sequence:

$$\alpha^{(1)} X^{(0)} = \left(\alpha^{(1)} x^{(0)}\,(2)\,, \alpha^{(1)}\,x^{(0)}\,(3)\,,\ \cdots\ , \alpha^{(1)}\,x^{(0)}\,(n)\right), \tag{23}$$

where $\alpha^{(1)} x^{(0)}\,(k) = x^{(0)}\,(k) - x^{(0)}\,(k-1)$ and $k = 2, 3,\ \dots, n$.

2. The GM(2,1) difference equation is

$$\alpha^{(1)} x^{(0)}\,(k) + c_1 x^{(0)}\,(k) + c_2 z^{(1)}\,(k) = g, \tag{24}$$

where $'c_1'$ and $'c_2'$ are the developing coefficient, and $'g'$ the Grey input value. The values of parameters $'c_1'$, $'c_2'$ and $'g'$ can be calculated by the least square method, resulting in Equation (25), Equation (26) and Equation (27).

$$c_1 = \frac{C_1}{C_2} \tag{25}$$

$$c_2 = \frac{C_3}{C_4} \tag{26}$$

with

$$\begin{aligned}
C_1 &= ((n-1)\,S_6 - S_4 \cdot S_0)\,S_3 - S_1^2 \cdot S_6 \\
&\quad + S_4 \cdot S_1 \cdot S_5 + ((1-n)\,S_5 + S_0 \cdot S_1)\,S_7 \\
C_2 &= \left((1-n)\,S_2 + S_0^2\right) S_3 + S_1^2 \cdot S_2 \\
&\quad + ((n-1)\,S_5 - 2S_1 \cdot S_0)\,S_5 \\
C_3 &= ((n-1)\,S_6 - S_4 \cdot S_1)\,S_2 + ((1-n)\,S_5 + S_0 \cdot S_1)\,S_6 \\
&\quad + S_0\,(S_4 \cdot S_5 - S_0 S_7) \\
C_4 &= \left((1-n)\,S_3 + S_1^2\right) S_2 + S_0^2 \cdot S_3 \\
&\quad + ((n-1)\,S_5 - 2S_1 \cdot S_0)\,S_5 \\
S_0 &= \sum_{k=2}^{n} x^{(0)}(k),\; S_1 = \sum_{k=2}^{n} z^{(1)}(k), S_2 = \sum_{k=2}^{n} \left[x^{(0)}(k)\right]^2, \\
S_3 &= \sum_{k=2}^{n} \left[z^{(1)}(k)\right]^2, S_4 = \sum_{k=2}^{n} \alpha^{(1)} x^{(0)}(k), \\
S_5 &= \sum_{k=2}^{n} x^{(0)}(k) \cdot z^{(1)}(k), S_6 = \sum_{k=2}^{n} x^{(0)}(k) \cdot \alpha^{(1)} x^{(0)}(k), \\
S_7 &= \sum_{k=2}^{n} z^{(1)}(k) \cdot \alpha^{(1)} x^{(0)}(k),
\end{aligned}$$

and

$$g = \frac{1}{n-1}\left[S_4 + c_1 S_0 + c_2 S_1\right]. \tag{27}$$

3. The solution of the second-order differential equation in Equation (24) is given by

$$\hat{x}^{(1)}(k+1) = \overline{x}^{(1)}(k+1) + \frac{g}{c_2}, \tag{28}$$

where $\overline{x}^{(1)}(k+1)$ is a general solution of the homogeneous equation in Equation (24). The value of $\overline{x}^{(1)}(k+1)$ can be calculated using the characteristic function, namely $\lambda^2 + c_1\lambda + c_2 = 0$ with $\Delta = c_1^2 - 4c_2$.
There are three possible situations for determining the value $\overline{x}^{(1)}(k+1)$, such as:
   (a) If $\Delta > 0$, then the value of $\overline{x}^{(1)}(k+1)$ can be obtained using the formula

$$\overline{x}^{(1)}(k+1) = v_1 e^{\lambda_1 k} + v_2 e^{\lambda_2 k}, \tag{29}$$

   where $\lambda_1 = \frac{-c_1+\sqrt{c_1^2-4c_2}}{2}$ and $\lambda_2 = \frac{-c_1-\sqrt{c_1^2-4c_2}}{2}$.
   (b) If $\Delta = 0$, then the value of $\overline{x}^{(1)}(k+1)$ can be obtained using the formula

$$\overline{x}^{(1)}(k+1) = e^{\lambda k}(v_1 + v_2 k), \tag{30}$$

   where $\lambda_1 = \lambda_2 = \lambda$.
   (c) If $\Delta < 0$, then the value of $\overline{x}^{(1)}(k+1)$ can be obtained using the formula

$$\overline{x}^{(1)}(k+1) = e^{\gamma k}(v_1 cos(\beta k) + v_2 sin(\beta k)), \tag{31}$$

   where $\gamma = -\frac{c_1}{2}$ and $\beta = \frac{\sqrt{4c_2 - c_1^2}}{2}$.

4. The forecast value of GM(2,1) can be given using Equation (9) in GM(1,1), which includes solving the solution of the second-order differential equation, $\hat{x}^{(1)}(k+1)$, with Equation (28).
5. The next step is to use the Markov chain and fuzzy logic concepts in the calculation to obtain the forecast value of FGMM(2,1), following the algorithm previously discussed in FGMM(1,1).

2.3. *Forecasting Accuracy*

Forecasting attempts to predict future conditions by examining past conditions [1]. The forecasting carried out is expected to minimize the value of forecasting errors, one of which can be measured using MAPE. The average of the error values, which considers the influence of the actual value on the model, expresses MAPE [27], which can be calculated using the formula

$$\text{MAPE} = \frac{\sum_{t=1}^{n}\left|\frac{Y(t)-\hat{Y}(t)}{Y(t)}\right|}{n}, \tag{32}$$

where $Y(t)$ is the actual value and $\hat{Y}(t)$ is the forecast value. According to Lewis (1982) in Lawrence [27], there is a scale to judge the accuracy of a model based on the MAPE measure, explained as follows:

Table 1. Accuracy scale based on MAPE

MAPE	Level of accuracy
$\leq 10\%$	Highly accurate forecast
$10\% < \text{MAPE} \leq 20\%$	Good forecast
$20\% < \text{MAPE} \leq 50\%$	Reasonable forecast
$> 50\%$	Inaccurate forecast

3. Results and Discussion

This study uses daily gold price data in Indonesia as the forecasting object. Based on data from https://logammulia.com, the price of gold from February 1, 2024, to March 1, 2024, experienced fluctuations so that sometimes it did not follow a consistent pattern of increases or decreases in the data. Therefore, gold price data is categorized as non-monotonic data. Under these conditions, the FGMM(2,1) method, which uses GM(2,1) as the initial forecasting initiation, is a forecasting method that can be considered for handling non-monotonic gold data, apart from using FGMM(1,1), which has been used in previous research.

3.1. *Fuzzy Grey Markov Model (1,1) Forecasting*

The first step in predicting gold prices is to calculate the forecast values using GM(1,1) from Equation (1) to Equation (10). These results are shown in Table 2, which also shows the value of the relative error between the actual value and the predicted value calculated using Equation (11).

Then, the value of relative error is used to determine the universe of discourse, state deviation, and the boundary of each state description for the FGMM(1,1). By employing Equation (12)–(14), five states are identified within the range $U = [-1.8\,,\ 1.4]$, thereby presenting the value of each limit as displayed in Table 3.

Table 2. GM(1,1) forecasting values

Days	Actual Value	GM (1,1)	Relative Error
1	1,143,000	1,143,000	0
2	1,151,000	1,135,630	1.3353
⋮	⋮	⋮	⋮
29	1,138,000	1,130,096	0.6945
30	1,142,000	1,129,892	1.0603

Table 3. State description for FGMM(1,1)

Fuzzy sets	Description	Range
A_1	Very low gold prices	$[-1.8\ ,\ -1.16]$
A_2	Low gold prices	$[-1.16\ ,\ -0.52]$
A_3	Medium gold prices	$[-0.52\ ,\ 0.12]$
A_2	High gold prices	$[0.12\ ,\ 0.76]$
A_3	Very high gold prices	$[0.76\ ,\ 1.4]$

Base on Equation (15)–(17), we construct the membership function for each fuzzy state as:

$$\mu_1(x)=\begin{cases}1 & ;\ -1.8\le x\le -1.48\\ \frac{-1.68-2x}{1.28} & ;\ -1.48\le x\le -0.84\\ 0 & ;\ \text{otherwise,}\end{cases}$$

$$\mu_2(x)=\begin{cases}\frac{2x+2.96}{1.28} & ;\ -1.48\le x\le -0.84\\ \frac{-0.4-2x}{1.28} & ;\ -0.84\le x\le -0.2\\ 0 & ;\ \text{otherwise,}\end{cases}$$

$$\mu_3(x)=\begin{cases}\frac{2x+1.68}{1.28} & ;\ -0.84\le x\le -0.2\\ \frac{0.88-2x}{1.28} & ;\ -0.2\le x\le 0.44\\ 0 & ;\ \text{otherwise,}\end{cases}$$

$$\mu_4(x)=\begin{cases}\frac{2x+0.4}{1.28} & ;\ -0.2\le x\le 0.44\\ \frac{2.16-2x}{1.28} & ;\ 0.44\le x\le 1.08\\ 0 & ;\ \text{otherwise,}\end{cases}$$

$$\mu_5(x)=\begin{cases}\frac{2x-0.88}{1.28} & ;\ 0.44\le x\le 1.08\\ 1 & ;\ 1.08\le x\le 1.4\\ 0 & ;\ \text{otherwise.}\end{cases}$$

Then, the fuzzy vector for day-1 is calculated by substituting the relative error value of day-1 into a fuzzy membership function. The relative error value for day-1 is 0, so using Equation (18), we obtain the membership degrees as $\mu_1(x)=0$, $\mu_2(x)=0$, $\mu_3(x)=0.6875$, $\mu_4(x)=0.3125$, and $\mu_5(x)=0$. Therefore, the fuzzy vector for day-1 is (0, 0, 0.6875, 0.3125, 0). Equation (19) gives the one-step transition probability matrix as

$$\mathbf{P}=\begin{bmatrix}p_{11} & p_{12} & 0 & 0 & 0\\ 0 & p_{22} & p_{23} & 0 & 0\\ p_{31} & 0 & p_{33} & p_{34} & p_{35}\\ 0 & 0 & p_{43} & p_{44} & p_{45}\\ 0 & 0 & 0 & p_{54} & 0\end{bmatrix},$$

with

$$\begin{aligned} p_{11}&=0.66666667, & p_{34}&=0.42857143,\\ p_{12}&=0.33333333, & p_{35}&=0.14285714,\\ p_{22}&=0.66666667, & p_{43}&=0.2,\\ p_{23}&=0.33333333, & p_{44}&=0.73333333,\\ p_{31}&=0.14285714, & p_{45}&=0.06666667,\\ p_{33}&=0.28571429, & p_{54}&=1.\end{aligned}$$

Then, the fuzzified form for day-2 can be calculated using Equation (20) by multiplying the day-1 fuzzy vector with the one-step transition probability matrix. So, the vector membership degree for day-2 is (0.0982, 0, 0.2589, 0.5238, 0.119). Consequently, the relative error for day-2 using Equation (21) is 0.1619.

Finally, the gold price prediction for day-2 using FGMM(1,1) in Equation (22) is given by

$$\hat{y}^{(0)}(k+1)=\frac{1,135,630.18}{1-0.001619}=1,137,471.801.$$

Furthermore, in the same way, the gold price forecast value can be calculated with FGMM(1,1) from day-3 until day-30. Table 4 shows the gold price forecast results with FGMM(1,1), obtained with the help of python.

Table 4. FGMM(1,1) forecasting values

Days	Actual Value	FGMM(1,1)
1	1,143,000	1,143,000
2	1,151,000	1,137,471.801
⋮	⋮	⋮
29	1,138,000	1,133,551.457
30	1,142,000	1,134,299.65

3.2. *Fuzzy Grey Markov Model (2,1) Forecasting*

We forecast gold prices in Indonesia using the FGMM(2,1) method, which is a combined forecasting method using derivatives of GM(1,1), namely GM(2,1), as the initial forecasting initiation. In FGMM(2,1), apart from the actual data sequences, 1-AGO and MGO from Equation (1), Equation (2), and Equation (3), it is also necessary to form the 1-IAGO sequence in Equation (22), which is the inverse of the 1-AGO sequence. From that, we are able to determine the parameter values c_1, c_2 and g using Equation (25)–(27), and carry out initial forecasting with GM(2,1), including finding a general solution of the homogeneous equation of Equation (24) with Equation (28). Table 5 shows the forecasting results with GM(2,1) and the relative error between the actual and the predicted value.

Table 5. GM(2,1) forecasting values

Days	Actual Value	GM (2,1)	Relative Error
1	1,143,000	1,143,000	0
2	1,151,000	1,143,872.650	0.6192
⋮	⋮	⋮	⋮
29	1,138,000	2,043,988.584	-79.6124
30	1,142,000	2,262,679.143	-98.133

Then, we continue forecasting FGMM(2,1) using the same technique as the previous method and obtain the following results as follows:

Table 6. FGMM(2,1) forecasting values

Days	Actual Value	FGMM(2,1)
1	1,143,000	1,143,000
2	1,151,000	1,039,042.688
⋮	⋮	⋮
29	1,138,000	1,119,864.290
30	1,142,000	1,201,571.421

3.3. Comparison of FGMM(1,1) and FGMM(2,1) in Forecasting Gold Prices in Indonesia

Based on the results of the three methods presented in Table 3 and Table 6, we compared the performance value between FGMM(1,1) and FGMM(2,1) using MAPE on Equation (32), shown in Table 7.

Table 7. Model evaluation base on MAPE

Model	MAPE	Judgment of Forecast Accuracy
FGMM(1,1)	0.28%	Highly accurate
FGMM(2,1)	4.32%	Highly accurate

Based on the model evaluation shown in Table 7, we can see that the MAPE value of the FGMM(1,1) method is smaller than that of FGMM(2,1), so we can conclude that the FGMM(1,1) method provides more accurate gold price forecasting results than FGMM(2,1). This difference may be due to the limited data, which might prevent FGMM(2,1) from performing optimally. In contrast, FGMM(1,1) is more straightforward to interpret with limited data and does not experience drastic changes. However, both methods are highly accurate in gold price forecasting, considering that both MAPE values remain in the category of accurate forecasting scale. Also, FGMM(2,1) can perform better than GM(2,1) and GMM(2,1) in forecasting future gold prices. A comparison graph of the results of gold price forecasting in Indonesia using FGMM(1,1) and FGMM(2,1) is presented in Figure 1.

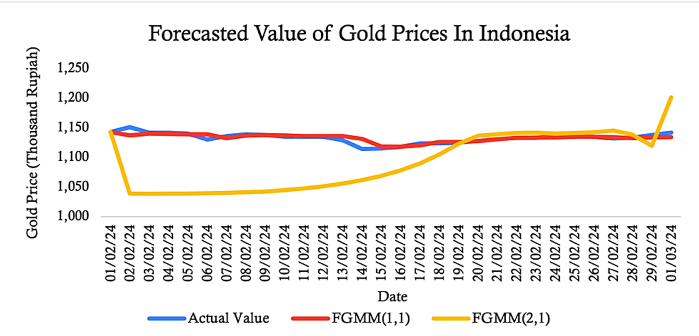


Figure 1. Comparison graph of actual and forecasted values

From the Figure 1, we can see that the graph of the forecasting results for gold prices in Indonesia using FGMM(1,1) almost coincides with the actual data. These results indicate that FGMM(1,1) provides more precise forecasting results than FGMM(2,1). So that, FGMM(1,1) was chosen as the best method for predicting gold prices in Indonesia. Although both methods are equally reliable for the prediction of gold prices.

4. Conclusion

The results of forecasting gold prices in Indonesia show that the FGMM(1,1) method provides a forecast value close to the actual value. Meanwhile, the FGMM(2,1) forecasting results are lower than the actual value at the beginning of the forecasting period. The accuracy of forecasting gold prices in Indonesia based on the MAPE value using the FGMM(1,1) method is 0.28%, and the FGMM(2,1) method is 4.32%. These results indicate that the FGMM(1,1) method provides more accurate forecasting values. Nevertheless, both methods are equally accurate in the forecast of gold prices, considering that both MAPE values remain on a highly accurate forecasting scale. Therefore, both the FGMM(1,1) and FGMM(2,1) are reliable for gold price forecasting, while FGMM(1,1) is slightly superior in terms of forecasting accuracy.

Author Contributions. **Arthamevia Najwa Soraya:** Conceptualization, methodology, formal analysis, resources, data curation, writing–original draft preparation, writing–review and editing. **Firdaniza:** Conceptualization, validation, writing–review and editing, supervision. **Kankan Parmikanti:** Conceptualization, validation, writing–review and editing, supervision. All authors have read and agreed to the published version of the manuscript.

Acknowledgement. The authors express sincere gratitude to the Department of Mathematics, Universitas Padjadjaran, for their invaluable assistance in the completion of this research, and to the editors and reviewers who have supported us in improving this manuscript.

Funding. This research received no external funding.

Conflict of interest. The authors declare no conflict of interest related to this article.

References

[1] J. Nazarudin, N. Gusriani, K. Parmikanti, and S. Susanti, "Article Application of Threshold Generalized Autoregressive Conditional Heteroscedastic (TGARCH) Model in Forecasting the LQ45 Stock Price Return," *Eksakta : Berkala Ilmiah Bidang MIPA*, vol. 24, no. 2, pp. 271–284, 2023, doi: 10.24036/eksakta/vol24-iss02/369.

[2] A. Manickam, S. Indrakala, and P. Kumar, "A Novel Mathematical Study on the Predictions of Volatile Price of Gold Using Grey Models," *Contemporary Mathematics (Singapore)*, vol. 4, no. 2, pp. 270–285, 2023, doi: 10.37256/cm.4220232389.

[3] R. Gupta, C. Pierdzioch, and W.-K. Wong, "A Note on Forecasting the Historical Realized Variance of Oil-Price Movements: The Role of Gold-to-Silver and Gold-to-Platinum Price Ratios," *Energies (Basel)*, vol. 14, no. 20, p. 6775, 2021, doi: 10.3390/en14206775.

[4] T. Andriyanto, "Sistem Peramalan Harga Emas Antam Menggunakan Double Exponential Smoothing," *Intensif*, vol. 1, no. 1, p. 1, 2017, doi: 10.29407/intensif.v1i1.531.

[5] S. Yurinanda, C. Multahadah, and R. Aryani, "Development of COVID-19 Case in District and City of Jambi Province with Exponential Smoothing Method," *Eksakta : Berkala Ilmiah Bidang MIPA*, vol. 21, no. 2, pp. 110–123, 2020, doi: 10.24036//eksakta/vol21-iss2/244.

[6] M. Yanto, S. Sanjaya, Yulasmi, D. Guswandi, and S. Arlis, "Implementation multiple linear regresion in neural network predict gold price," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 22, no. 3, pp. 1635–1642, 2021, doi: 10.11591/ijeecs.v22.i3.pp1635-1642.

[7] X. Li, D. Li, X. Zhang, G. Wei, L. Bai, and Y. Wei, "Forecasting regular and extreme gold price volatility: The roles of asymmetry, extreme event, and jump," *J Forecast*, vol. 40, no. 8, pp. 1501–1523, 2021, doi: 10.1002/for.2781.

[8] D. Makala and Z. Li, "Prediction of gold price with ARIMA and SVM," *J Phys Conf Ser*, vol. 1767, no. 1, p. 012022, 2021, doi: 10.1088/1742-6596/1767/1/012022.

[9] D. Ju-Long, “Control problems of grey systems,” *Syst Control Lett*, vol. 1, no. 5, pp. 288–294, 1982, doi: 10.1016/S0167-6911(82)80025-X.

[10] Y. H. Feng, X. Kong, L. Wang, Y. Zhang, J.Y. Zhang, P. D. Yuan, and Y. N. Li, “Prediction of Motorcycle Seat Styling Based on Grey Modelling (1,1).” *Computer-Aided Design & Applications*, vol. 16, no. 5, pp. 789-802, 2019, doi: 10.14733/cadaps.2019.789-802.

[11] B. Zeng, X. Ma, and J. Shi, “Modeling Method of the Grey GM(1,1) Model with Interval Grey Action Quantity and Its Application,” *Complexity*, pp. 1–10, 2019, doi: 10.5772/intechopen.83801.

[12] J. Lin, K. Zhu, Z. Liu, J. Lieu, and X. Tan, “Study on a simple model to forecast the electricity demand under China’s new normal situation,” *Energies*, vol. 12, no. 11: 2220, 2019, doi: 10.3390/en12112220.

[13] Y. Yang, C. Ming, and Q. Liu, “Research on Forecasting the Number of Tourists in Anyang Based on Grey System GM (1, 1),” *Scholars Journal of Physics, Mathematics and Statistics*, vol. 7, pp. 76-81, 2020, doi: 10.36347/sjpms.2020.v07i06.001.

[14] T. Škrinjaric and M. Cimešija, “Selected Applications of Grey Models in Stock Price Prediction,” *Recent Applications of Financial Risk Modelling and Portfolio Management*, pp. 346-326, 2021, doi: 10.4018/978-1-7998-5083-0.ch017.

[15] Z. Q. Jia, Z. F. Zhou, H. J. Zhang, B. Li, and Y. X. Zhang, “Forecast of coal consumption in Gansu Province based on Grey-Markov chain model,” *Energy*, vo. 199, 2020, doi: 10.1016/j.energy.2020.117444.

[16] P. S. Kumar, and D. Thamaraiselvi, “Prediction Of Estro-Progestin Supplimentation Enhances Growth Hormone Secretion In Postmenopausal Women By Using GM(2,1) And Grey-Markov(2,1) Model,” *Journal of Xidian University*, vol. 14, no. 4, pp. 803-810, 2020, doi: 10.37896/jxu14.4/098.

[17] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965, doi: 10.1016/S0019-9958(65)90241-X.

[18] N. Geng, Y. Zhang, Y. Sun, Y. Jiang, and D. Chen, “Forecasting China’s Annual Biofuel Production Using an Improved Grey Model,” *Energies (Basel)*, vol. 8, no. 10, pp. 12080–12099, 2015, doi: 10.3390/en81012080.

[19] K. Govindan, S. Ramalingam, and S. Broumi, “Traffic volume prediction using intuitionistic fuzzy Grey-Markov model,” *Neural Comput Appl*, vol. 33, no. 19, pp. 12905–12920, 2021, doi: 10.1007/s00521-021-05940-9.

[20] D. Nagarajan, R. Sujatha, G. Kuppuswami, and J. Kavikumar, “Real-time forecasting of the COVID 19 using fuzzy grey Markov: a different approach in decision-making,” *Computational and Applied Mathematics*, vol. 41, no. 6, p. 248, 2022, doi: 10.1007/s40314-022-01949-5.

[21] S. Liu and Y. Lin, *Grey information : theory and practical applications*. London: Springer-Verlag, 2006.

[22] R.-C. Tsaur, “A fuzzy time series-markov chain model with an application to forecast the exchange rate between the taiwan and us dollar,” *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 7, pp. 4931–4942, 2012. [Online]. Available at: http://www.ijicic.org/ijicic-11-04029.pdf.

[23] S. Kusumadewi and H. Purnomo, *Aplikasi Logika Fuzzy : untuk Pendukung Keputusan*, 2nd ed. Yogyakarta: Graha Ilmu, 2020.

[24] S. Osaki, *Applied Stochastic System Modeling*. Japan: Springer, 1992.

[25] L.-X. Wang, *A Course in Fuzzy Systems and Control*. New Jearsy: Prentice Hall,1997.

[26] D. Jain, S. Sharma, and P. Dhiman, “Comparative Analysis of Defuzzification Techniques for Fuzzy Output,” *J Algebr Stat*, vol. 13, no. 2, pp. 874–882, 2022, doi: 10.52783/jas.v13i2.234.

[27] K. D. Lawrence, R. K. Klimberg, and S. M. Lawrence, *Fundamentals of forecasting using Excel*. Industrial Press, 2009.