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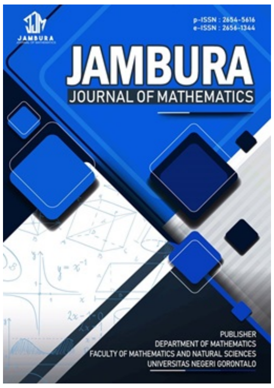

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Research Article

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Analysis of Optimal Portfolio Formation Using Multi-Objective Optimization Method and Nadir Compromise Programming

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ABSTRACT. *A portfolio is a collection of financial assets in the stocks owned by a company or individual. An optimal portfolio is a selected portfolio that aligns with the investor's preferences, drawn from a set of efficient portfolios that have been formed. This research aims to create an optimal portfolio using the Multi-Objective Optimization method and the Nadir Compromise Programming (NCP) method. Additionally, Value at Risk (VaR) analysis is applied to determine the maximum risk an investor will bear for the portfolio. The data used consists of closing stock prices on the IDX30 Index from February 2022 to July 2023. The findings indicate that the optimization approach produces portfolios that align with investor risk-return preferences. The comparison of Multi-Objective Optimization and NCP methods provides insights into their effectiveness in portfolio selection. Furthermore, the VaR analysis helps investors understand potential risk levels, offering a comprehensive perspective on portfolio performance.*

1. Introduction

Investment is the responsible allocation of capital or resources with the expectation of future profits [1]. Given the large number of stocks listed on the stock exchange, investors need to be cautious and rational when selecting stocks in order to achieve maximum returns within a certain level of risk [2, 3]. Risk and return are the most important factors in building an optimal portfolio for investors [4]. Risk can be defined as the probability that the expected return from a security will not be realized, while return indicates whether an investment generates a profit or loss over a specific period of time [5, 6]. An optimal portfolio is one selected according to the preferences of investors, derived from a set of efficient portfolios that have been constructed, and portfolio performance is assessed based on the level of profit and risk incurred [7].

The main problem in portfolio formation is to determine the optimal combination of risk and return because stock price fluctuations are difficult to predict [8]. The existence of diversification can help predict the level of stock returns so as to facilitate the investment decision-making process [9]. The best portfolio is one that can provide an optimal balance between the returns earned and the level of risk faced, in accordance with their investment preferences and objectives [8]. The key to achieving an investor's goals is to provide an optimal portfolio strategy that shows the investor how much should be invested in each asset within a given portfolio.

Portfolio optimization can use several variations of strategies, including portfolio formation with the Multi-Objective Optimization method and the Nadir Compromise Programming (NCP) method. Multi-objective optimization developed by French-Italian economist V. Pareto aims to maximize expected return and minimize portfolio risk simultaneously [10]. The multi-objective optimization method focuses on optimization by considering more than one perspective. The goal of this method is to maximize the expected return while simultaneously minimizing the risk from the combination of assets in the portfolio [11]. Investment through this method does not allow short selling or the allocation of all funds to a single stock in the portfolio. An additional strategy is the formation of a portfolio through the use of Nadir Compromise Programming (NCP). This NCP method was first published in 2011 by [12], NCP is used to solve multi-objective problems with optimization based on the nadir value. It aims to find a solution that is close to the nadir of each objective to be achieved.

Risk measurement for investment portfolios needs to be conducted so that investors can measure and assess risk. One of the commonly used techniques in risk management is the Value at Risk (VaR) method, which estimates the maximum amount of loss (risk) under normal market conditions and within a time span with a certain level of confidence [13]. Here are three ways to calculate VaR, namely, Monte Carlo Simulation, Variant-Covariance, and Historical Simulation [14]. Historical simulation was chosen in this study because this method directly estimates risk based on stock or portfolio return data over a certain period of time without making assumptions about the parametric shape of the risk factor return distribution.

Some research has been done to optimize portfolios. Septiyanto [15] in his research, he analyzed how to use the multi-objective optimization method to create an optimal portfolio,

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and determine the number of stocks selected in the optimal portfolio. Meanwhile Indriani [16] conducted research with the aim of knowing how the Nadir Compromise Programming (NCP) method is used to solve multi-objective optimization problems in stock portfolios. Solihatun et al. [17] in his research to understand how to calculate VaR on single assets and stock portfolios. Moreover, portfolio optimization can also be conducted using the Single Index Model, as studied by Abdjul et al. [18], this method focuses on calculating the optimal portfolio return using the Single Index Model, assessing risk with VaR (Value at Risk), and implementing the approach using a GUI (Graphical User Interface) in MATLAB (Matrix Laboratory).

Based on the explanation above, the use of the Multi-Objective Optimization method helps investors in maximizing profits while minimizing risks at the same time, while the Nadir Compromise Programming method is designed based on the nadir point obtained from the objective function. Multi-Objective optimization and Nadir Compromise Programming methods have something in common, namely, focusing on finding solutions by considering several objectives at once. Therefore, research was conducted to compare the optimal portfolio using the Multi-Objective Optimization method and the Nadir Compromise Programming method. The purpose of doing this comparison helps in understanding the characteristics of a particular problem and deciding which one is more appropriate. Furthermore, a Value at Risk (VaR) analysis is conducted to determine the maximum risk that an investor will bear for the portfolio.

2. Methods

The method used in this research begins with a literature study by tracing sources from scientific articles, scientific journals, books and other references related to the formation of optimal portfolios using the Multi-Objective Optimization method, Nadir Compromise Programming and Value at Risk. The goal is to obtain information and methods used in the discussion of related issues. In this study, the data used is secondary data for the closing price value of shares. Stock price reports listed on the IDX30 index are sourced from [19] and [20]. The case study used in this research is the monthly closing price of IDX30 index stocks taken from the period February 2022 - July 2023. This research sample uses purposive sampling technique, which is a sampling technique on consideration of certain criteria. Sampling is used if it meets the following criteria:

a. Companies that are consistently included in the IDX30 Index during the period February 2022-July 2023.
b. The companies are listed on the Indonesia Stock Exchange.
c. The company has complete trading data during the study period.

The steps to form an optimal portfolio using the Multi-Objective Optimization method are as follows:

1. Return and expected return
   The first step in processing data to form a portfolio is calculating stock returns. Return is defined as the result or income generated from investment activities over a certain period. It is important to note that investment activities during a given period do not always yield profits; investors may also experience losses. Returns can be classified as the actual return, which has already been realized from an investment, and the expected return, which reflects investors expectations of how well the investment will perform.
   This calculation can be performed using the following equation:
   $$R_{i,j} = \frac{S_{i,j} - S_{i,j-1}}{S_{i,j-1}}, \tag{1}$$
   with
   $R_{i,j}$ = Return rate of stock i at time j,
   $S_{i,j}$ = The value of the stock at time j,
   S_{j-1} = The value of the stock at the previously determined time $j-1$.
   The next step is to calculate the expected stock return, where the calculation of this value of stock return is completed using the following equation:
   $$E(R_i) = \frac{\sum_{j=1}^{N} R_{i,j}}{N}, \tag{2}$$
   with
   $E(R_i)$ = Expected return on stock i,
   N = The number of observation periods.
2. Next conduct a normality test using the Shapiro-wilk test.
   The importance of this test lies in its ability to determine whether the sample data belongs to a population with a normal distribution or whether it deviates from a normal distribution. Furthermore, normality tests play a crucial role in portfolio formation by helping to identify whether asset return data follows a normal distribution, which is a fundamental assumption in many financial and risk models. Therefore, this study employs the Shapiro-Wilk normality test to determine whether the stock return data meets the normality assumption.
   The research hypothesis is as follows:
   H_0 : Data is normally distributed
   H_1 : Data is not normally distributed
   a significance level $(\alpha) = 0.05$
   critical region
   - if $p > \alpha$, then H_0 is not rejected, which means the data follows a normal distribution.
   - if $p < \alpha$, then H_0 is rejected, which means the data does not follow a normal distribution.

   The test statistic formula is as follows:
   $$W = \frac{(\sum_{N}^{i=1} \alpha_i (y_{n+1-i} - y_i))^2}{(\sum_{N}^{i=1} (y_i - \bar{y}))^2} \tag{3}$$
   with
   α_i = Shapiro-Wilk test coefficient,
   N = Number of periods.
3. After obtaining the expected return values, the next step is to calculate the risk (variance and standard deviation). Stock investment risk is higher when there is greater variance, whereas lower variance indicates lower investment risk. The following equation can be used to calculate variance:
   $$\sigma_i^2 = \sum_{j=1}^{n} \frac{(R_{i,j} - E(R_i))^2}{n-1} \tag{4}$$
   While the standard deviation which is the square root of the

variance can be calculated using the following equation:

$$\sigma_i = \sqrt{\sum_{j=1}^{n} \frac{(R_{i,j} - E(R_i))^2}{n-1}} \tag{5}$$

with
σ_i^2 = Variance of the i-th stock,
n = Number of periods,
σ_i = The standard deviation of the i-th stock.

4. The next step is to create a covariance matrix from the selected stock returns. Covariance describes the level of relationship between the returns of two assets, indicating the extent to which the returns of the two assets move together. The covariance of an asset is expressed as follows:

$$\sum = \begin{bmatrix} \sigma_{X_1X_1} & \cdots & \sigma_{X_1X_k} \\ \vdots & \ddots & \vdots \\ \sigma_{X_kX_1} & \cdots & \sigma_{X_kX_k} \end{bmatrix},$$

with

$$\sigma_{X_iX_k} = \frac{1}{n-1}\sum_{j=1}^{n}[(x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k)], \tag{6}$$

for $i = 1, 2, ..., n$ with $k = 1, 2, ..., n$.

5. Determining the inverse value of the variance-covariance matrix.
6. The Multi-Objective Optimization method offers advantages over methods that use only one or two constraints. Specifically, incorporating more constraint indicators with varying k values results in a more optimized portfolio and provides a greater diversity of alternative choices. Selecting a weighting coefficient (k value) close to zero indicates that the investor tends to be risk-seeking, while a weighting coefficient approaching infinity indicates that the investor is more risk-averse. using the following equation:

$$w = \frac{1}{2k}\sum^{-1}\left(\mu - \left(\frac{1^T\sum^{-1}\mu - 2k}{1^T\sum^{-1}1}\right)1\right), \tag{7}$$

with
w = Portfolio weight,
k = Weighting coefficient,
μ = a column vector consisting of expected return.

7. A portfolio is a linear combination of multiple assets. The expected return of the portfolio can be calculated using the following equation:

$$E(R_p) = \mu^T w = \sum_{i=1}^{n} w_i\mu_i, \tag{8}$$

with
$E(R_p)$ = The expected return value of the portfolio,
w_i = The value of the weight of the fund formed on the $i-$th stock,
μ_i = $E(R_i)$ where μ_i expected return of the $i-$th stock.

8. As a final step, calculate the portfolio risk. To find the value of portfolio risk using the following equation:

$$\sigma_p^2 = w^T\sum w = \sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_{i,j}, \tag{9}$$

with
$\sum$ = Variance-covariance matrix $n \times n$,
w = The weight matrix of each security $n \times 1$.

Furthermore, the formation of an optimal portfolio using the Nadir Compromise Programming method is as follows:

1. The calculation of stock returns and expected returns can be done using eq. (1) and eq. (2).
2. Then calculate the market return and expected market return. The market return is represented by the Jakarta Composite Index (JCI).
3. Then calculate the market variance. The variance is calculated by looking at the return, the expected return and the time period of the stock price or market price. This equation is used to calculate the variance of a data set of N returns:

$$\sigma_m^2 = \sum_{j=1}^{N} \frac{(R_{mj} - E(R_m))^2}{N}, \tag{10}$$

with
σ_m^2 = Variance of the market index,
R_{mj} = The return of the market index in period j, and so on,
$E(R_m)$ = Expected return of the market index.

4. The next step is to calculate the covariance between stock returns and market returns. Covariance is a measure of the extent to which two assets move together over time, i.e., how often they move up or down together. In this case, moving together means that they are generally above or below the average at the same time. The covariance between asset i and market m is defined as:

$$\text{Cov}(R_i, R_m) = \sum_{j=1}^{N} \frac{(R_{ij} - E(R_i))(R_{mj} - E(R_m))}{N}, \tag{11}$$

with
R_{ij} = Return of stock i in period j, and so on,
$E(R_i)$ = Expected return of stock i,
R_i = Return of stock i,
R_m = Return of the market.

5. After calculating the covariance values of stock returns, market returns, and market variance, the next step is to calculate the risk coefficient (β). The risk coefficient indicates the level of risk that the investor will be responsible for. To determine the risk coefficient, the following equation can be used:

$$\begin{aligned}\beta_i &= \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} \\ &= \frac{\sum_{j=1}^{N}(R_{ij} - E(R_i))(R_{mj} - E(R_m))}{\sum_{j=1}^{N}(R_{mj} - E(R_m))^2}.\end{aligned} \tag{12}$$

6. Next, formulate the objective function and constraint function. When optimizing a portfolio, there are several factors

to consider, namely the expected return and the risk level. However, the outcome variable is the amount of investment allocated to each stock, represented by x_i.

7. As a final step, the LINGO software is used to calculate the proportion of funds, expected return, and risk.

To calculate the risk of each portfolio, Value at Risk (VaR) is used with the following steps:

1. The first step in processing data to form a portfolio is to calculate the weight value using the following equation:

$$w = \frac{\Sigma^{-1} i}{i^T \Sigma^{-1} i}, \tag{13}$$

with
Σ^{-1} = Inverse of the variance covariance matrix,
i = The matrix contains the number 1 in each element, with the number of columns corresponding to the number of expected returns,
i^T = The transpose of a matrix containing the number 1 for each column of expected returns.

2. The next step is to calculate the portfolio return. The portfolio return is the average return of individual assets, calculated by considering the weight assigned to each asset in the portfolio. Therefore, the portfolio return can be formulated as follows:

$$R_p = \sum_{i=1}^{n} (w_i R_i), \tag{14}$$

with
R_p = Portfolio return value,
w_i = The value of the fund weight allocated to the i-th stock,
R_i = The return of the i-th asset.

3. Calculating the VaR value of the portfolio as follows equation

$$VaR = V_0 \times P_\alpha \times \sqrt{t}, \tag{15}$$

with
VaR = The maximum possible loss value,
V_0 = The initial value of the investment,
P_α = $\alpha-$th percentile,
$\sqrt{t}$ = Predefined period.

3. Results and Discussion

Based on the predetermined criteria, there are 23 companies listed in the IDX30 Index. These companies were actively operating during the period February 2022 - July 2023.

3.1. Portfolio Formation Using the Multi-Objective Optimization Method

The first step in data analysis to form a portfolio is to calculate stock returns. The return and expected return values are used to evaluate the performance of stocks in the IDX30 Index to identify optimal profit opportunities during the period 2022 to July 2023. The calculation of the return of each stock is obtained through calculations using eq. (1) and for the calculation of the expected return of each stock using eq. (2).

Table 1 obtained 15 stocks that have a positive expected return value. As well as 8 stocks with negative expected return values. BMRI (PT Bank Mandiri (Persero) Tbk) shares provide the highest expected return value of 0.02521 or 2.521% and TOWR (PT Sarana Menara Nusantara Tbk) shares have the lowest expected return value of 0.00075 or 0.07%. This portfolio formation involves stocks that have a positive expected return value. This step aims to prevent losses in the portfolio formation process. Stock with positive expected return values are expected to provide favorable results, while stocks with negative expected return values can signal losses.

Table 1. IDX30 Stock Data

Stock	Return				Expected Return
	1	2	...	17	
ADRO	0.09796	0.24164	...	0.08072	0.00962
ANTM	0.09910	0.06557	...	0.01795	-0.00103
ASII	0.13362	0.15209	...	0.01107	0.01266
BBCA	-0.00932	0.01881	...	-0.00273	0.00839
BBNI	0.03125	0.11818	...	-0.03005	0.00802
BBRI	0.02418	0.04506	...	0.04147	0.01418
BMRI	0.02597	0.13291	...	0.10096	0.02521
BRPT	0.01685	-0.06077	...	0.04027	-0.00461
BUKA	0.13018	0	...	0.01887	-0.02228
CPIN	-0.02586	-0.08850	...	-0.01896	-0.00489
EMTK	0.18357	0.22041	...	-0.08451	-0.05176
INCO	0.24074	0.08955	...	0.09127	0.02167
INDF	-0.04032	0.05882	...	-0.00340	0.01109
KLBF	-0.02128	0.01863	...	-0.06585	0.00987
MDKA	0.17054	0.19924	...	0.14706	0.00424
PGAS	-0.02431	0.03203	...	0.04598	0.00111
PTBA	0.04777	0.16109	...	0.03358	-0.00057
SMGR	-0.07639	-0.03759	...	0.14815	0.00225
TBIG	-0.02381	0.04878	...	-0.03980	-0.02323
TLKM	0.05530	0.00873	...	-0.07000	-0.00766
TOWR	0.03382	-0.05607	...	-0.03791	0.00075
UNTR	0.02610	0.18493	...	0.18260	0.01202
UNVR	-0.00543	0.06284	...	-0.09624	0.00506

Furthermore, the multi-objective optimization method used for optimal portfolio formation assumes that stock returns are normally distributed. The normality test applied is the Shapiro-Wilk test, with a significance level (α) of 5%.

The research hypothesis is as follows:
H_0 : Data is normally distributed,
H_1 : Data is not normally distributed.
The calculation results show thal all 15 stocks are normally distributed.

The next step is to calculate the variance and standard deviation performed on 15 stocks with positive expected return values. Variance and standard deviation describe how much spread there is between the expected value and the actual value, as well as the spread in the probability distribution. The purpose of this measure of dispersion is to measure the extent to which the value obtained may deviate from the expected value. The risk that investors must face when investing in certain stocks can be described through the variance and standard deviation values.

From Table 2, it can be observed that the highest variance and standard deviation are found in ADRO (Adaro Energy Tbk) stock, with a variance value of 0.02008 or 2.008% and a standard deviation of 0.14170 or 14.17%. On the other hand, the lowest

Table 2. The Results of Variance and Standard Deviation Calculation deviasi

No	Stock	Variance (σ_i^2)	Standard Deviation (σ_i)
1.	ADRO	0.02008	0.14170
2.	ASII	0.00619	0.07868
3.	BBCA	0.00212	0.04604
4.	BBNI	0.00396	0.06293
5.	BBRI	0.00285	0.05339
6.	BMRI	0.00349	0.05908
7.	INCO	0.01437	0.11987
8.	INDF	0.00265	0.05148
9.	KLBF	0.00196	0.04427
10.	MDKA	0.01711	0.13081
11.	PGAS	0.00938	0.09685
12.	SMGR	0.00853	0.09236
13.	TOWR	0.00403	0.06348
14.	UNTR	0.01269	0.11265
15.	UNVR	0.00535	0.07314

Table 3. Stock Weights

k	0.005	0.1	1	10	50	100	150	1000	20000
ADRO	3.00%	3.00%	2.95%	2.42%	0.04%	0.00%	0.00%	0.00%	0.00%
ASII	1.63%	1.63%	1.63%	1.61%	1.48%	1.21%	0.98%	0.00%	0.00%
BBCA	38.34%	38.34%	38.30%	37.89%	34.68%	28.31%	22.61%	0.00%	0.00%
BBNI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
BBRI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	19.67%	22.44%
BMRI	20.71%	20.71%	20.77%	21.37%	23.12%	23.12%	22.53%	12.12%	8.85%
INCO	15.23%	15.22%	15.20%	14.92%	13.16%	10.16%	7.56%	0.00%	0.00%
INDF	11.39%	11.39%	11.37%	11.19%	9.93%	7.75%	5.85%	0.00%	0.00%
KLBF	0.00%	0.00%	0.00%	0.00%	2.93%	9.84%	15.00%	24.84%	24.84%
MDKA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.58%	7.68%	8.38%
PGAS	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
SMGR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
TOWR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.93%	11.50%	12.88%
UNTR	0.00%	0.00%	0.00%	0.00%	0.95%	3.66%	5.69%	9.60%	9.67%
UNVR	9.70%	9.71%	9.79%	10.60%	13.70%	15.94%	17.28%	14.70%	12.93%

variance and standard deviation are found in KLBF (Kalbe Farma Tbk) stock, with a variance value of 0.00196 or 0.196% and a standard deviation of 0.04427 or 4.427%.

The next step is to form the matrix and inverse covariance matrix of the selected stock returns. After obtaining the inverse variance covariance matrix, the next step is to determine the stock weights for different k values. The weighting coefficient k reflects the level of risk an investor takes on the expected return where the selection of the weighting coefficient (k value) close to zero indicates that the investor tends to like risk, while the weighting coefficient close to an infinite number indicates that the investor has a tendency to avoid risk.

Based on Table 3, at a weighting coefficient $k < 1$, the stock BBCA (Bank Central Asia Tbk) holds the highest weight, particularly at $k = 0.005$, and $k = 0.1$, with a weight of 38.34%. This This weighting coefficient is suitable for risk-seeking investors to meet their needs.

Furthermore, the identification of the optimal portfolio is carried out in accordance with the characteristics of investors. The following table presents the final results of the calculation of portfolio weights, expected return and portfolio risk.

Referring to Table 4, when the weighting coefficient $k < 1$ (close to zero), the portfolio formed consists of ADRO, ASII, BBCA, BMRI, INCO, INDF, and UNVR stocks.

In this condition, the portfolio with a coefficient of $k = 0.005$ provides the highest expected return of 0.013987 with a portfolio risk of 0.001480. This portfolio is considered optimal for investors who have a preference for risk (risk seekers). As concrete evidence, it is assumed that an investor will invest his capital in a portfolio of IDR 50,000,000 with $k = 0.005$ in the seven stocks that have been analyzed, it is found that in ADRO shares the amount of capital to be allocated is IDR 1,500,000, in ASII of IDR 815,000, on BBCA shares of IDR 19,170,000, on BMRI shares of IDR 10,355,000, on INCO shares of IDR 7,615,000, on INDF shares of IDR 5,695,000 and IDR 4,850,000 on UNVR shares. The expected return generated is 1.3987% or IDR 699,350 and the resulting risk is 0.1480% or IDR 7,400.

As the value of the weighting coefficient increases, the portfolio composition becomes more varied. When $1 \leq k \leq 150$, the portfolio includes ADRO, ASII, BBCA, BMRI, INDF, KLBF,

Table 4. Final Results of Portfolio Weights and Expected Returns Calculation

Investor Type	k	$E(R_p)$	σ_p^2	Stocks
risk seeker	0.005	0.013987	0.001480	ADRO, ASII, BBCA, BMRI, INCO, INDF, UNVR
	0.1	0.013985	0.001479	
risk indifference	1	0.013990	0.001476	ADRO, ASII, BBCA, BMRI, INCO, INDF, UNVR
	10	0.014013	0.001441	
	50	0.013979	0.001278	ADRO, ASII, BBCA, BMRI, INCO, INDF, KLBF, UNTR, UNVR
	100	0.013636	0.001182	ASII, BBCA, BMRI, INCO, INDF, KLBF, UNTR, UNVR
	150	0.013010	0.001155	ASII, BBCA, BMRI, INCO, INDF, KLBF, MDKA, TOWR, UNTR, UNVR
risk averse	1000	0.010595	0.000837	BBRI, BMRI, KLBF, MDKA, TOWR, UNTR, UNVR
	20000	0.010133	0.000811	

MDKA, TOWR, UNTR, and UNVR stocks, with expected returns ranging from 1.3010% to 1.3990%, and a portfolio risk ranging from 0.1155% to 0.1476%. In this interval, the portfolio is suitable for investors who are risk indifferent, where there are no special conditions that affect the selection of the optimal portfolio.

Furthermore, when the coefficient $k > 150$, the lowest expected return is achieved at $k = 20000$ with a portfolio consisting of BBRI, BMRI, KLBF, MDKA, TOWR, UNTR, and UNVR stocks. In this condition, risk averse investors can allocate capital to this portfolio with an expected return of 1.0133% and a portfolio risk of 0.0811%.

3.2. Portfolio Formation Using the Nadir Compromise Programming Method

Based on the calculation of stock returns and expected stock returns, the next step is to calculate market returns and expected market returns, where market returns are represented by the Jakarta Composite Index. The next stage involves calculation the risk coefficient, which is obtained by comparing the covariance between stock returns and the expected market return, resulting in a variance value of 0.0005088117. The results of the risk coefficient calculation for each stock are presented in Table 5.

Table 5 shows that the risk coefficient value of MDKA stock has the highest risk level compared to other stocks. In addition, it can be seen that eight stocks have a risk coefficient value above 1, which indicates that changes in the price of these stocks are more sensitive than changes in market prices. In contrast, BBRI, INDF, KLBF, SMGR, TOWR AND UNVR stocks have a risk coefficient value below 1, which indicates that the prices of these stocks tend to be more stable and not easily subject to changes in market prices.

Furthermore, several aspects need to be determined in the formation of a portfolio optimization model, including the decision variables, the formulation of the objective function, and the formulation of constraints. The definitions of the decision variables are presented in Table 6.

After determining the decision variables, the next step is to formulate the objective function. In portfolio formation, two aspects need to be considered, namely risk and expected return.

Table 5. Covariance Values and Risk Coefficients

No	Stock	Cov(R_i, R_m)	β_i
1.	ADRO	0.00198316	3.89759
2.	ASII	0.001243624	2.44415
3.	BBCA	0.000531402	1.04439
4.	BBNI	0.000748745	1.47154
5.	BBRI	0.000260463	0.511899
6.	BMRI	0.000904149	1.77696
7.	INCO	0.001538036	3.02277
8.	INDF	-0.00053319	-1.0479
9.	KLBF	-0.000049993	-0.0982528
10.	MDKA	0.002089245	4.10608
11.	PGAS	0.000691499	1.35903
12.	SMGR	0.000321245	0.631357
13.	TOWR	0.0000933815	0.183527
14.	UNTR	0.001891355	3.71716
15.	UNVR	-0.00043379	-0.852554

Table 6. Definition of Decision Variables

Variable	Stock Codes	Variable	Stock Codes
x_1	ADRO	x_9	KLBF
x_2	ASII	x_{10}	MDKA
x_3	BBCA	x_{11}	PGAS
x_4	BBNI	x_{12}	SMGR
x_5	BBRI	x_{13}	TOWR
x_6	BMRI	x_{14}	UNTR
x_7	INCO	x_{15}	UNVR
x_8	INDF		

Therefore, the objective function of this portfolio optimization model is:

1. Objective Function for Optimal Risk

$$\min f_1 = \sum_{i=1}^{15} \beta_i x_i, \tag{16}$$

β_i denotes the risk coefficient of stock i, and x_i being the decision variable indicating the proportion i-th stock.

2. Objective Function for Maximizing Expected Return

$$\max\ f_2 = \sum_{i=1}^{15} E(R_i)x_i, \tag{17}$$

$E(R_i$ is the expected return of the portfolio.

To achieve the goals of portfolio optimization, there are several constraints as follows:

1. Constraint Function for Total Fund Proportion

$$\sum_{i=1}^{15} x_i = 1. \tag{18}$$

2. Constraint Function for Fund Prportion Limits

$$0 \leq x_i \leq 0.5, i = 1, 2, 3, 4, \ldots, 15. \tag{19}$$

Before formulating in Nadir Compromise Programming, it is necessary to determine the nadir values for the maximum and minimum objective functions by optimizing each objective function with the existing constraints. Therefore, the calculation for the nadir value of the objective function to maximize expected return can be formulated as follows:

$$\begin{aligned} \max\ f_2 = {} & 0.00962x_1 + 0.01266x_2 + 0.00839x_3 + 0.00802x_4 \\ & + 0.01418x_5 + 0.02521x_6 + 0.02167x_7 + 0.01109x_8 \\ & + 0.00987x_9 + 0.00424x_{10} + 0.00111x_{11} \\ & + 0.00225x_{12} + 0.00075x_{13} + 0.01202x_{14} \\ & + 0.00506x_{15}, \end{aligned} \tag{20}$$

with constraint function in Eq. (18) and Eq. (19). Solving eq. (20) gives $f_{2*} = 0.00093$, which is the nadir value of the expected return for the 15 stocks formed in the portfolio.

In the Nadir Compromise Programming (NCP) calculation, it is assumed that all objective functions have equal weights, with each weight (w_k) being the same for each objective function. Given that the total sum of the weights is equal to 1, the weights are set as follows: $w_1 = \frac{1}{2}$ (risk) and $w_2 = \frac{1}{2}$ (*expected return*). The value p is also assumed to be 1. Thus, the NCP model for the portfolio selection problem is formulated as follows:

$$\min \frac{1}{2}(\delta_1^+ + \delta_1^-) + \frac{1}{2}(-\delta_2^+), \tag{21}$$

with constraint function.

$$\sum_{i=1}^{15} \beta_i x_i - \delta_1^+ = 1,$$

$$\sum_{i=1}^{15} \beta_i x_i + \delta_1^- = 1,$$

$$\sum_{i=1}^{15} E(R_i)x_i - \delta_2^+ = 0.00093,$$

$$x_1 + x_2 + x_3 +_4 + x_5 + x_6 + x_7 + x_8 + \\ x_9 + x_{10} + x_{11} +_{12} + x_{13} + x_{14} + x_{15} = 1,$$

$$0 \leq x_i \leq 0.5, i = 1, 2, 3, 4, .., 15,$$

$$\delta_1^+, \delta_1^-, \delta_2^+ \geq 0.$$

The calculation results of the Nadir Compromise Programming model are $\delta_1^+ = \delta_1^- = 0$, dan $\delta_2^+ = 0.01887164$. then the proportion of funds invested in the selected stocks is presented in Table 7.

Table 7. Investment Proportions

Variable	Stock	Investment Proportions
x_6	BMRI	0.5
x_7	INCO	0.1561094
x_8	INDF	0.3438906

Based on Table 7, variables x_6, x_7 and x_8 are obtained as the best stocks in the formation of the optimal stock portfolio. The proportion of funds obtained from the NCP method is for x_6 is PT Bank Mandiri (Persero) Tbk (BMRI) shares of 0.5, x_7 is PT Vale Indonesia Tbk (INCO) shares of 0.1561094 and x_8 is PT Indofood Sukses Makmur Tbk (INDF) shares of 0.3438906.

From the results of the fund proportions for the selected stocks in forming the optimal portfolio, the values f_1^* and f_2^* can be calculated.

$$\begin{aligned} f_1^* &= (x_6 \times \beta_6) + (x_7 \times \beta_7) + (x_8 \times \beta_8) \\ &= (0.5 \times 1.77696) + (0.1561094 \times 3.02277) \\ &\quad + (0.3438906 \times (-1.0479)) \\ &= 1. \\ f_2^* &= (x_6 \times E(R_6)) + (x_7 \times E(R_7)) + (x_8 \times E(R_8)) \\ &= (0.5 \times 0.02521) + (0.1561094 \times 0.02167) \\ &\quad + (0.3438906 \times 0.01109) \\ &= 0.01980164. \end{aligned}$$

The results of the optimization for portfolio selection using Nadir Compromise Programming are an optimal risk coefficient of 1, an expected return of 0.01980164.

3.3. Determining the Value at Risk (VaR) using the Historical Simulation Method

Based on the optimal stocks formed using the Multi-Objective Optimization method and the NCP method, the Value at Risk (VaR) for the portfolio can be calculated. The initial step involves determining the weight of each stock using eq. (13). VaR for the portfolio can then be calculated using eq. (15). The returns of the portfolio have been sorted, and the 5th percentile value has been obtained. With an initial fund of IDR 50,000,000 and a one-month time period, at a 95% confidence level, the VaR values for portfolios with optimal stocks formed by the Multi-Objective Optimization method are IDR 934,370, IDR 3,365,300, IDR 987,300, IDR 1,063,100, and IDR 937,400. For the portfolio with optimal stocks formed using the NCP method, the VaR is 2.37% or IDR 1,182,900.

4. Conclusion

The optimal portfolio formation with the Multi-Objective Method obtained on the IDX30 Index contains 12 stocks, namely ADRO, ASII, BBCA, BBRI, BMRI, INCO, INDF, KLBF, MDKA, TOWR, UNTR and UNVR stocks. The formation of the optimal portfolio with the Nadir Compromise Programming Method resulted in 3 selected stocks to form an optimal portfolio. These stocks are BMRI, INCO and INDF. Then, based on the results of the calculation of the Value at Risk (VaR) value on a portfolio with initial funds of IDR 50,000,000 and a time period of one month, as well as a confidence level of 95%, obtained the VaR value of the portfolio based on the optimal stocks formed by the Multi-Objective Optimization method is IDR 934,370, IDR 3,365,300, IDR 987,300, IDR 1,063,100, and IDR 937,400, respectively. As for the VaR value in the portfolio using the optimal shares formed by the NCP method of 2.37% or IDR 1,182,900.

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