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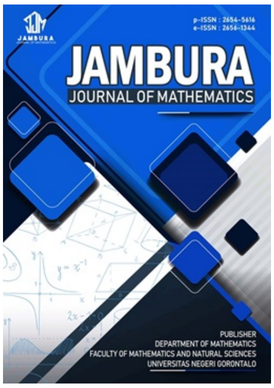

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Research Article

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Conditional Value at Risk Portfolio with Monte Carlo Control Variates

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ABSTRACT. *Stock investment is one of the instruments investors favor due to its potential for high returns, but the risks stemming from stock price volatility cannot be overlooked. Value at Risk (VaR) is commonly used as a standard approach to measure and manage these risks. However, VaR has limitations in handling extreme risks, making Conditional Value at Risk (CVaR) a more effective choice. This research measures the application of CVaR to a portfolio of banking sector stocks in Indonesia using the Monte Carlo Control Variates (MCCV) technique, with the Indonesia Composite Index (ICI) as the control variable. The portfolio consists of stocks of PT Bank Rakyat Indonesia Tbk (BBRI) and PT Bank Negara Indonesia Tbk (BBNI). The purpose of this research is to compare CVaR calculation results using Standard Monte Carlo Simulation (MCS) and MCCV simulations. The data used includes the daily closing prices of BBRI, BBNI, and ICI stocks for the period from March 1, 2023, to February 29, 2024. The VaR and CVaR calculated in this study are for one day. The results of the analysis show that the MCS CVaR values at 90%, 95%, and 99% confidence levels are 1.730%, 2.050%, and 2.569%, respectively, while the MCCV CVaR values at 90%, 95%, and 99% confidence levels are 1.400%, 1.662%, and 2.084%, respectively. These values indicate that using the ICI as a control variable has successfully improved risk estimation.*

1. Introduction

Stock investment is one of the most popular financial instruments among investors due to its high-profit potential. However, this potential comes with high risk [1], as stock market volatility is influenced by various external factors such as economic changes, government policies, and market sentiment. Sharp and unexpected price fluctuations can lead to significant losses if not properly managed.

To manage these risks, it is necessary to employ methods that can provide accurate risk estimates. One commonly used method is Value at Risk (VaR), which measures the maximum potential loss over a specified time period at a given confidence level [2, 3]. However, VaR provides limited guidance in analyzing distribution tail functions [4]. Conditional Value at Risk (CVaR) is a better alternative for measuring tail risk [5]. CVaR calculates the average loss incurred more than VaR, providing a more complete picture of risk in extreme market situation [6].

The Monte Carlo simulation method is widely used for this purpose due to its ability to simulate diverse market scenarios [7]. However, Monte Carlo simulations can be computationally intensive. To improve the efficiency of these simulations, the Control Variates (CV) technique is employed as a variance reduction method. Accurate CVaR estimation requires an approach capable of handling complex return distributions. A key assumption in Monte Carlo simulations is that the data used should follow a normal distribution. To ensure this assumption is met, the Kolmogorov-Smirnov Normality Test is applied to test whether the data is normally distributed. If the data does not meet the normality assumption, alternative distribution assumptions are considered for the simulations.

Several previous studies have applied the Control Variates technique in Monte Carlo simulations for financial risk estimation. For instance, the use of market indices such as the LQ45 has demonstrated improvements in estimation efficiency [8, 9]. However, the LQ45 index, which consists of the 45 most liquid stocks, may not fully represent the overall Indonesian stock market. Therefore, using the Indonesia Composite Index (ICI), which includes all stocks listed on the Indonesia Stock Exchange, as a control variable could provide a more comprehensive reflection of the market's conditions.

In portfolio optimization, the Mean Variance Efficient Portfolio (MVEP) theory is commonly used to determine the optimal portfolio allocation that balances risk and return [10]. This approach is particularly relevant in the context of stock returns, where the objective is to minimize risk while maximizing expected returns. The use of MVEP in this study will help identify an efficient portfolio of stocks that minimizes potential losses, thus contributing to improved risk management.

Furthermore, the Pearson Correlation Test is used to determine whether there is a significant correlation between the control variable (ICI) and the target variable (stock returns). This test ensures that the ICI can serve as an effective control variable in the Monte Carlo simulation, enhancing the accuracy of the CVaR estimation.

This study aims to enhance the accuracy of Conditional Value at Risk (CVaR) measurement by applying the Monte Carlo Control Variates method using the Indonesia Composite Index

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(ICI) as a control variable. By utilizing ICI, the study expects to achieve more accurate and efficient CVaR estimates for Indonesian stock portfolios, thereby contributing to improved investment risk management practices in the Indonesian capital market.

2. Methods

The analytical process for this research involves the following steps:

1. Calculate the return of BBRI, BBNI, and ICI, followed by a normality test on the stock returns. If the return data do not follow a normal distribution, adjustments are made accordingly.
2. Calculate the combined portfolio return by determining the optimal portfolio weights using MVEP method.
3. Assess the suitability of ICI as a control variable using the Pearson correlation method. If ICI does not meet the criteria, an alternative control variable is selected.
4. Calculate the CV portfolio return, determine the mean and standard deviation parameters for both the standard portfolio and the CV portfolio, and conduct Monte Carlo Simulation (MCS) and Monte Carlo Control Variates (MCCV) for portfolio returns.
5. Compute the VaR and CVaR values from MCS and MCCV simulations then, conclude the findings based on the comparison of the average VaR and CVaR values between the MCS and MCCV methods.

2.1. Stock Return

Stock is a type of financial instrument that shows ownership of a company and represents a claim to a portion of the assets and profits obtained by the Company [1]. In investing in stocks, investors will expect a high rate of return. Return stocks are the profits or losses that investors make from changes in the stock price over a certain period. Expected return is calculated based on the historical average of return obtained, this value helps investors determine whether a stock is worthy of inclusion in an investment portfolio. To calculate the return on stock, the following formula is used [11]:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right). \tag{1}$$

2.2. Kolmogorov-Smirnov Normality Test

A normality test is a statistical method to assess whether a dataset conforms to a normal distribution [12]. One of the commonly used normality tests is the Kolmogorov-Smirnov test. The steps in conducting the Kolmogorov-Smirnov test to test the normality of the data are as follows [13]:

a. Hypothesis
   - H_0: Data is distributed normally.
   - H_1: Data is not distributed normally.

b. Test Statistic

$$D_{\text{calculated}} = \max\left|F_0(x) - S_n(x)\right|, \tag{2}$$

where $F_0(x)$ is the theoretical cumulative frequency distribution and $S_n(x)$ is the cumulative frequency distribution of the observation score.

c. Test Criteria If the $D_{\text{calculated}}$ is less than the D_{table}, or the p-value exceeds the significance level ($\alpha = 5\%$), then H_0 is accepted, indicating that the data is distributed normally. On the other hand, if the $D_{\text{calculated}}$ is greater than the D_{table} or the p-value is below the significance level ($\alpha = 5\%$), H_0 is rejected, suggesting that the data is not distributed normally.

2.3. Mean Variance Efficient Portfolio

A stock portfolio is a collection of various stocks owned by investors to minimize risk and maximize returns [10]. Portfolio diversification is done by spreading investments into various stocks that have different risk characteristics. In this way, the specific risk of one stock can be offset by other stocks that may not be perfectly correlated [14]. To calculate return from the stock portfolio, the following formula is used [15]:

$$R_{p,t} = \sum_{i=1}^{n} R_{i,t} \cdot w_i, \tag{3}$$

where $R_{p,t}$ is the return of the portfolio in period t, $R_{i,t}$ is the return of the i-th stock in period t, w_i is the weight of the i-th stock, and n is the total number of stocks in the portfolio.

MVEP is one of the techniques that can be used to calculate the optimal weight of a portfolio of various assets [16]. MVEP refers to a portfolio that offers the highest expected return for a certain level of risk or the lowest risk for a certain level of expected return. The portfolio is formed based on the calculation of variance and covariance of asset returns, which allows for optimal risk diversification.

Portfolio risk depends not only on the individual risk of the asset but also on the correlation between the assets in the portfolio [14]. Therefore, by choosing the right combination of assets, an investor can minimize risk without having to sacrifice expected returns. Portfolio weighting with the MVEP method can be calculated using the following formula [17]:

$$\boldsymbol{w} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}_N}{\mathbf{1}_N{}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}_N}, \tag{4}$$

where $\boldsymbol{w}$ is the weight or proportion of stocks, $\boldsymbol{\Sigma}^{-1}$ is the inverse variance-covariance matrix of stock returns, $\mathbf{1}_N$ is a unit vector with dimension $N \times 1$ and $\mathbf{1}_N{}^T$ is a unit vector with dimension $1 \times N$.

To form a variance-covariance matrix of a portfolio consisting of two stocks, the following formula can be used [17]:

$$\Sigma = \begin{bmatrix} \text{var}(R_1) & \text{cov}(R_1, R_2) \\ \text{cov}(R_1, R_2) & \text{var}(R_2) \end{bmatrix}, \tag{5}$$

where Σ is the variance-covariance matrix of stock returns, var(R_1) is the variance of the first stock return, var(R_2) is the variance of the second stock return, and cov(R_1, R_2) is the covariance between the first and second stock returns.

2.4. Pearson Corellation Test

The Pearson correlation test is a statistical technique used to evaluate the relationship between two variables. A statistical method that measures the extent of the linear relationship between two quantitative variables. This correlation test is used when the two variables tested are normally distributed [18]. The correlation value ranges from -1 to 1. The formula for Pearson's correlation coefficient is as follows [19]:

$$r = \frac{\text{cov}(R_1, R_2)}{s(R_1) \times s(R_2)}, \tag{6}$$

where r is the Pearson correlation value, $\text{cov}(R_1, R_2)$ is the covariance of variables R_1 and R_2, $s(R_1)$ is the standard deviation of the first stock return, and $s(R_2)$ is the standard deviation of the second stock return. Covariance between R_1 and R_2 can be calculated using eq. (7) [19]:

$$\text{cov}(R_1, R_2) = \frac{\sum_{t=1}^{n}(R_{1,t} - \bar{R}_1)(R_{2,t} - \bar{R}_2)}{n-1}, \tag{7}$$

where $R_{1,t}$ is the first stock return in period t, $R_{2,t}$ is the second stock return in period t, $\bar{R}_1$ is the average of the first stock return, $\bar{R}_2$ is the average of the second stock return, t is the time period, and n is the number of data points.

2.5. Value at Risk and Conditional Value at Risk

VaR has been widely used as a measure of risk in financial risk management during a certain period which is generally relatively short [4]. This method estimates the maximum possible loss within a specific period at a certain confidence level [20]. Mathematically, VaR for a period with confidence level $(1-\alpha)$ can be expressed as [21]:

$$\text{VaR}_{(1-\alpha)} = V_0 R^* \sqrt{t}, \tag{8}$$

where $\text{VaR}_{(1-\alpha)}$ is the maximum potential loss with a confidence level of $(1-\alpha)$, V_0 is the initial investment fund, R^* is the α-th quantile of the return distribution, and t is the period.

CVaR is a development of VaR designed to overcome the limitations of measuring tail risk [22]. CVaR calculates the average loss that exceeds VaR, thus providing a more comprehensive picture of the risks faced by the portfolio in extreme market situations [23]. CVaR becomes a solution to handle asymmetrical return distribution. CVaR can be calculated using the following formula [24]:

$$\text{CVaR}_{(1-\alpha)} = \mathbb{E}(X \mid X \leq \text{VaR}_{(1-\alpha)}), \tag{9}$$

where $\text{CVaR}_{(1-\alpha)}$ is the potential loss exceeding VaR with a confidence level of $(1-\alpha)$, $\mathbb{E}$ is the expectation function, and X is a random variable representing the loss experienced by the portfolio.

2.6. Monte Carlo Simulation

Monte Carlo simulation is a numerical method used to model the various possible outcomes of a complex process [20]. In the context of financial risk measurement, Monte Carlo simulation is used to estimate VaR and CVaR by simulating the distribution of portfolio returns in various market conditions. This method is used to analyze the propagation of uncertainty with the aim of determining how random variance or error affects the sensitivity, performance, or reliability of the system being modelled [25].

2.7. Control Variates

Control Variates (CV) is used to improve efficiency in VaR estimation by using a variable that correlates with the target variable as a control variable [26]. This technique works by utilizing additional information from the control variable to reduce the variability of the simulation results [7]. CV is very effective when the value of the parameter $c = \mathbb{E}(h(X))$, where $h(X) \approx f(X)$, with $h(X)$ being the return of the control variable and $f(X)$ being the return of the target variable [26]. This means that the stronger the correlation between the control variable return and the target variable return, the more effective the CV method is in reducing the variance of the Monte Carlo estimate. CV can be expressed as [8]:

$$R_{cv} = R_p + c\left(\mathbb{E}(h(X)) - h(X)\right), \tag{10}$$

where R_{cv} is the CV portfolio return, R_p is the portfolio return, c is the CV parameter, $\mathbb{E}(h(X))$ is the expected return of the control variable, and $h(X)$ is the return of the control variable.

The coefficient c can be calculated using the following formula [8]:

$$c = \frac{\text{Cov}(R_p, h(X))}{\sigma^2_{h(X)}}, \tag{11}$$

where $\text{Cov}(R_p, h(X))$ is the covariance between the portfolio return and the control variable return, and $\sigma^2_{h(X)}$ is the variance of the control variable return.

3. Results and Discussion

3.1. Data Description

This study uses secondary data obtained from the www.yahoofinance.com in the form of the daily closing price of BBRI and BBNI stock from the period of March 1, 2023, to February 29, 2024. The methods used include MCS and MCCV simulations, using ICI as a control variable. The following is a plot of BBRI and BBNI's daily closing price data:

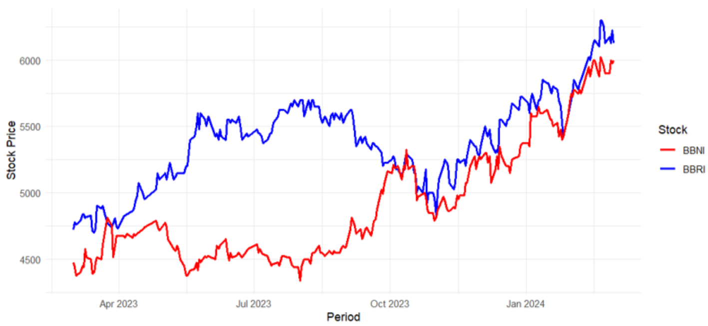

Figure 1. Plot of BBRI and BBNI stock prices

Figure 1 shows the movement of BBRI and BBNI stock prices. In May 2023, BBRI's stock price showed a significant uptrend, while BBNI's stock decreased. The increase in BBRI's stock price is influenced by several fundamental factors, such as the announcement of positive financial report results or broader market movements that benefit large issuers such as BBRI. On the other hand, the decline in BBNI's stock price was caused by external factors such as profit-taking activities carried out by investors and investor sentiment.

3.2. *Return of BBRI and BBNI Stock*

Stock closing price data is used to calculate the daily stock returns of BBRI and BBNI using eq. (1).

BBRI Stock:

$$R_2 = \ln\left(\frac{S_2}{S_1}\right) = \ln\left(\frac{4780}{4720}\right) = 0.01263.$$

BBNI Stock:

$$R_2 = \ln\left(\frac{S_2}{S_1}\right) = \ln\left(\frac{4438}{4475}\right) = -0.00841.$$

The plot of return on BBRI and BBNI stock can be seen in Figure 2.

Figure 2. Plot return of BBRI and BBNI stocks

From Figure 2, it can be seen that the movement of BBRI and BBNI stock returns during the observed period shows quite high fluctuations where the return price movements of the two stocks have similar characteristics. Both stocks had extreme spikes in April. In May, BBNI stock had a small surge until October, although there was a high spike at several points. Meanwhile, in May, BBRI stock still showed high volatility until June. These two stocks show similar return movements, but BBRI more often experiences high price changes. This indicates that BBRI stock is more sensitive to market changes compared to BBNI stock. The return on the first day for BBRI stock is 0.01263, while for BBNI stock is -0.008415. This value shows that on the first day, BBRI stock increased by 1.26%, while BBNI stock decreased by 0.84%. Descriptive statistics of stock returns can be seen in Table 1.

Table 1 shows that the returns of BBRI and BBNI stock have similar characteristics. The minimum return for BBRI is -0.05053, while for BBNI is -0.05129, indicating that the worst decline in BBNI is slightly larger than that of BBRI. The maximum return for BBRI was 0,04609, while BBNI recorded 0.04567, indicating that the biggest upside potential is almost the same. The average return of BBRI of 0.00110 and BBNI of 0.00124 indicates

Table 1. Descriptive statistics of BBRI and BBNI

Descriptive Statistics	BBRI	BBNI
Min	-0.05053	-0.05129
Max	0.04609	0.04567
Mean	0.00110	0.00124
Variance	0.00018	0.00017
Standard deviation	0.01340	0.01306

that both stocks provide small positive returns in the analysis period. In terms of volatility, the variance of the return of BBRI (0.00018) is slightly higher than that of BBNI (0.00017), and this is also reflected in the standard deviation of BBRI (0.01340) which is slightly larger than BBNI (0.01306). This indicates that BBRI has slightly higher price fluctuations than BBNI.

3.3. *Stock Return Normality Test*

In this study, the normality test was carried out using the Kolmogorov-Smirnov test, which was tested with the assistance of RStudio to calculate the p-value of stock return data. The following are the results of the normality test on BBRI and BBNI stock return data. The results of the calculation using Rstudio are presented in Table 2.

Table 2. Kolmogorov-Smirnov test results

Stock	p-value
BBRI	0.24620
BBNI	0.11320

Based on the results of the Kolmogorov-Smirnov test calculated using RStudio, the p-value for BBRI and BBNI stock returns is 0.24620 and 0.11320 respectively. The p-value of each stock return is greater than the significance level of 5% ($a = 0.05$), so the zero hypothesis (H_0) is accepted for BBRI and BBNI stocks, which shows that the return data of BBRI and BBNI stocks are normally distributed.

3.4. *MVEP Stock Weight*

Calculating the the optimal weight of each BBRI and BBNI stock can be calculated using eq. (4).

$$\begin{bmatrix} \boldsymbol{w_1} \\ \boldsymbol{w_2} \end{bmatrix} = \frac{\begin{bmatrix} 6087.602 & -1816.565 \\ -1816.565 & 6402.303 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 6087.602 & -1816.565 \\ -1816.565 & 6402.303 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \begin{bmatrix} 0.48223 \\ 0.51776 \end{bmatrix}.$$

The optimal weight obtained shows that for a portfolio consisting of BBRI and BBNI stocks, around 48.22% of the total investment should be allocated to BBRI stock, while the remaining 51.78% is allocated to BBNI stock. This combination is based on the goal of minimizing the total risk of the portfolio while maintaining optimal expected returns. The greater weight on BBNI stock indicates that this stock contributes more significantly to the overall portfolio risk.

3.5. Return Portfolio

After determining the optimal weight for BBRI and BBNI stocks, the next step is to calculate the portfolio return. The portfolio return is calculated as a weighted average of the returns of each stock with a predetermined weight using eq. (3):

$$R_{p,t} = \sum_{i=1}^{k} R_{i,t} \cdot w_i$$

For the first period:

$$\begin{aligned} R_{p,1} &= R_{1,1} \cdot w_1 + R_{2,1} \cdot w_2 \\ &= 0.01263 \times 0.4822 + (-0.00841) \times 0.5178 \\ &= 0.00173. \end{aligned}$$

The daily return in the first period of BBRI and BBNI stocks is 0.01263 and -0.00841, respectively. Thus, the portfolio return for the first period is 0.00173. This result reflects the performance of the optimal combination of BBRI and BBNI stock returns based on calculated weights.

3.6. Portfolio Return with Control Variates

Before applying the Control Variates (CV) technique, the return of the control variable (Indonesia Composite Index, ICI) is calculated using eq. (1), and a correlation test is performed using eq. (6):

$$R_{2,\text{ICI}} = \ln\left(\frac{6857}{6845}\right) = 0.00182$$

The ICI return in the first period was 0.00182, indicating a 1.82% increase.

Table 3. Descriptive statistics of ICI return

Descriptive Statistics	ICI
Min	-0.02161
Max	0.01699
Mean	0.00028
Variance	0.00003
Standard Deviation	0.00602

Descriptive statistics show moderate fluctuation and low volatility of the ICI index, making it a suitable control variable. Covariance between R_p and R_{ICI} is calculated using eq. (7):

$$\begin{aligned} \text{cov}(R_p, R_{\text{ICI}}) &= \frac{\sum_{t=1}^{238}(R_{1,t} - 0.00173)(R_{2,t} - 0.00028)}{237} \\ &= 0.00003664. \end{aligned}$$

Using eq. (6), the Pearson correlation value is calculated as:

$$r = \frac{0.00003664}{0.01063 \times 0.00603} = 0.57206.$$

This moderate positive correlation confirms ICI is a suitable control variable. The CV coefficient c is then calculated using eq. (11):

$$c = \frac{\text{cov}(R_p, R_{\text{ICI}})}{s^2_{R_{\text{ICI}}}} = \frac{0.00003664}{0.00003635} = 1.00821.$$

Using eq. (10), the CV-adjusted portfolio return is calculated as:

$$\begin{aligned} R_{cv,1} &= R_{p,1} + c(\bar{R}_{\text{ICI}} - R_{\text{ICI},1}) \\ &= 0.00173 + 1.00821(0.00028 - 0.00182) \\ &= 0.00018. \end{aligned}$$

This return shows the result after variance reduction through the CV technique, making it more stable.

3.7. VaR and CVaR using MCS and MCCV

One-day VaR and CVaR were calculated using Monte Carlo Simulation (MCS) and Monte Carlo Control Variates (MCCV). The simulation was run 1000 times in RStudio, using mean and standard deviation parameters. VaR was calculated using eq. (8) and CVaR using eq. (9). The average results across iterations are shown in Table 4.

Table 4. Average results of VaR and CVaR

Confidence Level	VaR		CVaR	
	MCS	MCCV	MCS	MCCV
99%	2.271%	1.849%	2.569%	2.084%
95%	1.611%	1.298%	2.050%	1.662%
90%	1.229%	0.993%	1.730%	1.400%

At a 99% confidence level, the VaR from MCS is 2.271%, meaning there's a 1% chance the loss exceeds 2.271%. CVaR from MCS is 2.569%, representing the average loss beyond that threshold. With MCCV, the VaR is reduced to 1.849%, and CVaR to 2.084%, showing that the use of ICI as a control variable improves risk estimation. At 90% and 95% confidence levels, the MCCV method consistently yields lower VaR and CVaR values compared to MCS, confirming that the CV technique effectively reduces variance in portfolio risk estimation.

4. Conclusion

The Standard Monte Carlo Simulation (MCS) is used to calculate the Conditional Value at Risk (CVaR) on BBRI and BBNI's stock portfolios. The calculation results show that the CVaR value of MCS at the 90% confidence level is 1.730%, at the 95% confidence level is 2.050%, and at the 99% confidence level is 2.569%. The Monte Carlo Control Variates (MCCV) technique is also applied to calculate CVaR on the same portfolio. The CVaR value of MCCV shows that at a 90% confidence level of 1.400%, at a 95% confidence level of 1.662%, and at a 99% confidence level of 2.084%.

The calculation results showed a clear difference between the CVaR values produced by MCS and MCCV, with MCCV consistently yielding lower values across all confidence levels. This suggests that the MCCV technique is more effective in reducing portfolio risk due to its ability to optimize the use of ICI control variables. For instance, if an investor allocates IDR 100,000,000 to this portfolio, the estimated maximum one-day loss at the 99% confidence level is IDR 2,569,000 using MCS, compared to a lower loss of IDR 2,084,000 with MCCV. Similar reductions are observed at other confidence levels, reinforcing MCCV's reliability in providing more conservative and accurate risk estimates. Further research could explore the comparative effectiveness of other variance reduction methods, such as Antithetic Variates or Latin Hypercube Sampling, in portfolio risk estimation.

Author Contributions. **Fahmi Giovani Maga**: Conceptualization, methodology, software, validation, formal analysis, resources, data curation, writing–original draft preparation, writing—review and editing, visualization. **Evy Sulistianingsih**: Conceptualization, methodology, writing–review and editing, supervision, funding acquisition. **Neva Satyahadewi**: Conceptualization, methodology, writing–review and editing, supervision, funding acquisition. All authors have read and agreed to the published version of the manuscript.

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