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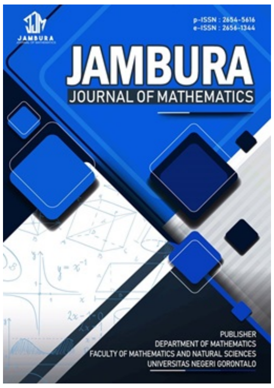

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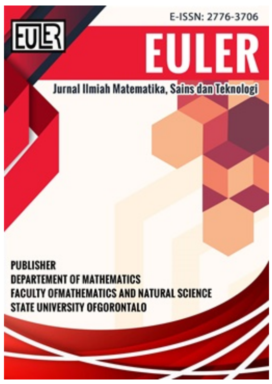

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Research Article

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Dynamical Analysis of Online Shopping with Beauty Influencers

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ABSTRACT. *This study develops a modified nonlinear mathematical model of online shopping dynamics that explicitly incorporates the direct influenced of beauty influencers through promotional content* (F) *and a parameter* α *for transition from offline to online shopping without the influenced of influencers. The model comprises offline shoppers* (L)*, online shoppers* (O)*, and the amount of promotional content created by beauty influencers* (F)*. Stability analysis shows two equilibrium points: an online shopping free state* E_1 *locally asymptotically stable when* $R_0 < 1$ *and an endemic state* E_2 *locally asymptotically stable when* $R_0 > 1$*. Numerical simulations using MATLAB R2013a confirm the analysis, revealing that higher promotional content growth rates* (k) *and lower decline rates* (θ) *increase* R_0 *and the online shoppers population. The novelty lies in explicitly modeling influencers based promotional content as a driver of shopping behavior, offering strategic insights for sustained engagement and customer retention in beauty sector digital marketing.*

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1. Introduction

Online shopping has increasingly become an integral component of contemporary consumer behavior, transforming not only the way people purchase goods and services but also redefining broader patterns of economic interaction. The popularity of online commerce is closely associated with the conveniences it offers, including competitive pricing, diverse product availability, and the elimination of physical queues for consumers [1]. Globally, more than 2.64 billion individuals engaged in online purchases in 2023, illustrating the vast scale of this phenomenon [2]. In Indonesia, the rapid growth of internet penetration has been a key driver for the expansion of e-commerce. By 2023, the number of internet users surpassed 215 million, while e-commerce users reached approximately 58.63 million [3–5]. These statistics not only highlight the technological progress achieved but also signal a broader societal shift, in which efficiency, accessibility, and flexibility are increasingly prioritized in everyday transactions. Online shopping, therefore, is no longer considered merely an alternative to conventional retail but has evolved into a dominant and embedded lifestyle choice. This rapid expansion reflects a structural transformation in consumption patterns rather than a passing trend.

While affordability and convenience remain primary drivers of online purchasing, digital promotional strategies have emerged as a powerful influence on consumer decision-making, particularly through the role of social media influencers [6]. Influencers, by leveraging credibility, relatability, and constant digital engagement, have created a new marketing paradigm that differs substantially from traditional advertising. A survey conducted by Rakuten Insight in 2023 reported that approximately 68% of Indonesian respondents purchased beauty products after viewing endorsements from influencers [5]. Such data illustrate the tangible economic effect of influencer-driven marketing. Content created on platforms like Instagram, TikTok, and YouTube exerts a significant psychological and social influence, as purchasing decisions are increasingly shaped by social validation and perceived trust rather than by intrinsic product features [7]. Consequently, the phenomenon marks a cultural shift in consumer behavior, wherein influencer-driven promotion operates not as a peripheral strategy but as a central mechanism of online marketing dynamics.

Despite its evident importance, mathematical investigations of online shopping dynamics have thus far remained limited in scope. Most existing models focus on information diffusion or general advertising effects, without explicitly incorporating influencer-driven marketing as a distinct variable. Nonlinear mathematical models, particularly systems of ordinary differential equations, have proven effective in capturing the complexities of socio-economic interactions [8, 9]. For instance, Sharma [1] introduced a nonlinear system, referred to as the NSP model, where N denotes offline shoppers, S online shoppers, and P the information generated by online shoppers. This framework demonstrated how the dissemination of positive information accelerates the transition of consumers toward online shopping platforms. Similarly, Misra et al. [10] examined the influence of social media advertising using comparable modeling techniques. These contributions underscore the centrality of information and advertising in shaping consumer dynamics, yet they fall short of addressing the distinct and increasingly influential role of social media influencers.

To address this gap, the present study extends Sharma's NSP model [1] by explicitly integrating the effect of promotional content created by beauty influencers. A new parameter, α, is in-

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Table 1. List of variable

Variabel	Description	Unit	Condition
$L(t)$	Subpopulation of offline shoppers	Individual	$L(t) \geq 0$
$O(t)$	Subpopulation of online shoppers	Individual	$O(t) \geq 0$
$F(t)$	The amount of promotional content created by beauty influencers	Promotion content	$F(t) \geq 0$

troduced to describe consumer transition from offline to online shopping independent of influencer effects, while the variable P is replaced by F, representing influencer-generated promotional content. Through this modification, the model provides a more nuanced framework to capture the dynamics of consumer behavior in the digital age. Specifically, it allows for a deeper analysis of how influencer-driven promotions accelerate shifts in consumption from offline to online platforms. In doing so, the study contributes both to the theoretical development of socio-economic modeling and to practical insights for e-commerce marketing strategies in Indonesia and beyond.

2. Methods

This research uses a descriptive method with the following stages:

1. Identify the problem.
2. Literature studies.
3. Determination of model assumptions, variables, and parameters.
4. Mathematical Model Building.
5. Dynamic Analysis of Mathematical Models.
   Analyze by determining the equilibrium point and stability of the equilibrium point of the model.
   a Equilibrium Point
   An equilibrium point is a state where all variables in the system remain constant over time. For an autonomous system defined as $\frac{dx}{dt} = f(x)$, the equilibrium point x^* satisfies $f(x^*) = 0$.
   b Basic Reproduction Number (R_0)
   The Basic Reproduction Number R_0, quantifies the influenced of promotional content by beauty influencers on consumers switching from offline to online shopping. It is calculated via the next generation matrix method.
   c Equilibrium Point Stability
   Stability analysis is conducted using the Jacobian matrix, which examines system behavior near the equilibrium point. The Jacobian matrix is:

$$J(f(x)) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x^*) & \frac{\partial f_1}{\partial x_2}(x^*) & \cdots & \frac{\partial f_1}{\partial x_n}(x^*) \\ \frac{\partial f_2}{\partial x_1}(x^*) & \frac{\partial f_2}{\partial x_2}(x^*) & \cdots & \frac{\partial f_2}{\partial x_n}(x^*) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1}(x^*) & \frac{\partial f_n}{\partial x_2}(x^*) & \cdots & \frac{\partial f_n}{\partial x_n}(x^*) \end{bmatrix}.$$

   The eigenvalues of $J(f(x))$ are found by solving $|J(f(x)) - \lambda I| = 0$. According to [11], the stability conditions for an equilibrium point x^* are:
   i. $x^* = 0$ is stable if and only if the real part of all eigenvalues is negative or equal to zero.
   ii. $x^* = 0$ is asymptotically stable if and only if the real part of all eigenvalues is negative.
   iii. $x^* = 0$ is unstable if at least one eigenvalue has a positive real part.
6. Numerical simulation and interpretation
   Numerical simulations were performed using MATLAB R2013a with the fourth-order Runge–Kutta method implemented with a fixed step size of 0.1 days, over a simulation period of 0–450 days. No adaptive convergence criteria were applied, as the method was executed for the entire time range with the specified step size. The results were then analyzed and interpreted to examine the influenced of parameters on the system's dynamics.

3. Results and Discussion

3.1. Mathematical Model of Online Shopping Dynamics with Beauty Influencers

To construct a mathematical model that describes the dynamics of online shopping under the influenced of beauty influencers, it is necessary to first define the variables, model assumptions, and parameters. The variables used are presented in Table 1.

The model is constructed based on the following assumptions:

1. New individuals entering the system are assumed to initiate offline shopping activities at a rate A. An example is a purchase at a physical store, market, or mall. In this model, there is no direct move to online shopping without first making an offline purchase.
2. Offline shoppers can switch to online shopping due to the influenced of promotional content from beauty influencers, at the rate of β and can switch independently, without the influenced of beauty influencers, for example factors such as convenience, price, personal needs, or word-of-mouth recommendations, at the rate of α [12, 13].
3. Online shoppers may return to being offline shoppers at the rate of δ. This can happen due to negative experiences, such as unsuitable product quality, not being able to try the product directly, or late delivery [14].
4. Both offline and online shoppers can exit the system at the μ rate. This describes situations where individuals stop shopping altogether or for other reasons.
5. The amount of promotional content increases as the number of online shoppers increases, at a rate of k. A larger online shoppers increases the opportunity for promotional activities by influencers, as the effectiveness of digital marketing relies heavily on influencers engagement [15].
6. Promotional content may decline at a rate of θ. This can be caused by audience saturation, controversies involving influencers, decreased content relevance, or loss of audience trust [16, 17].

Subsequently, the parameters used in the model are presented in Table 2.

Table 2. List of parameters

Parameter	Description	Unit	Condition
A	Individual who have just entered the shopping system	$\frac{\text{individual}}{\text{day}}$	$A \geq 0$
α	Transition rate from individual in $L(t)$ to $O(t)$ without the influenced of beauty influencers	$\frac{1}{\text{individual}\cdot\text{day}}$	$\alpha \geq 0$
β	Transition rate from individual in $L(t)$ to $O(t)$ due to the influenced of beauty influencers	$\frac{1}{\text{promotional content}\cdot\text{day}}$	$\beta \geq 0$
δ	Transition rate from individual in $O(t)$ to $L(t)$	$\frac{1}{\text{day}}$	$\delta \geq 0$
μ	Rate at which individual exit the shopping system	$\frac{1}{\text{day}}$	$\mu \geq 0$
k	Growth rate of promotional content by beauty influencers	$\frac{\text{promotional content}}{\text{individual}\cdot\text{day}}$	$k \geq 0$
θ	Decline rate of promotional content by beauty influencers due to various factors	$\frac{1}{\text{day}}$	$\theta \geq 0$

Based on the assumptions that have been set, the schematic compartment diagram of the mathematical model of online shopping dynamics with the influence of beauty influencers is presented in Figure 1.

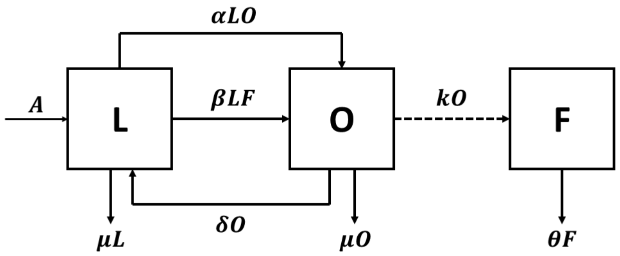

Figure 1. Mathematical model of online shopping dynamics with beauty influencers

Based on the Figure 1, the mathematical model describing online shopping dynamics influenced by beauty influencers is expressed as a nonlinear autonomous system as follows:

$$\begin{aligned} \frac{dL}{dt} &= A - \beta LF - \alpha LO + \delta O - \mu L, \\ \frac{dO}{dt} &= \beta LF + \alpha LO - \delta O - \mu O, \\ \frac{dF}{dt} &= kO - \theta F. \end{aligned} \tag{1}$$

with the initial condition $L(0), O(0), F(0) \geq 0$, and the number of individuals expressed by $N(t) = L(t) + O(t)$, the solution of the system (1) is confined to the region:

$$\Pi = \left\{ (L(t), O(t), F(t)) \mid L(t), O(t) \leq \frac{A}{\mu},\ 0 \leq F(t) \leq \frac{kA}{\mu\theta} \right\}.$$

3.2. Analysis of Mathematical Models of Online Shopping Dynamics with Beauty Influencers

In analyzing the dynamics of online shopping with the influenced of beauty influencers, the equilibrium point, equilibrium point stability, and simulation will be determined in the analysis of the mathematical model.

3.2.1. Equilibrium Point

The equilibrium point is a state in the system of equations (1) that satisfies the condition

$$\frac{dL}{dt} = \frac{dO}{dt} = \frac{dF}{dt} = 0. \tag{2}$$

Based on eq. (2), two equilibrium points of system (1) are obtained, namely: the online shopping free equilibrium (when there are no online shoppers)

$$E_1 = (L_1^*, O_1^*, F_1^*) = \left(\frac{A}{\mu}, 0, 0 \right), \tag{3}$$

and the endemic online shopping equilibrium (when online shoppers are present)

$$E_2 = (L_2^*, O_2^*, F_2^*), \tag{4}$$

where:

$$\begin{aligned} L_2^* &= \frac{(\delta + \mu)\theta}{\beta k + \alpha\theta}, \\ O_2^* &= \frac{\alpha A\theta + \beta Ak - \delta\mu\theta - \mu^2\theta}{\mu(\beta k + \alpha\theta)}, \\ F_2^* &= \frac{k(\alpha A\theta + \beta Ak - \delta\mu\theta - \mu\delta\theta - \mu^2\theta)}{\mu(\beta k + \alpha\theta)\theta}. \end{aligned}$$

The equilibrium point E_2 will exist if $\alpha A\theta + \beta Ak \geq \delta\mu\theta + \mu^2\theta$.

3.2.2. Basic Reproduction Number (R_0)

The basic reproduction number, denoted by R_0, serves as a measure of the extent to which promotional content by beauty influencers can encourage the transition of consumers from offline shopping to online shopping. The value of R_0 is derived using the next-generation matrix method. From system (1), the equation describing the dynamics of the growth in online shopping is extracted as follows:

$$\begin{aligned} \frac{dO}{dt} &= \beta LF + \alpha LO - \delta O - \mu O, \\ \frac{dF}{dt} &= kO - \theta F. \end{aligned} \tag{5}$$

Based on the eq. (5), the W and Z matrices are obtained as follows:

$$W = \begin{bmatrix} \beta LF + \alpha LO \\ 0 \end{bmatrix},\ Z = \begin{bmatrix} (\delta + \mu)O \\ -kO + \theta F \end{bmatrix}. \tag{6}$$

The Jacobian matrices of the eq. (6) are expressed by the matrices $W = J(W)$ and $Z = J(Z)$, resulting in

$$J(W) = \begin{bmatrix} \alpha L & \beta L \\ 0 & 0 \end{bmatrix},\ J(Z) = \begin{bmatrix} \delta + \mu & 0 \\ -k & \theta \end{bmatrix}. \tag{7}$$

Next, substitute the equilibrium point of free online shopping E_1 (eq. (3)) into the eq. (7), obtained:

$$J(W_{E_1}) = \begin{bmatrix} \frac{\alpha A}{\mu} & \frac{\beta A}{\mu} \\ 0 & 0 \end{bmatrix},\ J(Z_{E_1}) = \begin{bmatrix} \delta+\mu & 0 \\ -k & \theta \end{bmatrix}. \tag{8}$$

The next generation matrix (G) is defined based on the eq. (8) with $G = J(W_{E_1})J(Z_{E_1})^{-1}$, namely:

$$G = \begin{bmatrix} \frac{\alpha A}{(\delta+\mu)\mu} + \frac{\beta Ak}{\mu(\delta+\mu)\theta} & \frac{\beta A}{\mu\theta} \\ 0 & 0 \end{bmatrix}.$$

The basic reproduction number (R_0) is the largest positive eigenvalue of the matrix G namely $R_0 = \rho(G)$ with ρ the largest eigenvalue. To determine the eigenvalue of G, the characteristic equation of matrix G will be find the characteristic equation of the G matrix, namely $|G - \lambda I| = 0$, obtained:

$$\left(\frac{\alpha A}{(\delta+\mu)\mu} + \frac{\beta Ak}{\mu(\delta+\mu)\theta} - \lambda\right)(-\lambda) = 0.$$

So we get $\lambda_1 = \frac{\alpha A\theta+\beta Ak}{\delta\mu\theta+\mu^2\theta}$ and $\lambda_2 = 0$. So the basic reproduction number of the system (1) is:

$$R_0 = \frac{\alpha A\theta + \beta Ak}{\delta\mu\theta + \mu^2\theta}.$$

3.2.3. Analysis of Local Stability of Equilibrium Point

Based on the system, the Jacobian matrix (9) is obtained as follows:

$$J = \begin{bmatrix} -\beta F - \alpha O - \mu & -\alpha L + \delta & -\beta L \\ \beta F + \alpha O & \alpha L - \delta - \mu & \beta L \\ 0 & k & -\theta \end{bmatrix}. \tag{9}$$

The analysis of the local stability of the online shopping free equilibrium point can be analyzed by referring to the following theorem.

Theorem 1. *The equilibrium point E_1 is locally asymptotically stable if $R_0 < 1$*

Proof. Substitute E_1 into the Jacobian matrix (9), obtained:

$$J(E_1) = \begin{bmatrix} -\mu & -\frac{\alpha A}{\mu} + \delta & -\frac{\beta A}{\mu} \\ 0 & \frac{\alpha A}{\mu} - \delta - \mu & \frac{\beta A}{\mu} \\ 0 & k & -\theta \end{bmatrix} \tag{10}$$

Using cofactor expansion, the characteristic equation (10) is

$$(-\mu - \lambda)\left[\left(\frac{\alpha A}{\mu} - \delta - \mu - \lambda\right)(-\theta - \lambda) - \frac{\beta Ak}{\mu}\right] = 0.$$

The eigenvalue $\lambda_1 = -\mu$ and the other eigenvalues λ_2 and λ_3 are as follows:

$$\lambda^2 + \left(\delta + \mu + \theta - \frac{\alpha A}{\mu}\right)\lambda + \frac{\delta\mu\theta + \mu^2\theta - \alpha A\theta - \beta Ak}{\mu} = 0. \tag{11}$$

Previously obtained $R_0 = \frac{\alpha A\theta+\beta Ak}{\delta\mu\theta+\mu^2\theta}$, can be rewritten as $\alpha A\theta + \beta Ak = R_0(\delta\mu\theta + \mu^2\theta)$. Then substitute to eq. (11), obtained:

$$a\lambda^2 + b\lambda + c = 0, \tag{12}$$

with $a = 1; b = \delta + \mu + \theta - \frac{\alpha A}{\mu}; c = (\delta\theta + \mu\theta)(1 - R_0)$.

Based on the Descrates sign rule, eq. (12) has two negative roots when $b > 0$ and $c > 0$. The condition $c > 0$ is satisfied if $R_0 < 1$, so the eigenvalues λ_2 and λ_3 are negative when $R_0 < 1$. So the Theorem 1 is proven. □

Furthermore, the local stability of the endemic equilibrium point of online shopping can be analyzed by referring to the following theorem.

Theorem 2. *The equilibrium point E_2 is locally asymptotically stable if $R_0 > 1$*

Proof. Substituting E_2 into the Jacobian matrix (9), we get:

$$J(E_2) = \begin{bmatrix} -\beta F_2^* - \alpha O_2^* - \mu & -\alpha L_2^* + \delta & -\beta L_2^* \\ \beta F_2^* + \alpha O_2^* & \alpha L_2^* - \delta - \mu & \beta L_2^* \\ 0 & k & -\theta \end{bmatrix}. \tag{13}$$

Next, determine the eigenvalues of the matrix $J(E_2)$, by finding the characteristic equation $|J(E_2) - \lambda| = 0$, obtained:

$$\begin{vmatrix} -\beta F_2^* - \alpha O_2^* - \mu - \lambda & -\alpha L_2^* + \delta & -\beta L_2^* \\ \beta F_2^* + \alpha O_2^* & \alpha L_2^* - \delta - \mu - \lambda & \beta L_2^* \\ 0 & k & -\theta - \lambda \end{vmatrix} = 0. \tag{14}$$

Using cofactor expansion, the characteristic equation of eq. (14) is

$$m_0\lambda^3 + m_1\lambda^2 + m_2\lambda + m_3 = 0, \tag{15}$$

with

$$\begin{aligned} m_0 &= 1 > 0, \\ m_1 &= -(-\beta F_2^* + \alpha L_2^* - \alpha O_2^* - \delta - 2\mu - \theta) \\ &= \beta F_2^* - \alpha L_2^* + \alpha O_2^* + \delta + 2\mu + \theta, \\ m_2 &= -(-\beta\mu F_2^* - \beta\theta F_2^* + \alpha\mu L_2^* + \alpha\theta L_2^* + \beta kL_2^* - \alpha\mu O_2^* \\ &\quad - \alpha\theta O_2^* - \delta\mu - \delta\theta - \mu^2 - 2\mu\theta) \\ &= \beta\mu F_2^* + \beta\theta F_2^* - \alpha\mu L_2^* - \alpha\theta L_2^* - \beta kL_2^* + \alpha\mu O_2^* \\ &\quad + \alpha\theta O_2^* + \delta\mu + \delta\theta + \mu^2 + 2\mu\theta, \\ m_3 &= \beta\mu\theta F_2^* - \alpha\mu\theta L_2^* - \beta k\mu L_2^* + \alpha\mu\theta O_2^* + \delta\mu\theta + \mu^2\theta. \end{aligned}$$

Next, to show that all the eigenvalues of the real part of the eq. (15) are negative, the Routh-Hurwitz criterion is if the following conditions are met:

$$m_1 > 0,\ m_1m_2 - m_3 > 0.$$

By substituting the equilibrium points (L_2^*, O_2^*, F_2^*) into each coefficient, it can be proved that all conditions are satisfied if $R_0 > 1$. Therefore, all real parts of the eigenvalues are negative, so the endemic equilibrium point E_2 is locally asymptotically stable. So the theorem 2 is proved. □

Table 3. Parameter values (1)

Parameter	Value	Unit	Description	Source
A	10	$\frac{\text{individual}}{\text{day}}$	Recruitment rate of new consumers	[1]
α	0.00001	$\frac{1}{\text{individual}\cdot\text{day}}$	Rate at which individuals move from offline shopping to online shopping independently of beauty influencers	[18]
β	0.0002	$\frac{1}{\text{promotional content}\cdot\text{day}}$	The effectiveness rate of promotional content in attracting consumers	[1]
δ	0.03	$\frac{1}{\text{day}}$	Rate of consumers returning from online to offline	[1]
μ	0.02	$\frac{1}{\text{day}}$	Natural exit rate of consumers	[1]
k	0.03	$\frac{\text{promotional content}}{\text{individual}\cdot\text{day}}$	Content production rate by beauty influencers that scales linearly with the online shopper population.	[10]
θ	0.2	$\frac{1}{\text{day}}$	Decay rate of promotional content	[19]

3.3. Numerical Simulation and Interpretation

Numerical simulation aims to verify the results of the equilibrium point stability analysis and provide additional insights into behavioral dynamics that are not fully captured by equilibrium analysis. The simulation process is carried out by entering predetermined parameter values. There are two types of simulations performed, namely numerical simulations for online shopping free conditions and numerical simulations for online shopping endemic conditions.

3.3.1. Numerical Simulation under Online Shopping-Free Conditions

In accordance with Brauer & Castillo-Chavez [20], numerical simulations require parameter values that can be obtained from literature, field studies, or laboratory reports. When empirical measurements are unavailable, parameters can be estimated from related studies or statistical data, with mean values used for deterministic simulations and ranges applied in stochastic simulations to assess variability. The initial values and parameter values used in the model are presented in Table 3 and Table 4.

Table 4. List of initial values

Variable	Value	Unit	Source
$L(t)$	150	Individual	[1]
$O(t)$	150	Individual	[1]
$F(t)$	10	Promotional content	Assumption

Based on Table 3, the value of $R_0 = \frac{\alpha A\theta+\beta Ak}{\delta\mu\theta+\mu^2\theta} = 0.4 < 1$, then using Matlab 2013a software, the numerical simulation for the online shopping free equilibrium point is shown in Figure 2 with a time span of up to 450 days as follows.

The analytical findings indicate that the online shopping free equilibrium point (E_1) is locally asymptotically stable when $R_0 < 1$. This is consistent with the numerical simulations, where the chosen parameter values yielding $R_0 < 1$ lead the trajectories to converge to the shopping free equilibrium point, and the number of online shoppers approaches zero over time. Based on Figure 2, it can be observed that the subpopulation of offline shoppers increases toward the equilibrium point of 500, while the subpopulation of online shoppers and the amount of promotional content by beauty influencers decline to zero and stabilize at that point. This integrated analysis highlights that when the combined effect of influencer driven promotions and independent switching is too weak, online shopping cannot sustain itself

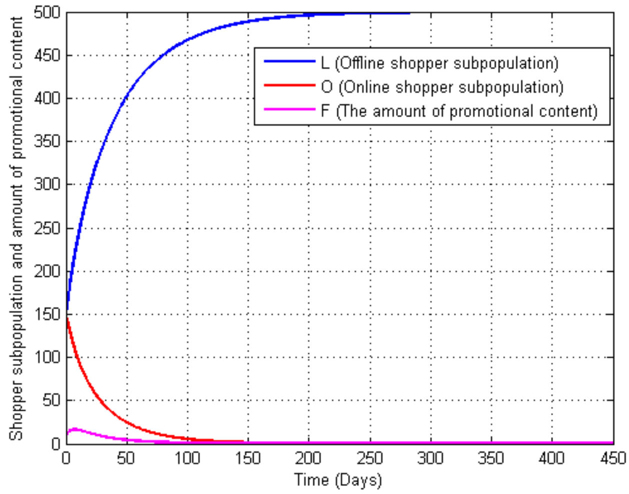

Figure 2. Numerical simulation under online shopping free conditions

in the market, leading consumer behavior to shift entirely toward offline channels.

3.3.2. Numerical Simulation under Endemic Online Shopping Conditions

For the simulation of the endemic online shopping condition, the parameters changed from the online shopping free condition are α and β, while all other parameters remain unchanged. The values of α and β used in the simulation are shown in Table 5.

In Table 5, the parameter β is assumed based on primary data collection. The calculation was carried out using the number of promotional contents posted by seven beauty influencers on Instagram, YouTube, and TikTok in 2023, totaling 251 posts or approximately 0.687 promotional content per day. According to [5], 68% of Indonesian respondents purchased beauty products based on influencers endorsements. In the same year, the number of e-commerce users in Indonesia was 58.63 million, while the total population reached 278.7 million. Therefore, the estimated non e-commerce population was 220.06 million. Of this group, approximately 68% (149.65 million people) were influenced by beauty influencers to switch to online shopping, with an average daily shift of $409,986$ people. Thus, β is calculated as the ratio between the daily shift and the product of the non e-commerce

Table 5. Parameter values (2)

Parameter	Value	Unit	Description	Source
α	0.002	$\frac{1}{\text{individual.day}}$	Rate at which individuals move from offline shopping to online shopping independently of beauty influencers	[10]
β	0.003	$\frac{1}{\text{promotional content.day}}$	The effectiveness rate of promotional content in attracting consumers	Assumption

population with the average daily promotional content:

$$\beta = \frac{409,986}{220,066,200 \times 0.687} = 0.003.$$

Numerical simulations for the endemic equilibrium point of online shopping carried out with the initial values presented in Table 4 and a time span of up to 450 days are as follows.

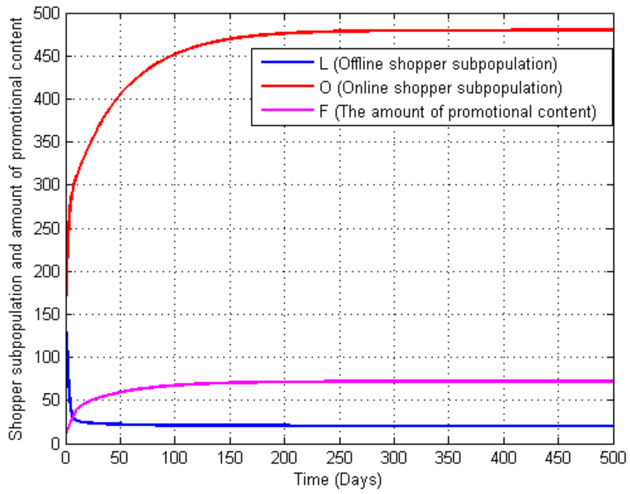

Figure 3. Numerical simulation under endemic online shopping conditions

The analytical findings indicate that the online shopping endemic equilibrium point (E_2) is locally asymptotically stable when $R_0 > 1$, and the simulations consistently show the trajectory to converge to the shopping endemic equilibrium point, and the number of online shoppers continues to increase. Figure 3 illustrates that offline shoppers decline to an equilibrium of 20.4, while online shoppers and influencers driven promotional content increase to 479.6 and 71.9, respectively. Together, these results validate the robustness of the model and highlight a long term balance where online shopping dominates without fully displacing offline shopping, reflecting real world conditions in which digital commerce expands rapidly while traditional retail maintains a niche presence.

Furthermore, numerical simulations were carried out for variations in the parameter values k (the growth rate of promotional content by beauty influencers) and θ (the rate of decline in promotional content by beauty inluencers), to review the effect of both parameters on the growth and stability of the number of online shoppers in the system. The following variations of parameter values k, θ and R_0 are used.

The following is a graph with different parameter values of k and θ.

Based on Figure 4 and Figure 5, the numerical simulation results show that the parameters k and θ significantly affect the

Table 6. Variation of parameter value k

Parameter Value	R_0 Value	Source
k=0.03	24.5	[10]
k=0.05	27.5	[1]
k=0.002	20.3	Assumption

Table 7. Variation of parameter value θ

Parameter Value	R_0 Value	Source
0.2	24.5	[19]
0.06	35	[1]
0.02	65	[10]

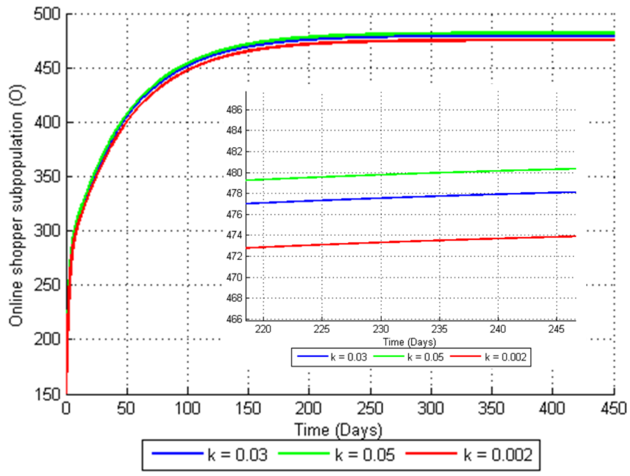

Figure 4. Effect of parameter k on online shopping subpopulation

number of online shoppers. A larger value of k and a smaller value of θ lead to an increase in the online shopper population, indicating that more frequent promotional content creation by influencers expands the reach of online shopping, while longer lasting promotional effects help sustain it over time. The implication is that marketing strategies should balance frequency and longevity of content, as frequent posts increase exposure, while high quality, evergreen content maintains consumer interest and purchase behavior in the long run. Furthermore, minor deviations in transient dynamics, such as oscillations before stabilization, highlight the sensitivity of the system to parameter choices, particularly the growth rate k and the decline rate θ, but they do not alter the long term stability conditions.

The findings of this study are consistent with prior research. Liu [21] emphasizes the role of influencers in shaping consumer purchasing decisions, while Lailia and Andriana [22] and Yoebrilianti [23] highlight how online promotions and social media strategies enhance consumer interest and purchase inten-

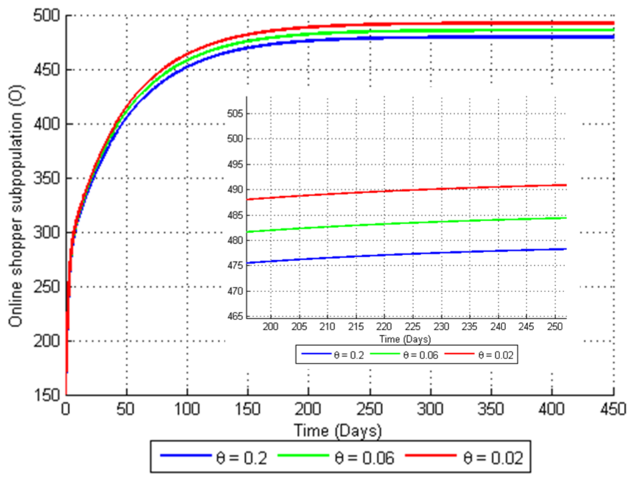

Figure 5. Effect of parameter θ on online shopping subpopulation

tions. These similarities validate the robustness of the proposed model in capturing real consumer dynamics under the influence of beauty influencers.

4. Conclusion

This study proposed a novel mathematical model to describe the dynamics of online shopping behavior under the influenced of beauty influencers. The model introduces a feedback mechanism between online shoppers and promotional content, capturing how influencers activity accelerates consumer adoption while the decline of promotional content reduces it. The analysis revealed the existence of two possible long term conditions: the absence of online shopping activity and the persistence of online shopping adoption. The transition between these conditions depends on the relative balance between the growth and decline of promotional content as well as the behavioral shifts of consumers between offline and online shopping. Numerical simulations demonstrated that an increase in the growth of promotional content, accompanied by a decrease in its decline, enhances the long term prevalence of online shopping. These findings highlight the strategic role of influencer based marketing in shaping consumer dynamics.

From a practical perspective, these results provide strategic insights for e-commerce platforms, brands, and marketing agencies. Specifically:

1. Sustained Engagement
   Maintaining a continuous stream of relevant and creative influencers content can extend consumer interest and reduce the decline rate θ.
2. Targeted Campaigns
   Allocating resources to influencers who closely align with brand identity can amplify the effect of k, accelerating the growth of the online shoppers base.
3. Consumer Retention Strategies
   Addressing issues that drive online shoppers back to offline shopping (e.g., delivery delays, product quality mismatches) can reduce δ, strengthening the long term stability of online shopping adoption.

These managerial insights highlight how mathematical modeling can be applied not only for theoretical understanding but also for designing data driven, influencers based marketing strategies that maximize long-term customer conversion and retention.

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