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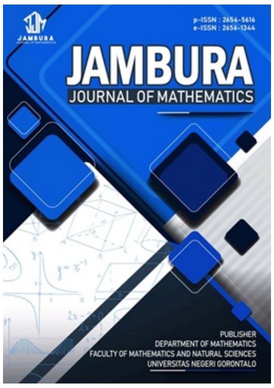

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Research Article

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The Connection Between Jordan Derivations and Nilpotent Derivations on 2-Torsion-Free Semiprime Rings

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ABSTRACT. *Let R be a ring. A derivation on a ring is an additive map that satisfies the Leibniz rule $d(ab) = d(a)b + ad(b)$, for every $a, b \in R$. A derivation d on a ring R is called a nilpotent derivation if there exists a natural number n such that $d^n(R) = 0$. This research studies nilpotent Jordan derivations on 2-torsion free semiprime rings. Motivated by the results of Bresar, Chung, and Luh on Jordan derivations and nilpotent derivations, the study aims to analyze their nilpotency patterns and the behavior of inner derivations generated by nilpotent elements. Using a deductive literature-study approach through mathematical proofs and examples, we show that if $d^{2n}(R) = 0$ for some $n \in \mathbb{N}$, then $d^{2n-1}(R) = 0$. Consequently, every nonzero nilpotent Jordan derivation has an odd nilpotency index. In addition, for a nilpotent element $P \in R$ with nilpotency index n, the inner derivation $d_p(x) = [p, x]$ is nilpotent with nilpotency index at most $2n - 1$. These results contribute to a deeper understanding of derivation structures on 2-torsion free semiprime rings.*

1. Introduction

The study of derivations plays a fundamental role in ring theory and noncommutative algebra. A derivation on a ring can be viewed as an algebraic analogue of differentiation in calculus and is characterized by the well-known Leibniz rule [1]. Since their introduction, derivations have become an essential tool for investigating the internal structure of rings, particularly in relation to ideal structure, commutativity conditions, and polynomial identities. Consequently, the theory of derivations has developed into one of the central topics in modern algebra.

Classical results have revealed that the behavior of derivations is strongly influenced by the structural properties of the underlying ring. For instance, investigations on commutator identities in simple rings carried out by Herstein [2] showed that certain nilpotency phenomena arise naturally in connection with commutator operations. These ideas motivated further studies on the interaction between derivations and nilpotent behavior in rings. In particular, Chung and Luh [3] studied nilpotent derivations and proved that on a 2-torsion-free semiprime ring the nilpotency index of a nilpotent derivation must be an odd integer. This result highlights the restrictive influence of semiprime and 2-torsion-free conditions on the behavior of derivations.

Another important generalization of derivations is the Jordan derivations. A Jordan derivation is an additive mapping satisfying the Jordan identity $d(x^2) = d(x)x + xd(x)$ for every x in a ring. Although this condition is formally weaker than the Leibniz rule, Jordan derivations often coincide with ordinary derivations under suitable structural assumptions. A fundamental result in this direction was obtained by Bresar [4], who proved that every Jordan derivation on a 2-torsion-free semiprime ring is in fact a derivation. This theorem reveals a deep structural connection between Jordan identities and the classical derivation property.

Over the past decades, derivation theory has expanded considerably and has been investigated in a wide variety of algebraic structures. For instance, Ern [5] examined the role of derivations in the structural analysis of factor rings, highlighting how derivations can reflect important properties of ring quotients. Generalizations of derivations have also been introduced in different module settings; for example, Fitriani [6] studied f-derivations on polynomial modules and demonstrated how derivation-type mappings can be extended beyond classical ring structures. More recent investigations have emphasized that the behavior of derivations is strongly influenced by the underlying algebraic framework [1, 7]. In particular, derivations and related mappings have been explored in several specialized systems, including skew generalized power series modules [8], polynomial rings with nil derivations [9], (σ, τ)-derivations on group rings [10], and (α', β')-derivations on polynomial rings $\mathcal{K}[x]$ [11]. These studies illustrate that derivation theory continues to evolve through various generalizations and applications in modern algebra.

Closely related to the theory of derivations, considerable attention has also been devoted to Jordan-type mappings and their generalizations. Jordan derivations arise naturally as a weaker form of derivations and have been extensively studied in various algebraic structures. For instance, Hongan and Rehman [12] investigated generalized Jordan $*$-derivations on semiprime rings with involution. Further generalizations have been studied in semiprime Γ-rings [13], prime rings through gen-

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eralized Jordan centralizers [14], and higher-order Jordan derivations [15]. Additional developments include centrally extended Jordan derivations [16], Jordan homoderivations on prime rings [17], and generalized Jordan derivations in arbitrary rings [18]. More recent contributions continue to extend the framework of Jordan-type mappings to broader algebraic contexts [19–25]. These studies confirm that Jordan derivations remain an active area of research in ring theory.

Besides Jordan derivations, the study of nilpotent derivations has also attracted considerable attention. Nilpotent derivations are closely related to the structure of rings, especially with respect to the generation of ideals and the stability of algebraic systems. For example, Hattori and Kojima [26] investigated rings whose nilpotent elements arise from monomial derivations on polynomial rings. Similarly, studies on locally nilpotent derivations have revealed important structural properties of polynomial algebras [27, 28]. These works indicate that nilpotent derivations play a crucial role in understanding algebraic structures and their invariants. However, relatively little attention has been devoted to the behavior of nilpotent mappings arising from Jordan derivations, particularly in the context of semiprime rings.

Despite the extensive research on derivations and Jordan derivations, the interaction between Jordan derivations and nilpotent derivations has not been systematically investigated in the context of 2-torsion-free semiprime rings. Existing studies typically examine either the structural characterization of Jordan derivations or the properties of nilpotent derivations separately. However, the combination of two classical results, namely Bresar's theorem on Jordan derivations [4] and the result of Chung and Luh on nilpotent derivations [3], suggests that a deeper relationship between these two types of mappings may exist under the 2-torsion-free semiprime condition

Motivated by this observation, this paper investigates the relationship between Jordan derivations and nilpotent derivations on 2-torsion-free semiprime rings. In particular, we analyze the consequences of combining the structural characterization of Jordan derivations with the odd nilpotency property of derivations. The main result shows that if a Jordan derivation on a 2-torsion-free semiprime ring is nilpotent, then its nilpotency index must be an odd integer. Furthermore, several examples are provided to illustrate the theoretical results and to clarify the role of the semiprime and 2-torsion-free conditions in determining the behavior of such derivations.

The results obtained in this paper contribute to a better understanding of the structural interaction between Jordan identities and nilpotent behavior in rings. They also provide additional insight into how classical results in derivation theory can be combined to yield further properties of algebraic mappings on semiprime rings.

2. Preliminaries

This section presents several basic concepts and preliminary results that will be used throughout the paper. In particular, we review fundamental notions related to rings, semiprime rings, 2-torsion-free rings, derivations, and Jordan derivations. These concepts form the theoretical foundation for the results developed in the subsequent sections. The definitions and classical results used in this paper follow standard references in ring theory such as [2, 4, 5, 29].

2.1. Rings and Semiprime Rings

We begin by recalling the definition of a ring and some related concepts that are essential for the discussion in this paper.

Definition 1. [29] Let R be a nonempty set equipped with two binary operations $+$ and $\cdot$. The structure $\langle R, +, \cdot\rangle$ is called a *ring* if the following conditions hold:

1. $\langle R, +\rangle$ is an Abelian group;
2. multiplication is associative, that is, $(a\cdot b)\cdot c = a\cdot(b\cdot c)$ for all $a, b, c \in R$;
3. the distributive laws hold:
$$a\cdot(b+c) = a\cdot b + a\cdot c, \qquad (a+b)\cdot c = a\cdot c + b\cdot c$$
for all $a, b, c \in R$.

Furthermore, it is conventional to write ab instead of $a\cdot b$. We now review one important class of rings frequently studied in derivation theory, namely semiprime rings.

Definition 2. [4] A ring R is called a *semiprime ring* if for any $a \in R$,
$$aRa = \{0\} \Rightarrow a = 0.$$
Equivalently, R contains no nonzero nilpotent ideals.

Another structural condition used in this study is the notion of a 2-torsion-free ring.

Definition 3. [4] A ring R is said to be 2*-torsion-free* if
$$2a = 0 \Rightarrow a = 0$$
for every $a \in R$.

The assumptions that the ring is semiprime and 2-torsion-free play an important role in establishing several properties of derivations and Jordan derivations.

Now, we review the definition of a nilpotent element in a ring.

Definition 4. [30] Let R be a ring with identity and let $a \in R$. The element a is said to be nilpotent if $a^n = 0$ for some $n \in \mathbb{N}$.

2.2. Derivations and Jordan Derivations

In ring theory, derivations are algebraic mappings that generalize the notion of differentiation in calculus.

Definition 5. [5] Let R be a ring. A mapping $d : R \to R$ is called a *derivation* if it satisfies

1. $d(a+b) = d(a) + d(b)$ for all $a, b \in R$;
2. $d(ab) = d(a)b + ad(b)$ for all $a, b \in R$.

A related concept is the Jordan derivation, which satisfies a weaker condition involving squares of elements.

Definition 6. [2] Let R be a ring. An additive mapping $d : R \to R$ is called a *Jordan derivation* if

$$d(a^2) = d(a)a + ad(a)$$

for every $a \in R$.

Now, we review the definition of a nilpotent derivation on a ring R.

Definition 7. [1] A derivation d defined on a ring R is said to form a nilpotent derivation if $d^n(R) = 0$ for some natural number n.

The relationship between derivations and Jordan derivations has been widely studied in ring theory.

Theorem 1. [4] *Let R be a 2-torsion-free semiprime ring. Then every Jordan derivation on R is a derivation.*

This result plays an important role in the study of Jordan derivations on semiprime rings and serves as one of the main theoretical references in this paper.

2.3. Research Procedure

This research is conducted using a theoretical and literature-based approach. Various references related to ring theory, semiprime rings, and derivation theory are examined in order to establish the theoretical framework of the study. Based on these foundations, the properties of Jordan derivations on 2-torsion-free semiprime rings are analyzed using deductive mathematical reasoning.

The main steps of the research are summarized as follows:

1. analyzing the relationship between Jordan derivations and derivations on 2-torsion-free semiprime rings through mathematical proofs;
2. examining the obtained results within the framework of semiprime rings;
3. providing illustrative examples to support the theoretical results obtained in this study.

3. Results and Discussion

It is well known that every Jordan derivation on a 2-torsion-free semiprime ring is a derivation [4]. The following example illustrates this phenomenon in the matrix rings.

Example 1. Consider the ring $M_2(\mathbb{R})$ of all 2×2 real matrices,

$$M_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}.$$

It is well known that $M_2(\mathbb{R})$ is a semiprime ring and is 2-torsion-free. Define a mapping $d : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by the inner derivation

$$d(X) = [P, X] = PX - XP,$$

where

$$P = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

We show that d satisfies the Jordan identity and hence is a derivation.

i. Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}).$$

Since every inner derivation satisfies the derivation identity, it follows that

$$d(X^2) = d(X)X + Xd(X),$$

and therefore d is a Jordan derivation.

ii. Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad Y = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \in M_2(\mathbb{R}).$$

We verify the derivation identity

$$d(XY) = d(X)Y + Xd(Y).$$

First,

$$\begin{aligned} d(XY) &= P(XY) - (XY)P \\ &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} - \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ ae+bg & af+bh \end{bmatrix} - \begin{bmatrix} af+bh & 0 \\ cf+dh & 0 \end{bmatrix} \\ &= \begin{bmatrix} -(af+bh) & 0 \\ (ae+bg)-(cf+dh) & af+bh \end{bmatrix}. \end{aligned}$$

On the other hand,

$$\begin{aligned} d(X)Y + Xd(Y) &= \left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right) \begin{bmatrix} e & f \\ g & h \end{bmatrix} \\ &\quad + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} -b & 0 \\ a-d & b \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -f & 0 \\ e-h & f \end{bmatrix} \\ &= \begin{bmatrix} -(af+bh) & 0 \\ (ae+bg)-(cf+dh) & af+bh \end{bmatrix}. \end{aligned}$$

Thus,

$$d(XY) = d(X)Y + Xd(Y),$$

which shows that d is a derivation on $M_2(\mathbb{R})$.

This example illustrates that, in a 2-torsion-free semiprime ring such as $M_2(\mathbb{R})$, a Jordan derivation indeed coincides with a derivation. In a semiprime 2-torsion-free ring, the nilpotency index of a derivation cannot be even. More precisely, Chung and Luh [3] proved that if a derivation d satisfies

$$d^{2n}(R) = 0$$

for some positive integer n, then it necessarily follows that

$$d^{2n-1}(R) = 0.$$

Thus, the nilpotency index of a nilpotent derivation on a semiprime 2-torsion-free ring must be an odd integer. Moreover, the assumptions that the ring is semiprime and 2-torsion free are essential for this conclusion. In the previous example we illustrated the construction of a derivation on a matrix ring. In the following example we further analyze such a derivation and show that its nilpotency behavior is consistent with the result of Chung and Luh.

Example 2. Let $M_2(\mathbb{R})$ be the ring

$$M_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}.$$

Define the inner derivation $d : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by

$$d(X) = [P, X] = PX - XP,$$

where

$$P = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}).$$

Then

$$\begin{aligned} d(X) &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} - \begin{bmatrix} b & 0 \\ d & 0 \end{bmatrix} \\ &= \begin{bmatrix} -b & 0 \\ a-d & b \end{bmatrix}. \end{aligned}$$

Next, we compute the second iterate of d:

$$\begin{aligned} d^2(X) &= Pd(X) - d(X)P \\ &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -b & 0 \\ a-d & b \end{bmatrix} - \begin{bmatrix} -b & 0 \\ a-d & b \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ -b & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ b & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ -2b & 0 \end{bmatrix}. \end{aligned}$$

Similarly, the third iterate satisfies

$$\begin{aligned} d^3(X) &= Pd^2(X) - d^2(X)P \\ &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -2b & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ -2b & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Hence $d^3(X) = 0$ for every $X \in M_2(\mathbb{R})$, and therefore $d^3 = 0$. This shows that d is a nilpotent derivation with nilpotency index 3.
Since $3 = 2n - 1$ for $n = 2$, the nilpotency index is an odd integer. Consequently, this example illustrates the result of Chung and Luh [3], which states that for derivations on semiprime 2-torsion-free rings, the nilpotency index cannot be even.

The previous example illustrates the behavior of nilpotent derivations on a semiprime 2-torsion-free ring. In this class of rings, Jordan derivations exhibit an even stronger structural property. It is well known that every Jordan derivation on a 2-torsion-free semiprime ring is actually a derivation. On the other hand, Chung and Luh [3] proved that nilpotent derivations on such rings necessarily have odd nilpotency index. By combining these two fundamental results, we obtain the following theorem.

Theorem 2. *Let R be a 2-torsion-free semiprime ring and let $d : R \to R$ be a Jordan derivation. If d is nilpotent, then the nilpotency index of d is odd. In particular, for every $n \in \mathbb{N}$,*

$$d^{2n}(R) = 0 \;\Rightarrow\; d^{2n-1}(R) = 0.$$

Proof. Let R be a 2-torsion-free semiprime ring and let $d : R \to R$ be a nilpotent Jordan derivation. By a result of Bresar [4], every Jordan derivation on a 2-torsion-free semiprime ring is a derivation. Hence d is a derivation on R.
Since d is assumed to be nilpotent, there exists a positive integer k such that $d^k(R) = 0$. Applying the result of Chung and Luh [3], we obtain that for every $n \in \mathbb{N}$,

$$d^{2n}(R) = 0 \quad \Rightarrow \quad d^{2n-1}(R) = 0.$$

Therefore, the nilpotency index of d must be an odd integer. This completes the proof. □

Theorem 2 shows that every nilpotent Jordan derivation on a 2-torsion-free semiprime ring must have an odd nilpotency index. We now present an example that illustrates this result concretely. In particular, we construct an inner derivation on a matrix ring and show that it is nilpotent with an odd nilpotency index.

Example 3. Let R be the ring of all 3×3 real matrices, namely

$$R = M_3(\mathbb{R}) = \left\{ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \middle| \; a_{ij} \in \mathbb{R},\ 1 \le i, j \le 3 \right\}.$$

Define the mapping $d : R \to R$ by

$$d(X) = [P, X] = PX - XP,$$

where

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \in R.$$

Observe that P is nilpotent of index 3, that is, $P^3 = 0$ but $P^2 \neq 0$.

(i) We first show that d is a Jordan derivation. Let $X \in R$. Then

$$d(X^2) = PX^2 - X^2P.$$

On the other hand,

$$\begin{aligned} d(X)X + Xd(X) &= (PX - XP)X + X(PX - XP) \\ &= PX^2 - XPX + XPX - X^2P \\ &= PX^2 - X^2P. \end{aligned}$$

Hence

$$d(X^2) = d(X)X + Xd(X),$$

so d is a Jordan derivation.

(ii) Next we show that d is a derivation. For any $X, Y \in R$ we compute

$$\begin{aligned} d(XY) &= P(XY) - (XY)P \\ &= (PX)Y - X(YP) \\ &= (PX)Y - X(PY) + X(PY - YP) \\ &= (PX - XP)Y + X(PY - YP) \\ &= d(X)Y + Xd(Y). \end{aligned}$$

Thus d satisfies the derivation property.

(iii) Finally we examine the nilpotency of d. Since P is nilpotent, repeated commutators with P eventually vanish. A direct computation gives

$$\begin{aligned} d(X) &= PX - XP \\ &= \begin{bmatrix} a_{21} & -a_{11} + a_{22} & -a_{12} + a_{23} \\ a_{31} & -a_{21} + a_{32} & -a_{22} + a_{33} \\ 0 & -a_{31} & -a_{32} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} d^2(X) &= d(d(X)) = P(d(X)) - (d(X))P \\ &= \begin{bmatrix} a_{31} & -2a_{21} + a_{32} & a_{11} - 2a_{22} + a_{33} \\ 0 & -2a_{31} & a_{21} - 2a_{32} \\ 0 & 0 & a_{31} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} d^3(X) &= d(d^2(X)) = P(d^2(X)) - (d^2(X))P \\ &= \begin{bmatrix} 0 & -3a_{31} & 3a_{21} - 3a_{32} \\ 0 & 0 & 3a_{31} \\ 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} d^4(X) &= d(d^3(X)) = P(d^3(X)) - (d^3(X))P \\ &= \begin{bmatrix} 0 & 0 & 6a_{31} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

$$d^5(X) = d(d^4(X)) = P(d^4(X)) - (d^4(X))P = 0.$$

A direct computation shows that for every $X \in R$

$$d^5(X) = 0.$$

Hence d is a nilpotent derivation with nilpotency index 5. According to Theorem 2, if $d^{2n} = 0$ then $d^{2n-1} = 0$, which implies that a nilpotent derivation cannot have an even nilpotency index. As shown in our computations, $d^4 \neq 0$ while $d^5 = 0$.

Therefore, d is both a Jordan derivation and a derivation on $M_3(\mathbb{R})$. Moreover, d is nilpotent with odd nilpotency index 5, which is consistent with Theorem 2.

As a consequence of Theorem 2, the following result follows.

Corollary 1. *Every nilpotent derivation on a 2-torsion-free semiprime ring has an odd nilpotency index.*

Proof. Since every nilpotent derivation d is, in particular, a nilpotent Jordan derivation, it follows from Theorem 2 that the nilpotency index of d is odd. □

Corollary 1 states that every derivation that is also a Jordan derivation on a torsion-free semiprime ring of order 2 is nilpotent, and its nilpotency index must be an odd integer. The following example provides a concrete illustration of this result.

Example 4. Let $R = M_3(\mathbb{R})$ be the ring of all 3×3 real matrices. To illustrate the corollary, we construct a mapping $d : R \to R$ that satisfies the Jordan identity.

Let

$$X = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \in R.$$

Define the mapping $d : R \to R$ by

$$d(X) = \begin{bmatrix} d_{21} & d_{22} - d_{11} & d_{23} - d_{12} \\ d_{31} & d_{32} - d_{21} & d_{33} - d_{22} \\ 0 & -d_{31} & -d_{32} \end{bmatrix}.$$

The mapping d defined above is additive on R. Moreover, by Example 3, it satisfies the Jordan identity

$$d(X^2) = d(X)X + Xd(X), \qquad X \in R,$$

and therefore d is a Jordan derivation on R.

Next, we examine the nilpotency of d by computing successive iterations of the mapping. A direct computation gives

$$d(X) = \begin{bmatrix} d_{21} & d_{22} - d_{11} & d_{23} - d_{12} \\ d_{31} & d_{32} - d_{21} & d_{33} - d_{22} \\ 0 & -d_{31} & -d_{32} \end{bmatrix},$$

$$d^2(X) = \begin{bmatrix} d_{31} & d_{32} - 2d_{21} & d_{33} - 2d_{22} + d_{11} \\ 0 & -2d_{31} & -2d_{32} + d_{21} \\ 0 & 0 & d_{31} \end{bmatrix},$$

$$d^3(X) = \begin{bmatrix} 0 & -3d_{31} & -3d_{32}+3d_{21} \\ 0 & 0 & 3d_{31} \\ 0 & 0 & 0 \end{bmatrix},$$

$$d^4(X) = \begin{bmatrix} 0 & 0 & 6d_{31} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$d^5(X) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence $d^5(X) = 0$ for all $X \in R$. By definition, the condition $d^4(X) \neq 0$ implies that $d^3(X)$ is also not zero. Therefore, d is a nilpotent Jordan derivation with nilpotency index of 5. Since 5 is an odd integer, this example confirms Corollary 1.

Example 4 shows that the nilpotency index of a nilpotent Jordan derivation on a 2-torsion-free semiprime ring is odd, namely 5. This observation is consistent with Corollary 1.

Furthermore, Theorem 2 implies that on a 2-torsion-free semiprime ring a Jordan derivation cannot have an even nilpotency index. In particular, if a Jordan derivation d satisfies $d^2 = 0$, then by Theorem 2 we obtain

$$d^{2\cdot 1-1} = d = 0.$$

Thus, there exists no nonzero nilpotent Jordan derivation of index 2 on a 2-torsion-free semiprime ring. This fact is stated in the following corollary.

Corollary 2. *Let R be a 2-torsion-free semiprime ring and let $d : R \to R$ be a Jordan derivation. If $d^2(R) = 0$, then $d(R) = 0$.*

Proof. Assume that $d^2(R) = 0$. Then for every $x \in R$ we have

$$d^2(x) = 0.$$

Since d is a Jordan derivation on a 2-torsion-free semiprime ring, Theorem 2 implies that

$$d^{2n}(R) = 0 \Rightarrow d^{2n-1}(R) = 0$$

for all $n \in \mathbb{N}$. Taking $n = 1$, we obtain

$$d(R) = 0.$$

Hence $d(x) = 0$ for all $x \in R$, and therefore $d = 0$. □

It should be noted that Corollary 2 relies essentially on the assumptions that the ring is semiprime and 2-torsion free. If either of these conditions is removed, the conclusion of the corollary may fail. In particular, there may exist a nonzero Jordan derivation that is nilpotent of index 2. The following example illustrates this situation.

Example 5. Let

$$R = UT_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a,b,c \in \mathbb{R} \right\}$$

be the ring of all 2×2 upper triangular matrices over $\mathbb{R}$.
The ring R is not semiprime. Indeed, consider the subset

$$N = \left\{ \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \mid b \in \mathbb{R} \right\}.$$

It is easy to verify that N is a nonzero ideal of R satisfying $N^2 = 0$. Hence R is not semiprime.
Let

$$P = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \in R,$$

and define a mapping $d : R \to R$ by

$$d(X) = [P, X] = PX - XP, \qquad X \in R.$$

Since d is an inner derivation, it is both a derivation and a Jordan derivation.
For an arbitrary element

$$X = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \in R,$$

we compute

$$PX = \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix}, \qquad XP = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}.$$

Hence

$$d(X) = PX - XP = \begin{bmatrix} 0 & c-a \\ 0 & 0 \end{bmatrix}.$$

If $c \neq a$, then $d(X) \neq 0$, and therefore d is a nonzero mapping.
Next we compute the second iterate:

$$d^2(X) = d(d(X)) = [P, d(X)].$$

Since every matrix of the form $d(X)$ has zero diagonal entries, we obtain

$$Pd(X) = d(X)P = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Thus

$$d^2(X) = Pd(X) - d(X)P = 0$$

for all $X \in R$. Consequently, $d^2 = 0$ while $d \neq 0$.
Therefore, d is a nonzero Jordan derivation on R which is nilpotent of index 2. This example shows that the semiprime assumption in Corollary 2 is essential.

Example 5 highlights the importance of the semiprime condition. Indeed, in a non-semiprime ring there may exist nonzero nilpotent ideals that give rise to nilpotent derivations of even index.

We now describe a general construction of nilpotent inner derivations arising from nilpotent elements.

Proposition 1. *Let R be a unital associative ring and let $p \in R$*

be a nilpotent element of nilpotency index n, that is,

$$p^n = 0 \quad \text{and} \quad p^{n-1} \neq 0.$$

Define a mapping $d_p : R \to R$ by

$$d_p(x) = [p, x] = px - xp, \qquad x \in R.$$

Then d_p is an inner derivation and hence a Jordan derivation. Moreover, d_p is nilpotent and satisfies

$$d_p^{2n-1}(R) = 0.$$

Consequently, the nilpotency index of d_p is at most $2n - 1$.

Proof. Let $p \in R$ satisfy $p^n = 0$ and define $d_p(x) = px - xp$ for all $x \in R$.

1. For arbitrary $x, y \in R$ we compute

$$\begin{aligned} d_p(xy) &= p(xy) - (xy)p \\ &= (px)y - x(yp) \\ &= (px - xp)y + x(py - yp). \end{aligned}$$

Hence

$$d_p(xy) = d_p(x)y + xd_p(y),$$

so d_p satisfies the Leibniz rule and therefore is a derivation. Since every derivation is a Jordan derivation, d_p is also a Jordan derivation.

2. Next we show that for every integer $k \geq 1$,

$$d_p^k(x) = \sum_{i=0}^{k} (-1)^i \binom{k}{i} p^{k-i} x p^i, \qquad x \in R. \tag{1}$$

We prove this formula by induction on k.

(i) For $k = 1$,

$$d_p(x) = px - xp = \binom{1}{0} p^1 x p^0 - \binom{1}{1} p^0 x p^1,$$

so eq. (1) holds.

(ii) Assume that eq. (1) holds for some $k \geq 1$, namely

$$d_p^k(x) = \sum_{i=0}^{k} (-1)^i \binom{k}{i} p^{k-i} x p^i.$$

Then

$$\begin{aligned} d_p^{k+1}(x) &= d_p(d_p^k(x)) \\ &= p\, d_p^k(x) - d_p^k(x)p \\ &= \sum_{i=0}^{k} (-1)^i \binom{k}{i} p^{k+1-i} x p^i \\ &\quad - \sum_{i=0}^{k} (-1)^i \binom{k}{i} p^{k-i} x p^{i+1}. \end{aligned}$$

In the second sum, let $j = i + 1$. Then

$$\sum_{i=0}^{k} (-1)^i \binom{k}{i} p^{k-i} x p^{i+1} = \sum_{j=1}^{k+1} (-1)^{j-1} \binom{k}{j-1} p^{k+1-j} x p^j.$$

Combining the two sums and using the binomial identity

$$\binom{k}{i} + \binom{k}{i-1} = \binom{k+1}{i},$$

we obtain

$$d_p^{k+1}(x) = \sum_{i=0}^{k+1} (-1)^i \binom{k+1}{i} p^{k+1-i} x p^i.$$

Thus eq. (1) holds for all $k \geq 1$.

3. Taking $k = 2n - 1$ in eq. (1), we obtain

$$d_p^{2n-1}(x) = \sum_{i=0}^{2n-1} (-1)^i \binom{2n-1}{i} p^{2n-1-i} x p^i.$$

For each $i \in \{0, 1, \ldots, 2n-1\}$, at least one of the inequalities

$$i \geq n \quad \text{or} \quad 2n - 1 - i \geq n$$

holds. Hence every term in the sum contains a factor p^m with $m \geq n$ either on the left or on the right. Since $p^n = 0$, all terms vanish, and therefore

$$d_p^{2n-1}(x) = 0 \quad \text{for all } x \in R.$$

Thus $d_p^{2n-1}(R) = 0$, and d_p is nilpotent with nilpotency index at most $2n - 1$. □

Corollary 2 shows that on a 2-torsion free semiprime ring, a Jordan derivation cannot have nilpotency index 2. In other words, if $d^2 = 0$ then necessarily $d = 0$. Thus, every nonzero nilpotent Jordan derivation must have an odd nilpotency index greater than 1.

The following example illustrates this phenomenon. Using the construction from Proposition 1, we obtain a nonzero nilpotent derivation on a semiprime ring whose nilpotency index is 3.

Example 6. Let R be the ring of 2×2 matrices over the real numbers, that is,

$$R = M_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}.$$

Define a map $d : R \to R$ to be an *inner derivation* given by

$$d(X) = [P, X] = PX - XP, \text{ for all } X \in R,$$

where

$$P = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

We will show that d is a Jordan derivation and, moreover, a nilpotent derivation.

1. First, observe that

$$P^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Since $P \neq 0$ but $P^2 = 0$, it follows that P is a nilpotent element with nilpotency index 2.

2. Next, since d is an inner derivation, the map d is a derivation on R. Every derivation is a Jordan derivation. Hence, d is a Jordan derivation on R.
3. We now show that d is a nilpotent derivation with an odd nilpotency index. Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in R.$$

Then

$$\begin{aligned} d(X) &= PX - XP \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix} \\ &= \begin{bmatrix} c & d-a \\ 0 & -c \end{bmatrix}. \end{aligned}$$

The second iterate of d, namely $d^2(X) = d(d(X))$, is given by

$$\begin{aligned} d^2(X) &= P\,d(X) - d(X)\,P \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} c & d-a \\ 0 & -c \end{bmatrix} - \begin{bmatrix} c & d-a \\ 0 & -c \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -c \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -2c \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

The third iterate, $d^3(X) = d(d^2(X))$, is

$$\begin{aligned} d^3(X) &= P\,d^2(X) - d^2(X)\,P \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2c \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -2c \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Note that the nilpotency index of P is 2. Therefore, by the relation $2n - 1 = 2(2) - 1 = 3$, the nilpotency index of the derivation d is 3. Furthermore, as shown in the previous step, $d^2 \neq 0$. Consequently, $d^3(X) = 0$ for all $X \in R$, and hence $d^3 = 0$.
Thus, d is a nilpotent derivation with nilpotency index 3, which is an odd integer.

It should be noted that if p is not a nilpotent element, then the mapping $d(x) = [p, x]$ is not nilpotent. Consequently, d cannot be a nilpotent Jordan derivation. In other words, the nilpotency of p is a necessary condition for the induced inner derivation d to be nilpotent.

Remark 1. Let R be a semiprime ring and let $P \in R$. Define a mapping $d : R \to R$ by

$$d(X) = [P, X] = PX - XP, \qquad X \in R.$$

If P is not a nilpotent element and $d \neq 0$, then d cannot be a nilpotent Jordan derivation.

Since d is an inner derivation, the mapping d is a derivation on R. Every derivation is a Jordan derivation; hence d is also a Jordan derivation. To illustrate this fact and to show that such a mapping need not be nilpotent, we present the following example.

Example 7. Let R be the ring of 2×2 matrices over the real numbers,

$$R = M_2(\mathbb{R}).$$

Define an inner derivation $d : R \to R$ by

$$d(X) = [P, X] = PX - XP, \qquad X \in R,$$

where

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Clearly, P is not a nilpotent element. We show that d is a Jordan derivation but not nilpotent.

(i) Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in R.$$

A direct computation shows that

$$d(X^2) = d(X)X + Xd(X),$$

and therefore d satisfies the Jordan derivation identity.

(ii) For

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in R,$$

we obtain

$$d(X) = \begin{bmatrix} 0 & b \\ -c & 0 \end{bmatrix}.$$

Computing higher iterates gives

$$d^2(X) = \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}, \qquad d^3(X) = \begin{bmatrix} 0 & b \\ -c & 0 \end{bmatrix} = d(X).$$

Hence

$$d^3 = d.$$

Therefore, for every $n \in \mathbb{N}$,

$$d^{2n-1}(X) = d(X) \neq 0$$

for suitable $X \in R$. Consequently, there is no positive integer n such that $d^n = 0$.

Thus d is a Jordan derivation which is not nilpotent.

The results of this section clarify the behavior of nilpotent Jordan derivations on 2-torsion free semiprime rings. In particular, we proved that such derivations cannot have even nilpotency index, and hence every nonzero nilpotent Jordan derivation must have an odd nilpotency index. The examples presented above illustrate these phenomena and highlight the role of nilpotent elements in the construction of nilpotent inner derivations.

4. Conclusion

Our main result shows that nilpotent Jordan derivations on 2-torsion free semiprime rings necessarily exhibit an odd nilpotency pattern. More precisely, if $d^{2n}(R) = 0$ for some $n \in \mathbb{N}$, then $d^{2n-1}(R) = 0$. Consequently, every nonzero nilpotent Jordan derivation must have an odd nilpotency index. In particular, the condition $d^2 = 0$ implies $d = 0$, and therefore a nilpotent Jordan derivation cannot have even nilpotency index.

Furthermore, we considered inner derivations generated by nilpotent elements. If $P \in R$ is nilpotent with nilpotency index n, then the inner derivation $d_p(x) = [p, x]$ is also nilpotent and satisfies $d_p^{2n-1}(R) = 0$, so that its nilpotency index is at most $2n - 1$. On the other hand, if p is not nilpotent and $d_p \neq 0$, then the induced inner derivation is not a nilpotent Jordan derivation.

These results clarify the relationship between nilpotent elements and the nilpotency behavior of the corresponding inner derivations, and provide a structural description of nilpotent Jordan derivations on 2-torsion free semiprime rings.

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