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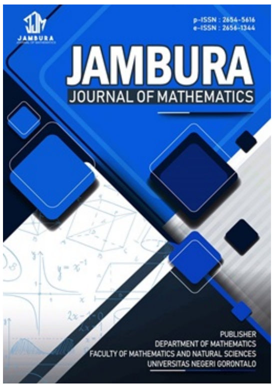

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Research Article

Geographically and Temporally Weighted Log-Logistic 3-Parameter Regression Model for Poverty Severity Index : A Case Study on East Java Province

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ABSTRACT. *This study develops the Geographically and Temporally Weighted Three-Parameter Log-Logistic Regression (GTWLL3R) model to accommodate spatial and temporal heterogeneity in the poverty severity index. The analysis used panel data from 38 regencies and cities in East Java Province during 2022–2024, comprising 114 observations. Local parameters were estimated using maximum likelihood estimation with a fixed Gaussian kernel weighting function based on spatial and temporal distances. The parameter estimates were obtained numerically using the Newton–Raphson algorithm, while the optimal bandwidth was selected using the minimum cross-validation criterion. Model performance was evaluated using the corrected Akaike Information Criterion (AICc). The results show that the three-period GTWLL3R model achieved the lowest AICc value of 18.311 compared with the one-period and two-period specifications. The estimated regression coefficients varied across regions and observation periods, indicating that the effects of the predictor variables on the poverty severity index were spatially and temporally heterogeneous. Based on the significant predictor variables, the regencies and cities were classified into three clusters. These findings indicate that the GTWLL3R model provides a flexible framework for identifying regional differences in poverty determinants and may support the formulation of more targeted poverty alleviation policies.*

1. Introduction

Statistical data show that the poverty severity index in East Java Province is non-negative, right-skewed, and exhibits significant variation across regions and over time. Based on data from the Central Statistics Agency, the poverty rate in rural areas remains higher than in urban areas, indicating the presence of spatial inequality. Furthermore, changes in the index values from year to year reveal temporal dynamics. These characteristics underscore that modeling the poverty severity index requires an approach capable of capturing both the asymmetric data distribution and spatial and temporal heterogeneity.

To handle non-negative continuous data that is not normally distributed and right-skewed [1], distribution-based approaches such as three-parameter log-logistic regression (LL3R) have been widely used [2]. This model provides more appropriate estimates for asymmetric data compared to classical regression based on the normality assumption. Sulistyaningsih et al.[3] demonstrated that LL3R is effective in modeling data with these characteristics. However, LL3R remains a global model that assumes constant regression parameters across all regions and observation periods, making it less capable of representing local differences between regions or changes over time.

These limitations led to the development of a spatially weighted approach, namely Geographically Weighted Regression (GWR), which allows for different regression parameters at each location [4]. Subsequently, this model was developed into Geographically and Temporally Weighted Regression (GTWR), which incorporates spatial and temporal dimensions through a weighting matrix based on spatial and temporal distances by Huang et al. and Brunsdon et al. [5, 6].

GTWR has proven effective for analyzing dynamic data with spatial-temporal heterogeneity. To extend the scope of GTWR to non-normal univariate responses, various models have been developed, such as Geographically and Temporally Weighted Bivariate Negative Binomial Regression (GTWBNBR) [7], Geographically and Temporally Weighted Bivariate Weibull Regression (GTWBWR) [8] Geographically temporally weighted negative binomial regression (GTWNBR)[9], Geographically and temporally weighted bivariate Poisson inverse Gaussian regression (GTWBPIGR)[10], Geographically and temporally weighted bivariate generalized Poisson regression (GTWBGPR)[11] and Geographically and temporally weighted bivariate Gamma regression (GTWBGR)[12]. Although these models handle skewness better than Gaussian models, they are typically limited to two-parameter distributions, which lack the flexibility to capture complex hazard shapes or extreme kurtosis often found in regional economic data [13].

Several studies have attempted to combine distribution-based approaches with spatial local models. Mawadah et al. [14] for example, developed a GWR-based log-logistic regression

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model capable of capturing inter-regional variations. However, that model only considers the spatial dimension and does not explicitly account for temporal changes. Based on this research gap, this study proposes the Geographically and Temporally Weighted Log-Logistic Three-Parameter Regression (GTWLL3R) model, a new development of the GTWR achieved through the integration of a local likelihood approach based on the three-parameter log-logistic distribution. Unlike previous studies that focused solely on the distribution aspect (LL3R) or only on spatial variation (GWLL3R), the GTWLL3R model simultaneously combines the flexibility of the log-logistic distribution with local parameter estimation based on both space and time. Therefore, the objective of this study is to identify significant variables influencing the poverty severity index in East Java Province for the 2022–2024 period using the GTWLL3R model, employing a fixed Gaussian kernel weighting function and determining the spatio-temporal bandwidth based on the minimum Cross Validation (CV) criterion.

2. Model

2.1. *Model Regresi Log Logistic 3-Parameter*

The Three-Parameter Log-Logistic (LL3) Regression model is a regression approach designed for continuous response data that are strictly positive and tend to be right-skewed. Based on the LL3 distribution, this model is suitable for analyzing data with asymmetric characteristics [15]. The model links its three distribution parameters to predictor variables, allowing researchers to examine how these predictors affect the distribution of the response variable and to better understand the relationship between predictors and response [16].

An LL3R regression model can be constructed based on the parameters used . This study uses location parameters as a regression model, as shown in eq. (1).

$$\ln(\mu) = \mathbf{x}^T\boldsymbol{\beta} \Rightarrow \mu = \exp\left(\mathbf{x}^T\boldsymbol{\beta}\right), \tag{1}$$

Assuming that the shape parameter (ψ) and the scale parameter (σ) are constant, μ is the location parameter, $\mathbf{x}^T = \begin{bmatrix}1 & x_1 & x_2 & \cdots & x_p\end{bmatrix}$ is the vector of predictor variables, and $\boldsymbol{\beta}^T = \begin{bmatrix}1 & \beta_1 & \beta_2 & \cdots & \beta_p\end{bmatrix}$ is the vector of regression model parameters [17]. Given n random samples $(y_i, x_{i1}, x_{i2}, \ldots, x_{ip})$ distributed LL3R $((\psi, \mu, \sigma)$. The LL3R model for the i-th observation is as shown in eq. (2).

$$\ln(\mu_i) = \mathbf{x}_i^T\boldsymbol{\beta} \Rightarrow \mu_i = \exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right), \tag{2}$$

where $\mathbf{x}_i^T = \begin{bmatrix}1 & x_{i1} & x_{i2} & \cdots & x_{ip}\end{bmatrix}; i = 1, 2, \ldots, n.$

The probability density function of the LL3R model [3] in eq. (3).

$$f(y_i|\psi,\boldsymbol{\beta},\sigma) = \left(\frac{\psi}{\sigma^\psi}(y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta}))^{\psi-1}\right) \times \left(1 + \left(\frac{y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta})}{\sigma}\right)^{\psi}\right)^{-2}, \tag{3}$$

where $\psi > 1$ is the shape parameter, $\sigma > 0$ is the scale parameter, and $y_i > \exp(\mathbf{x}_i^T\boldsymbol{\beta})$ for $i = 1, 2, \ldots, n$.

The ln form of the probability density of the LL3R model is as shown in eq. (4).

$$\begin{aligned}\ln f(y_i|\psi,\boldsymbol{\beta},\sigma) &= \ln\left(\frac{\psi}{\sigma^\psi}\left(y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta})\right)^{\psi-1}\right) \times \left(1 + \left(\frac{y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta})}{\sigma}\right)^{\psi}\right)^{-2}, \\ &= (\ln\psi - \psi\ln\sigma) \\ &\quad + (\psi-1)\ln\left(y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta})\right) \\ &\quad - 2\ln\left(1 + \left(\frac{y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta})}{\sigma}\right)^{\psi}\right),\end{aligned} \tag{4}$$

To statistically support this finding, the Kolmogorov–Smirnov (KS) test was applied, indicating that the data follow the LL3 distribution appropriately [18]. Prior to estimating the LL3 regression model, multicollinearity among predictor variables was examined using the Variance Inflation Factor (VIF), where values below 5 suggested no serious multicollinearity issues [19]. After ensuring that multicollinearity was not present. Next, to determine the presence of spatial heterogeneity, the Breusch-Pagan test was conducted, while temporal heterogeneity was assessed visually using box plots.

2.2. *Geographically and Temporally Weighted Log-Logistic 3-Parameter Regression*

The GTWLL3R model is an extension of the LL3R model that accounts for spatial and temporal effects. Based on the LL3D model in eq. (3), the probability density function is as shown in eq. (5).

$$Y \sim GTWLLR(\boldsymbol{\beta}_l(u_i,v_i,t_i), \psi_l, \sigma_l(u_i,v_i,t_i))$$

$$\begin{aligned}&f(y_{il}|\boldsymbol{\beta}_l(u_{il},v_{il},t_{il}), \psi_l, \sigma_l(u_i,v_i,t_i)) \\ &= \left(\frac{\psi_l}{(\sigma_l(u_i,v_i,t_i))^{\psi_l}}\left(y_{il} - \exp\left(\mathbf{x}_{il}^T\boldsymbol{\beta}_l(u_i,v_i,t_i)\right)\right)^{\psi_l-1}\right) \times \left(1 + \left(\frac{y_{il} - \exp\left(\mathbf{x}_{il}^T\boldsymbol{\beta}_l(u_i,v_i,t_i)\right)}{\sigma_l(u_i,v_i,t_i)}\right)^{\psi_l}\right)^{-2},\end{aligned} \tag{5}$$

where $y_{il} \geq \exp\left(\mathbf{x}_{il}^T\boldsymbol{\beta}_l(u_i,v_i,t_i)\right); \boldsymbol{\beta}_l(u_i,v_i,t_i), \psi_l, \sigma_l(u_i,v_i,t_i); l = 1, 2, \ldots, L; i = 1, 2, \ldots, n.$

Index l denotes the period. Parameter estimation in the GTWLL3R model using the MLE method is performed iteratively for each period, incorporating observations from the previous period. Parameter estimation for the first period uses n data points from the first period. Parameter estimation for the second period uses n data points from the first period and n data points from the second period. Parameter estimation for the L th period uses n data points from the first period, the second period, through the L th period.

The parameter estimates obtained are not in closed form, so we proceed with the Newton-Raphson numerical iteration. The likelihood function for n random samples $y_1, y_2, \ldots, y_i$ shown in eq. (6).

$$P = \prod_{i=1}^{n}\left(f\left(y_i|\theta^i_{GTWLL3R}\right)\right)$$

$$= \prod_{i=1}^{n} \left(\frac{\psi_l}{(A_{il})^{\psi_l}} B_{il} \times C_{il} \right), \tag{6}$$

where:

$$A_{il} = \sigma_l(u_i, v_i, t_i),$$

$$B_{il} = \left(y_{il} - \exp\left(\mathbf{x}_{il}^T \boldsymbol{\beta}_l(u_i, v_i, t_i)\right)\right)^{\psi_l - 1},$$

$$C_{il} = \left(1 + \left(\frac{y_{il} - \exp\left(\mathbf{x}_{il}^T \boldsymbol{\beta}_l(u_i, v_i, t_i)\right)}{A_{il}}\right)^{\psi_l}\right)^{-2},$$

The natural logarithm is taken of eq. (7) as shown below :

$$P_L = \ln L(\theta_{GTWLL3R}) = \sum_{l=1}^{L}\sum_{i=1}^{n} \ln\left(\frac{\psi_l}{(A_{il})^{\psi_l}}\right) + \sum_{l=1}^{L}\sum_{i=1}^{n} B_{il} - \sum_{l=1}^{L}\sum_{i=1}^{n} C_{il}, \tag{7}$$

where:

$$A_{il} = \sigma_l(u_i, v_i, t_i),$$

$$B_{il} = (\psi_l - 1)\ln\left(y_{il} - \exp\left(\mathbf{x}_{il}^T \boldsymbol{\beta}_l(u_i, v_i, t_i)\right)\right),$$

$$C_{il} = 2\ln\left(1 + \left(\frac{y_{il} - \exp\left(\mathbf{x}_{il}^T \boldsymbol{\beta}_l(u_i, v_i, t_i)\right)}{A_{il}}\right)^{\psi_l}\right),$$

Thus, the equation for the GTWLL3R model in period i, where $L = 1, 2, ..., L^*$, shown in eq. (8) as shown below :

$$P_{L^*} = \ln\left(\frac{\psi_l}{(A_{i^*})^{\psi_l}}\right) \sum_{l=1}^{L}\sum_{i=1}^{n} w_{ii^*l} + \sum_{i=1}^{n} w_{ii^*l} B_{il} - \sum_{l=1}^{L}\sum_{i=1}^{n} w_{ii^*l} C_{il}, \tag{8}$$

where:

$$A_{i^*} = \sigma_l(u_{i^*}, v_{i^*}, t_{i^*}),$$

$$B_{il} = (\psi_l - 1)\ln\left(y_{il} - \exp\left(\mathbf{x}_{il}^T \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*})\right)\right),$$

$$C_{il} = 2\ln\left(1 + \left(\frac{y_i - \exp\left(\mathbf{x}_{il}^T \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*})\right)}{A_{i^*}}\right)^{\psi_l}\right),$$

the likelihood function in eq. (7) does not have a closed-form expression, the solution to the log-likelihood eq. (9) cannot be found analytically to obtain the parameter estimators for the GTWLL3R model at the i-th location. Therefore, the GTWLL3R parameters at location i-th are estimated by finding the values that set the derivative of the log-likelihood function to zero, which must be solved numerically using the Newton Raphson algorithm.

$$\mathbf{g}\left(\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}\right) = \left[\frac{\partial P_{l^*}}{\partial \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*})} \quad \frac{\partial P_{l^*}}{\partial \psi_l} \quad \frac{\partial P_{l^*}}{\partial \sigma_l(u_{i^*}, v_{i^*}, t_{i^*})}\right]^T_{\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}},$$

The Hessian matrix is a matrix whose elements consist of the second partial derivatives of the function $P_(l^*)$ with respect to each parameter, as follows,

$$\mathbf{H}\left(\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}(u_{i^*}, v_{i^*}, t_{i^*})\right) = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ & H_{22} & H_{23} \\ \text{Simetris} & & H_{33} \end{bmatrix}^T_{\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}}$$

where:

$$H_{11} = \frac{\partial^2 \ln P_{l^*}}{\partial \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*}) \partial \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*})^T},$$

$$H_{12} = \frac{\partial^2 \ln P_{l^*}}{\partial \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*}) \partial \psi_l(u_{i^*}, v_{i^*}, t_{i^*})},$$

$$H_{13} = \frac{\partial^2 \ln P_{l^*}}{\partial \boldsymbol{\beta}_l(u_{i^*}, v_{i^*}, t_{i^*}) \partial \sigma_l(u_{i^*}, v_{i^*}, t_{i^*})},$$

$$H_{22} = \frac{\partial^2 \ln P_{l^*}}{\partial \psi_l^2(u_{i^*}, v_{i^*}, t_{i^*})},$$

$$H_{23} = \frac{\partial^2 \ln P_{l^*}}{\partial \psi_l(u_{i^*}, v_{i^*}, t_{i^*}) \partial \sigma_l(u_{i^*}, v_{i^*}, t_{i^*})},$$

$$H_{33} = \frac{\partial^2 \ln P_{l^*}}{\partial \sigma_l^2(u_{i^*}, v_{i^*}, t_{i^*})}.$$

Substituting the initial parameter estimates $\widehat{\boldsymbol{\theta}}_{l(0)} = \left[\widehat{\boldsymbol{\beta}}_l(u_{i^*}, v_{i^*}, t_{i^*})^T_{(0)} \quad \widehat{\sigma}_l(u_{i^*}, v_{i^*}, t_{i^*})^T_{(0)} \quad \widehat{\psi}^T_{l(0)}\right]$ with the gradient vector $\mathbf{g}\left(\widehat{\boldsymbol{\theta}}_l(u_{i^*}, v_{i^*}, t_{i^*})\right)$ and the matrix $\mathbf{H}\left(\widehat{\boldsymbol{\theta}}_l(u_{i^*}, v_{i^*}, t_{i^*})\right)$ so that the gradient vector $\mathbf{g}\left(\widehat{\boldsymbol{\theta}}_l(u_{i^*}, v_{i^*}, t_{i^*})_0\right)$ and the matrix $\mathbf{H}\left(\widehat{\boldsymbol{\theta}}_l(u_{i^*}, v_{i^*}, t_{i^*})_0\right)$. Iterations are performed for $i^* = 1, 2, \ldots, n$ and $l = 1, 2, \ldots, L$ starting from $m = 0$ in the equation below

$$\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m+1)} = \widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)} - \mathbf{H}^{-1}\left(\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}\right)\mathbf{g}\left(\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}\right),$$

where $\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}$ is a set of parameter estimates that converge to the i^*-th location and the L-th period at the m-th iteration, where $m = m + 1$ if a convergent parameter estimate has not yet been obtained. The iteration stops $\left\|\widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m+1)} - \widehat{\boldsymbol{\theta}}_{i^*,GTWLL3R}^{(m)}\right\| < \varepsilon$, where ε is a very small positive number.

After estimating the parameters, the next step is to test the parameters of the GTWLL3R model using the MLRT method. The parameter testing is conducted through simultaneous and partial hypothesis testing. However, prior to that, a test of model equivalence between the GTWLL3R and GWLL3R models will be performed. The hypotheses in the local and global model equivalence tests are as follows :

$$H_0 : \beta_j(u_i, v_i, t_i) = \beta_j; \quad j = 0, 1, 2, \ldots, k; \quad i = 1, 2, \ldots, n,$$

H_1 : at least one $\beta_j(u_i, v_i, t_i) \neq \beta_j$.

The test statistic used is the following eq. (9)

$$F_1 = \frac{\dfrac{G^2_{GWLL3R}}{df_1}}{\dfrac{G^2_{GTWLL3R}}{df_2}}, \tag{9}$$

where H_0 will be rejected if $F_1 > F_{\alpha;df_1;df_2}$.

If the model equivalence test results in a rejection of H_0, then a simultaneous parameter test is conducted on the GTWLL3R model. The purpose of conducting the simultaneous test is to determine the joint significance of the parameters in the model. The hypotheses for the simultaneous test of the GTWLL3R model are as follows.

$$H_0 : \beta_1(u_i, v_i, t_i) = \beta_2(u_i, v_i, t_i) = \cdots = \beta_p(u_i, v_i, t_i) = 0 \quad \text{for } i = 1, 2, \ldots, n$$

$$H_1 : \text{at least one } \beta_j(u_i, v_i, t_i) \neq 0 \quad \text{for } j = 1, 2, \ldots, p; \quad i = 1, 2, \ldots, n$$

Let the set of parameters under the null hypothesis be $\Omega_{GTWLL3R} = \{\boldsymbol{\beta}(u_i, v_i, t_i) \quad \psi \quad \sigma(u_i, v_i, t_i)\}$ and the population under H_0 be $\omega_{GTWLL3R} = \{\beta_{l_\omega 0}(u_{i^*}, v_{i^*}, t_{i^*}) \quad \psi_{l_\omega} \quad \sigma_{l_\omega}(u_{i^*}, v_{i^*}, t_{i^*})\}$. The values used to determine the test statistic are as shown in eq. (10).

$$G^2_{GTWLL3R} = -2\left(\ln L(\widehat{\omega}_{GTWLL3R}) - \ln L\left(\widehat{\Omega}_{GTWLL3R}\right)\right). \tag{10}$$

The simultaneous null hypothesis is rejected if $G^2_{\text{GTWLL3R}} > \chi^2_{1-\alpha,\text{df}}$, where df denotes the degrees of freedom. Partial parameter tests are conducted when the simultaneous test rejects H_0. These tests are used to determine the statistical significance of each regression parameter in the GTWLL3R model. The hypotheses, test statistics, and rejection regions are presented in Table 1.

Table 1. Partial hypothesis test for the regression parameters of the GTWLL3R model.

Parameter	Hypotheses	Test statistic	Rejection region
$\beta_j(u_i, v_i, t_l)$	$H_0 : \beta_j(u_i, v_i, t_l) = 0$ $H_1 : \beta_j(u_i, v_i, t_l) \neq 0$	$Z_{\beta_j} = \dfrac{\widehat{\beta}_j(u_i, v_i, t_l)}{\text{SE}\left(\widehat{\beta}_j(u_i, v_i, t_l)\right)}$	$\lvert Z_{\beta_j}\rvert > z_{1-\alpha/2}$

The estimated variance of $\widehat{\beta}_j(u_i, v_i, t_l)$ is obtained from the corresponding diagonal element of the estimated covariance matrix,

$$\widehat{\text{Cov}}\left(\widehat{\boldsymbol{\theta}}_{\text{GTWLL3R}}\right) = -\mathbf{H}^{-1}\left(\widehat{\boldsymbol{\theta}}_{\text{GTWLL3R}}\right).$$

The scale and shape parameters are estimated subject to the constraints $\sigma(u_i, v_i, t_l) > 0$ and $\psi > 1$, respectively.

After obtaining the local parameter estimates and their variances and conducting the Z-tests for parameter significance, the next step is to identify the model and the factors influencing the poverty severity index in East Java Province using the GTWLL3R model as follows:

1. Conduct a descriptive analysis.
2. Detect multicollinearity and address it if present.
3. Test the Log-Logistic distribution of the response variable using the Kolmogorov-Smirnov test.
4. Test for spatial heterogeneity using the Breusch-Pagan test and visually detect temporal heterogeneity using boxplots. Calculate the BP test statistic using the following eq. (11) below:
$$BP = \left(\frac{1}{2}\right)\mathbf{f}^T\mathbf{Z}(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{f}, \tag{11}$$
where $\mathbf{f} = (f_1, f_2, \ldots, f_n)^T$.
5. Determine the optimal δ_L.
6. Determine the Euclidean distance.
7. Determine the spatio-temporal weights using a fixed Gaussian kernel weighting function, as shown in eq. (12):
$$w_{ii^*l} = \begin{cases} \exp\left[-\dfrac{1}{2}\left(\dfrac{d_{ii^*l}}{h}\right)^2\right], & \text{if } d_{ii^*l} < h,\ i = 1, 2, \ldots, n, \\ 0, & \text{if } d_{ii^*l} \geq h,\ i = 1, 2, \ldots, n. \end{cases} \tag{12}$$
8. Perform GTWLL3R modeling
9. Determine the best model based on the smallest AICc value. The AICc formula, which accounts for spatiotemporal effects, is shown in eq. (13) below:
$$AICc = -2\ln L\left(\widehat{\boldsymbol{\beta}}((u_i, v_i, t_i)), \widehat{\psi}, \widehat{\sigma}(u_i, v_i, t_i);\ e\right) + 2k + \frac{2k(c+1)}{n-c-1} \tag{13}$$
where e represents $i = 1, 2, \ldots, n$.
10. Model interpretation.

3. Results and Discussion

This study uses secondary panel data obtained from the Central Statistics Agency (BPS) of East Java Province covering the period 2022–2024 [20]. The units of observation are 38 regencies/cities in East Java Province observed over three years, resulting in a balanced panel dataset with 114 observations.

3.1. Data Exploration

Descriptive statistics were computed for the response and predictor variables based on the panel dataset of 38 regencies/cities observed from 2022 to 2024. The results are presented in Table 2.

This Table 2 used panel data from 38 regencies/cities in East Java during 2022–2024. Overall, the results indicate improving regional welfare. The average Poverty Severity Index declined from 0.35 (2022) to 0.27 (2024), meaning that expenditure inequality among poor households became lower. The increase in adequate sanitation access to 88.42% in 2024 was followed by a decrease in health complaints from 33.43% to 19.19%, highlighting the importance of basic infrastructure for public health.

From an economic perspective, the unemployment rate decreased to 4.04%, while labor force participation increased, indicating a stronger labor market. In addition, the average life expectancy reached 72.60% with low variation, suggesting more equal health conditions across regions.Overall, these findings show that development in East Java during 2022–2024 not only

Table 2. Descriptive statistics of variables used in the study.

Year	Variables	Min (%)	Mean (%)	Max (%)	SD (%)
2022	Poverty Severity Index (Y)	0.06	0.35	1.16	0.22
	Percentage of households with access to adequate sanitation (X_1)	51.64	82.15	96.41	12.35
	Percentage of households with access to a safe drinking water source (X_2)	78.56	95.36	100	4.82
	Open unemployment rate (X_3)	1.36	5.27	8.80	1.77
	Percentage of the population with health complaints (X_4)	12.82	33.43	61.88	11.15
	Labor force participation rate (X_5)	63.08	71.28	82.99	3.49
	Life expectancy (X_6)	67.29	72.07	74.54	1.96
2023	Poverty Severity Index (Y)	0.06	0.33	1.42	0.24
	Percentage of households with access to adequate sanitation (X_1)	50.30	88.58	100	12.18
	Percentage of households with access to a safe drinking water source (X_2)	79.26	96.11	100	4.81
	Open unemployment rate (X_3)	1.71	4.66	8.05	1.42
	Percentage of the population with health complaints (X_4)	15.72	27.77	37.95	5.69
	Labor force participation rate (X_5)	66.89	73.15	81.64	3.76
	Life expectancy (X_6)	67.60	72.41	74.91	1.98
2024	Poverty Severity Index (Y)	0.06	0.27	1.53	0.25
	Percentage of households with access to adequate sanitation (X_1)	29.75	88.42	100	13.03
	Percentage of households with access to a safe drinking water source (X_2)	79.26	96.11	100	4.81
	Open unemployment rate (X_3)	1.56	4.04	6.49	1.17
	Percentage of the population with health complaints (X_4)	15.88	19.19	39.36	5.26
	Labor force participation rate (X_5)	67.52	73.66	86.62	3.73
	Life expectancy (X_6)	67.76	72.60	75.11	1.99

promoted economic growth, but also improved welfare quality and reduced poverty severity. Next, we detected multicollinearity among the variables by examining the VIF values. The VIF values for each predictor variable are shown in Table 3.

Table 3. Multicollinearity Detection Results.

Variable	X_1	X_2	X_3	X_4	X_5	X_6
VIF	1.493	1.630	2.688	1.364	2.955	1.554

Based on Table 3, it was found that the VIF values for all predictor variables were less than 5, so it can be concluded that there is no multicollinearity among the predictor variables. Therefore, each predictor variable can be included in the regression model without causing instability in parameter estimation.

Following the validation of the predictor variables, the analysis proceeded to examine the response variable. The response variable was examined to assess its conformity to the LL3 distribution using the Kolmogorov–Smirnov (KS) test. The null hypothesis assumed that the data followed an LL3 distribution, while the alternative hypothesis suggested a departure from it. For the poverty severity index variable, the KS test yielded a statistic of 0.0856 with a p-value of 0.9431. These results indicate insufficient evidence to reject the null hypothesis, implying that the poverty severity index data are consistent with the LL3 distribution.

Next, to determine the presence of spatial heterogeneity, the Breusch–Pagan test was conducted using eq. (11). The Breusch–Pagan test statistic for the first data period was greater than the critical value $\chi^2_{(0.95,6)} = 12.592$, so it can be concluded that H_0 was rejected. Meanwhile, temporal heterogeneity was assessed visually using box plots. Figure 1 shows a box plot of the Poverty Severity Index for the years 2022 to 2024.

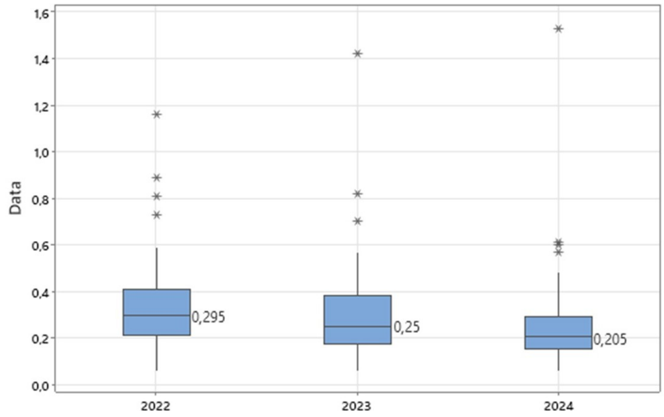

Figure 1. Boxplot of East Java poverty severity index.

The box plot in Figure 1 illustrates the variation in the data range from 2022 to 2024, as reflected in the differences in the median values for each year. This indicates the presence of temporal heterogeneity in the response variable. Based on these two tests, it can be concluded that there is spatiotemporal heterogeneity in the global model of the East Java Poverty Severity Index.

3.2. Modeling Using GTWLL3R Model

The GTWLL3R model was developed through parameter estimation and hypothesis testing of the model; the parameter estimation process required spatio-temporal weights that took into account spatio-temporal distance and optimum bandwidth. Optimum bandwidth shown in Table 4.

Table 4. Optimum δ_L

Period	Year Used	δ_L	AICc
1	2024	–	22.612
2	2024, 2023	0.2	19.044
3	2024, 2023, 2022	2.0	18.311

Based on Table 4, the AICc values calculated using eq. (13) indicate that the 3-period model with a fixed Gaussian kernel weighting function, as specified in eq. (12), has a lower AICc value than the 1-period and 2-period models with the same weighting scheme.

The LL3R model assumes the same predictor effects across all districts/cities and periods, making it less flexible. The GWLL3R model accounts for spatial differences but does not fully capture temporal changes. In contrast, the GTWLL3R model combines spatial and temporal effects, making it more adaptive to regional conditions and changes over time. Therefore, the

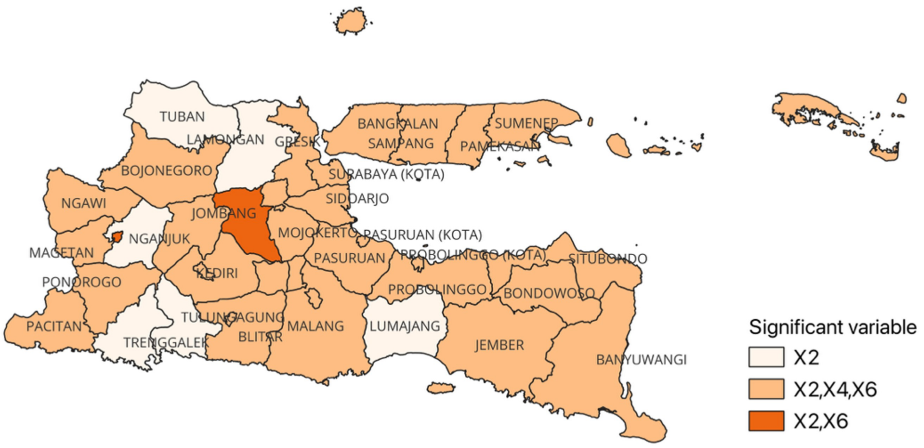

Figure 2. Clustering of districts in East Java Province based on variables significant for the poverty severity index in period 3.

GTWLL3R model provides more accurate estimates and achieves the lowest AICc value.

After obtaining the parameter estimates for the GTWLL3R model, an equivalence test between the GTWLL3R and GWLL3R models was conducted using eq. (9). The test yielded an F-statistic of 1.3955, which was greater than the critical value $F_{\text{table}} = 1.1895$. Therefore, H_0 was rejected, indicating that the GTWLL3R model is significantly different from the GWLL3R model. This result suggests that incorporating the temporal dimension significantly improves the model beyond the geographical effects captured by the GWLL3R model.

The parameters of the GTWLL3R model were tested simultaneously to determine whether the predictor variables were jointly significant or whether at least one predictor variable influenced the poverty severity index in East Java using eq. (10). At a significance level of 5%, the test statistic was $G^2 = 58.9001$, while the critical value from the chi-square distribution was $\chi^2 = 12.59159$. Since the test statistic exceeded the critical value, H_0 was rejected. Therefore, at least one predictor variable had a significant effect on the poverty severity index in East Java.

Furthermore, partial tests conducted across all districts and cities yielded varying conclusions regarding the significance of the predictor variables. The results of the partial tests are presented in Table 5.

Table 5. Clustering of regencies and cities in East Java Province based on predictor variables significant for the poverty severity index in period 3.

Period	Significant Variables	Regency/City
3	X_2, X_6	Trenggalek, Tulungagung, Lumajang, Madiun, Tuban, Lamongan
	X_2, X_4, X_6	Pacitan, Ponorogo, Blitar, Kediri, Malang, Jember, Banyuwangi, Bondowoso, Situbondo, Probolinggo, Pasuruan, Sidoarjo, Mojokerto, Nganjuk, Magetan, Ngawi, Bojonegoro, Gresik, Bangkalan, Sampang, Pamekasan, Sumenep, Kota Kediri, Kota Blitar, Kota Malang, Kota Probolinggo, Kota Pasuruan, Kota Mojokerto, Kota Surabaya, Kota Batu
	X_1, X_2, X_4, X_6	Jombang, Kota Madiun

Table 6. Estimated values of the GTWLL3R model parameters in Banyuwangi Regency (10th location) period 3.

Parameter	Estimate	SE	Z-value	P-value
$\beta_0(u_{10}, v_{10})$	-3.830	0.778	-4.920	0.000
$\beta_1(u_{10}, v_{10})$	-0.135	0.090	-1.505	0.132
$\beta_2(u_{10}, v_{10})$	-0.650	0.142	-4.578	0.000
$\beta_3(u_{10}, v_{10})$	0.163	0.165	0.989	0.322
$\beta_4(u_{10}, v_{10})$	-0.613	0.196	-3.116	0.001
$\beta_5(u_{10}, v_{10})$	-0.358	0.319	-1.123	0.261
$\beta_6(u_{10}, v_{10})$	-1.123	0.377	-2.975	0.002
$\sigma(u_{10}, v_{10})$	0.216	0.023	9.372	0.000

Table 5 shows that the significant predictor variables differ across regions in period 3, indicating that the relationship between the predictor variables and the Poverty Severity Index (Y) varies spatially. In other words, the factors influencing poverty severity are not the same across all regencies and cities. An illustration of the regional grouping based on the predictor variables that significantly affect the Poverty Severity Index in period 3 is presented in Figure 2.

Based on Figure 2, the influence of the predictor variables on the Poverty Severity Index varies across regencies and cities in East Java Province during period 3. In general, neighboring regions tend to share the same significant predictor variables, whereas more distant regions may exhibit different patterns. Table 6 presents an example of the partial test results for Banyuwangi Regency during period 3.

Based on the parameter estimates presented in Table 6, the predictor variables that significantly affect the Poverty Severity Index (Y) in Banyuwangi Regency are the percentage of households with access to a safe drinking water source (X_2), the percentage of the population with health complaints (X_4), and life expectancy (X_6).

4. Conclusion

This study confirms the superiority of the GTWLL3R model with a fixed Gaussian kernel weighting function in analyzing the Poverty Severity Index in East Java Province. The model effectively captures spatial and temporal heterogeneity, indicating that the poverty severity index varies across regions and years. It also identifies three regional clusters in which neighboring areas tend to exhibit similar patterns of significant predictor variables. Access to a safe drinking water source was consistently significant across many regencies and cities, indicating that the provision of basic services is an important factor associated with poverty severity. Therefore, government policies should prioritize improvements in public infrastructure and the implementation of region-specific poverty reduction programs. This study is limited by the relatively short observation period and the limited number of predictor variables. Future research should use longer-term panel data and incorporate additional socioeconomic indicators. In addition, future studies may consider modeling two response variables simultaneously by extending the proposed model to Geographically and Temporally Weighted Bivariate Three-Parameter Log-Logistic Regression (GTWBLL3R), thereby providing a more comprehensive understanding of regional poverty conditions.

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