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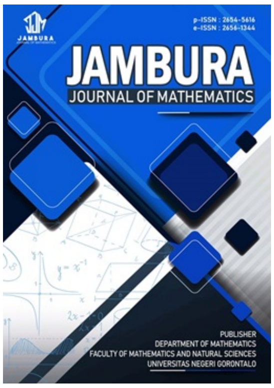
JAMBURA
JOURNAL OF MATHEMATICS

Volume 8, Issue 2, Pages 152–158, August 2026

Received 4 April 2026, Revised 25 June 2026, Accepted 1 July 2026, Published 20 July 2026

To Cite this Article : H. Khotimah, A. T. A. Nur, and P. N. Cayo,“A Construction of a Smooth Travel Groupoid on a Spanning Tree Associated with Lotus Graphs”, *Jambura J. Math*, vol. 8, no. 2, pp. 152–158, 2026, *https://doi.org/10.37905/jjom.v8i2.38029*

JOURNAL INFO • JAMBURA JOURNAL OF MATHEMATICS



Homepage	:	http://ejurnal.ung.ac.id/index.php/jjom/index
Journal Abbreviation	:	Jambura J. Math.
Frequency	:	Biannual (February and August)
Publication Language	:	English (preferable), Indonesia
DOI	:	https://doi.org/10.37905/jjom
Online ISSN	:	2656-1344
Editor-in-Chief	:	Hasan S. Panigoro
Publisher	:	Department of Mathematics, Universitas Negeri Gorontalo
Country	:	Indonesia
OAI Address	:	http://ejurnal.ung.ac.id/index.php/jjom/oai
Google Scholar ID	:	iWLjgaUAAAAJ
Email	:	info.jjom@ung.ac.id

Jambura Journal of Mathematics, Volume 8, Issue 2, Pages 152–158, August 2026 https://doi.org/10.37905/jjom.v8i2.38029

Research Article

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A Construction of a Smooth Travel Groupoid on a Spanning Tree Associated with Lotus Graphs

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ARTICLE HISTORY
Received 4 April 2026
Revised 25 June 2026
Accepted 1 July 2026
Published 20 July 2026

KEYWORDS
Travel Groupoid
Smooth Travel Groupoid
Lotus Graph
Spanning Tree
Binary Operation

ABSTRACT. *A travel groupoid is a binary system associated with a graph through an operation on its vertex set, while a smooth travel groupoid satisfies an additional smoothness condition. In this paper, we construct a smooth travel groupoid on a particular spanning tree associated with the lotus graph $L(n)$. The spanning tree is obtained by deleting the edges u_iv_{i+1}, for $1 \leq i \leq n-1$, from the lotus graph. Using the unique path between two vertices in this tree, we define a binary operation by assigning to each ordered pair the first step from one vertex toward the other. We prove that the resulting binary system satisfies the axioms of a travel groupoid and fulfills the smoothness condition. Explicit examples for $L(2)$ and $L(3)$ are also presented to illustrate the construction.*

1. Introduction

The interaction between graph theory and algebra has been an active topic of research for many years. On the one hand, various graphs have been constructed from algebraic structures in order to study algebraic properties through graph-theoretical methods. Examples of this direction include graphs defined on groups, such as power graphs in [1], enhanced power graphs in [2], coprime graphs in [3], commuting graphs in [4], and non-generating graphs in [5]. On the other hand, graphs may also be used to generate algebraic structures, for instance through automorphism groups or binary systems defined on their vertex sets. Several studies have investigated automorphism groups of special graph classes, including some families of bipartite graphs in [6], graphs related to cycle graphs in [7], Andrásfai graphs in [8], some token graphs in [9], polyhedral graphs in [10], and lotus graphs in [11].

One of the algebraic structures associated with graphs is a travel groupoid. This concept was introduced by Nebeský in [12] as a binary system related to a graph through two axioms that reflect movement on the graph. In particular, a travel groupoid associates a binary operation with the vertex set of a graph in a way that reflects adjacency and movement on the graph. Since its introduction, travel groupoids have attracted attention as an interesting bridge between graph theory and algebra.

A further development of this topic is the notion of a smooth travel groupoid, which satisfies an additional smoothness condition. Smooth travel groupoids provide a more structured class of travel groupoids and have been studied in several works. Matsumoto and Mizusawa introduced a method for constructing smooth travel groupoids on finite graphs in [13]. More recently, travel groupoids on special graph classes have also been investigated, including complete multipartite graphs in [14] and finite geodetic graphs in [15]. These results show that the study of travel groupoids remains open and meaningful, especially when considered on specific graph families.

Motivated by this line of research, in this paper we consider the construction of a smooth travel groupoid on a spanning tree naturally associated with lotus graphs. Lotus graphs have appeared in the literature as a special family of graphs and have been studied from several perspectives. For example, lotus-type graphs have been discussed in graph labeling problems in [16], and the automorphism groups of lotus graphs have also been investigated in [11]. However, to the best of our knowledge, an explicit construction of a smooth travel groupoid on a spanning tree derived from lotus graphs has not yet been discussed. Therefore, this topic provides a specific construction that is worth investigating.

The aim of this paper is to provide an explicit construction of a smooth travel groupoid on a spanning tree naturally associated with the lotus graph $L(n)$. To do this, we first select a spanning tree of $L(n)$ and then define a binary operation on its vertex set by using the first step along the unique path in the selected tree. We prove that the resulting binary system satisfies the axioms of a travel groupoid and the smoothness condition. Thus, the contribution of this paper lies in presenting a concrete and explicit construction for a tree structure derived from lotus graphs, rather than in proving the general existence of smooth travel groupoids on finite connected graphs.

2. Methods

This section presents the basic concepts and the approach used in this research. Throughout this paper, all graphs are finite, simple, and undirected. The definitions of travel groupoid and smooth travel groupoid are adapted from [12], while the definition of lotus graph follows [11]. After presenting the neces-

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sary definitions, the general procedure for constructing a smooth travel groupoid on a selected spanning tree associated with lotus graphs is described.

2.1. Basic Definitions

We first recall the concept of a binary operation. Let S be a nonempty set.

Definition 1. A *binary operation* on S is defined as a mapping $*: S \times S \to S$.

Definition 2 ([12]). A *binary system* is the ordered pair $(S, *)$, where $*$ is a binary operation defined on S.

The main algebraic structure considered in this paper is the travel groupoid. Consider a nonempty set V and a binary operation $*$ on V.

Definition 3 ([12]). The pair $(V, *)$ is called a *travel groupoid*, whenever for all $u, v \in V$, the following properties hold:

1. $(u * v) * u = u$;
2. if $(u * v) * v = u$, then $u = v$.

For a travel groupoid $(V, *)$, one can associate a graph $G_* = (V, E_*)$, where two distinct vertices $u, v \in V$ are adjacent whenever

$$u * v = v \quad \text{or} \quad v * u = u.$$

In this sense, a travel groupoid $(V, *)$ is said to be a travel groupoid on a graph $G = (V, E)$ if the associated graph G_* satisfies

$$E_* = E.$$

Thus, the binary operation must recover the edge set of the graph under consideration.

In the present work, the binary operation is constructed from a selected spanning tree T of the lotus graph $L(n)$. Consequently, the structure obtained in this paper is a smooth travel groupoid on the spanning tree T, which is naturally associated with $L(n)$.

In addition to the travel groupoid axioms, this paper also uses the smoothness condition given below.

Definition 4 ([12]). We call a travel groupoid $(V, *)$ *smooth* if the condition below is fulfilled for every $u, v, w \in V$:

$$u * v = u * w \Rightarrow u * (w * v) = u * v.$$

Below we recall several graph-theoretic concepts that will be used throughout the paper. The terminology is adopted from standard references in graph theory [17].

Definition 5 ([17]). Let G be a graph. A *path* in G is an ordered sequence of distinct vertices $v_0, v_1, \ldots, v_k$ where v_{i-1} and v_i are adjacent for each $i = 1, 2, \ldots, k$.

Definition 6 ([17]). A *tree* is a connected graph such that every pair of distinct vertices is joined by exactly one path.

Definition 7 ([17]). Let G be a connected graph. A subgraph T of G is said to be a *spanning tree* of G if T is a tree and $V(T) = V(G)$.

If T is a tree, then every two vertices of T are connected by exactly one path.

Next, we recall the family of graphs studied in this paper.

Definition 8 ([11]). For $n \geq 2$, the *lotus graph* $L(n)$ is the graph with vertex set

$$V(L(n)) = \{v_0\} \cup \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\},$$

and edge set

$$E(L(n)) = \{v_i u_i \mid 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} \mid 1 \leq i \leq n-1\} \\ \cup \{v_0 v_i \mid 1 \leq i \leq n\} \cup \{v_0 u_n\}.$$

From this definition, it follows that $L(n)$ has $2n+1$ vertices and $3n-1$ edges. The graph consists of one central vertex, a sequence of outer vertices connected in a chain-like pattern, and one pendant vertex attached to the center. An illustration of $L(n)$ is given in Figure 1.

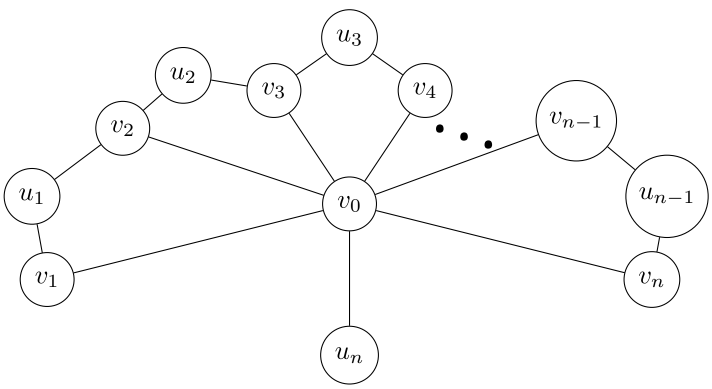

Figure 1. Lotus graph $L(n)$.

2.2. Construction Procedure

The construction carried out in this research is theoretical in nature and is based on the structure of lotus graphs. In general, the procedure is organized into the following steps:

1. Select a spanning tree of $L(n)$ that preserves all vertices of the graph.
2. Using the unique path joining two vertices in the chosen spanning tree, define a binary operation on the vertex set

of $L(n)$.

3. Verify that the resulting binary system defines a travel groupoid on the selected spanning tree.
4. Prove that the constructed travel groupoid also satisfies the smoothness condition.
5. Present examples for small values of n to illustrate the construction.

The detailed construction of the spanning tree, the induced binary operation, and the verification of the required properties are presented in the next section.

3. Results and Discussion

In this section, we present the construction of a smooth travel groupoid on a selected spanning tree associated with lotus graphs. The discussion begins with the selection of a spanning tree of the lotus graph $L(n)$, which will be used to define a binary operation on its vertex set.

Lemma 1 ([17]). *Let G be a connected graph with n vertices. If G has exactly $n-1$ edges, then G is a tree.*

Proposition 1. *Let $L(n)$ be the lotus graph with vertex set*

$$V(L(n)) = \{v_0\} \cup \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\},$$

and edge set

$$E(L(n)) = \{v_iu_i \mid 1 \le i \le n-1\} \cup \{u_iv_{i+1} \mid 1 \le i \le n-1\} \cup \{v_0v_i \mid 1 \le i \le n\} \cup \{v_0u_n\}.$$

Define a subgraph T of $L(n)$ by

$$V(T) = V(L(n))$$

and

$$E(T) = \{v_0v_i \mid 1 \le i \le n\} \cup \{v_iu_i \mid 1 \le i \le n-1\} \cup \{v_0u_n\}.$$

Then T is a spanning tree of $L(n)$.

Proof. By definition, $V(T) = V(L(n))$. Hence, T spans all vertices of $L(n)$. Next, we count the number of edges of T. Since

$$|\{v_0v_i \mid 1 \le i \le n\}| = n,$$

$$|\{v_iu_i \mid 1 \le i \le n-1\}| = n-1,$$

and $|\{v_0u_n\}| = 1$, it follows that

$$|E(T)| = n + (n-1) + 1 = 2n.$$

On the other hand,

$$|V(T)| = |V(L(n))| = 2n+1.$$

Thus,

$$|E(T)| = |V(T)| - 1.$$

We now show that T is connected. The vertex v_0 is adjacent to each vertex v_i for $1 \le i \le n$, and also to u_n. Moreover, for each $i = 1, 2, \ldots, n-1$, the vertex u_i is adjacent to v_i. Therefore, every vertex of T is connected to v_0, and so T is connected.

By Lemma 1, since T is connected and has exactly $|V(T)| - 1$ edges, T is a tree. Therefore, T is a spanning tree of $L(n)$. □

Remark 1. The spanning tree T is obtained from $L(n)$ by deleting the edges u_iv_{i+1} for $1 \le i \le n-1$. Hence, every two vertices in T are joined by a unique path.

The uniqueness of the path joining any two vertices in T allows us to introduce a binary operation on the vertex set of $L(n)$.

Definition 9. On $V(L(n))$, define the binary operation $*$ by

$$x * y = \begin{cases} x, & \text{if } x = y, \\ z, & \text{if } x \neq y, \end{cases}$$

where z is the vertex immediately following x on the unique path connecting x and y in T.

Based on Definition 9 and the structure of the spanning tree T, the operation $*$ can be written explicitly as follows.

$$v_0 * y = \begin{cases} v_0, & y = v_0, \\ v_i, & y = v_i,\ 1 \le i \le n, \\ v_i, & y = u_i,\ 1 \le i \le n-1, \\ u_n, & y = u_n, \end{cases}$$

for $1 \le i \le n-1$,

$$v_i * y = \begin{cases} v_i, & y = v_i, \\ u_i, & y = u_i, \\ v_0, & \text{otherwise}, \end{cases}$$

$$v_n * y = \begin{cases} v_n, & y = v_n, \\ v_0, & y \neq v_n, \end{cases}$$

for $1 \le i \le n-1$,

$$u_i * y = \begin{cases} u_i, & y = u_i, \\ v_i, & y \neq u_i, \end{cases}$$

and

$$u_n * y = \begin{cases} u_n, & y = u_n, \\ v_0, & y \neq u_n. \end{cases}$$

Using the binary operation defined above, we next show that the binary system induced on the selected spanning tree T satisfies the axioms of a travel groupoid.

Theorem 1. *Let T be the spanning tree of $L(n)$ defined in Proposition 1. Then the binary system $(V(T), *)$ is a travel groupoid on T.*

Proof. To prove that $(V(T),*)$ is a travel groupoid, it suffices to verify the following two axioms for all $x, y \in V(T) = V(L(n))$:

$$(x * y) * x = x, \tag{1}$$
$$(x * y) * y = x \implies x = y. \tag{2}$$

We first prove (1). The verification is carried out according to the possible choices of x.

Case 1.

Let $x = v_0$. If $y = v_0$, then

$$(v_0 * v_0) * v_0 = v_0 * v_0 = v_0.$$

If $y = v_i$ for some $1 \leq i \leq n$, then

$$(v_0 * v_i) * v_0 = v_i * v_0 = v_0.$$

If $y = u_i$ for some $1 \leq i \leq n-1$, then

$$(v_0 * u_i) * v_0 = v_i * v_0 = v_0.$$

If $y = u_n$, then

$$(v_0 * u_n) * v_0 = u_n * v_0 = v_0.$$

Thus, $(v_0 * y) * v_0 = v_0$ for all $y \in V(L(n))$.

Case 2.

Let $x = v_i$ for some $1 \leq i \leq n-1$. If $y = v_i$, then

$$(v_i * v_i) * v_i = v_i * v_i = v_i.$$

If $y = u_i$, then

$$(v_i * u_i) * v_i = u_i * v_i = v_i.$$

If $y \notin \{v_i, u_i\}$, then $v_i * y = v_0$, and hence

$$(v_i * y) * v_i = v_0 * v_i = v_i.$$

Therefore, $(v_i * y) * v_i = v_i$ for all $y \in V(L(n))$.

Case 3.

Let $x = v_n$. If $y = v_n$, then

$$(v_n * v_n) * v_n = v_n * v_n = v_n.$$

If $y \neq v_n$, then $v_n * y = v_0$, so

$$(v_n * y) * v_n = v_0 * v_n = v_n.$$

Hence, $(v_n * y) * v_n = v_n$ for all $y \in V(L(n))$.

Case 4.

Let $x = u_i$ for some $1 \leq i \leq n-1$. If $y = u_i$, then

$$(u_i * u_i) * u_i = u_i * u_i = u_i.$$

If $y \neq u_i$, then $u_i * y = v_i$, and therefore

$$(u_i * y) * u_i = v_i * u_i = u_i.$$

So, $(u_i * y) * u_i = u_i$ for all $y \in V(L(n))$.

Case 5.

Let $x = u_n$. If $y = u_n$, then

$$(u_n * u_n) * u_n = u_n * u_n = u_n.$$

If $y \neq u_n$, then $u_n * y = v_0$, and hence

$$(u_n * y) * u_n = v_0 * u_n = u_n.$$

Thus, $(u_n * y) * u_n = u_n$ for all $y \in V(L(n))$. Therefore, axiom (1) holds for every $x, y \in V(L(n))$.

Next, we prove (2). Assume that $x \neq y$. Since T is a spanning tree, there exists a unique path

$$x = x_0, x_1, \ldots, x_k = y$$

from x to y in T, where $k \geq 1$. By Definition 9,

$$x * y = x_1.$$

If $k = 1$, then in T, x and y are adjacent, and so $x * y = y$. Hence,

$$(x * y) * y = y * y = y \neq x.$$

If $k \geq 2$, then the unique path from x_1 to y is

$$x_1, x_2, \ldots, x_k.$$

Thus,

$$(x * y) * y = x_1 * y = x_2.$$

Since the path is simple, we have $x_2 \neq x_0 = x$. Therefore,

$$(x * y) * y \neq x.$$

We conclude that whenever $x \neq y$, it follows that $(x * y) * y \neq x$. Equivalently,

$$(x * y) * y = x \implies x = y.$$

Hence, both axioms of a travel groupoid are satisfied.

Indeed, for distinct vertices $x, y \in V(T)$, we have $x * y = y$ or $y * x = x$ if and only if x and y are adjacent in T. Thus, by the definition of the operation, the associated graph recovered from $*$ is precisely the selected spanning tree T. Accordingly, $(V(T),*)$ defines a travel groupoid on T. □

We have shown that $(V(T),*)$ is a travel groupoid on the selected spanning tree T. Next, we prove that it also satisfies the smoothness condition.

Theorem 2. *The travel groupoid $(V(T),*)$ is smooth.*

Proof. To prove that $(V(T),*)$ is smooth, it suffices to show that for all $u, v, w \in V(T) = V(L(n))$,

$$u * v = u * w \implies u * (w * v) = u * v.$$

Assume that $u * v = u * w$. Put $a = u * v = u * w$. If $a = u$, then by the definition of $*$, we have $u * v = u$, which implies $v = u$. Similarly, $u * w = u$ implies $w = u$. Hence,

$$u * (w * v) = u * (u * u) = u * u = u = u * v.$$

Thus, the smoothness condition holds in this case.

Now suppose that $a \neq u$. By Definition 9, the element a is the first vertex next to u on the unique u to v path in the spanning tree T, and also the first vertex next to u on the path uniquely determined by u to w in T.

Remove the vertex u from the tree T. Then the graph $T-u$ is divided into several connected components, each corresponding to one neighbor of u in T. Since the path from u to v begins with a, the vertex v belongs to the component of $T-u$ containing a. Similarly, since the path from u to w also begins with a, the vertex w belongs to the same component.

Let this component be denoted by C. Then

$$v, w \in V(C).$$

Since C is connected and C is also a subgraph of the tree T, the unique path from w to v lies entirely in C. Again by Definition 9, the vertex $w * v$ is the first vertex next to w on the path uniquely determined by w to v in T. Therefore,

$$w * v \in V(C).$$

Because $w * v$ lies in the same component C, the path uniquely determined by u to $w*v$ must also begin with a. Hence,

$$u * (w * v) = a.$$

But $a = u * v$, so we obtain

$$u * (w * v) = u * v.$$

Therefore, the smoothness condition holds for all $u, v, w \in V(L(n))$. Hence, $(V(T), *)$ is a smooth travel groupoid on the selected spanning tree T. □

As an illustration of the above construction, we first consider the lotus graph $L(2)$ and present the corresponding operation table.

Example 1. For $n = 2$, the lotus graph $L(2)$ has vertex set

$$V(L(2)) = \{v_0, v_1, v_2, u_1, u_2\},$$

and edge set

$$E(L(2)) = \{v_1u_1,\ u_1v_2,\ v_0v_1,\ v_0v_2,\ v_0u_2\}.$$

The lotus graph $L(2)$ is illustrated in Figure 2.

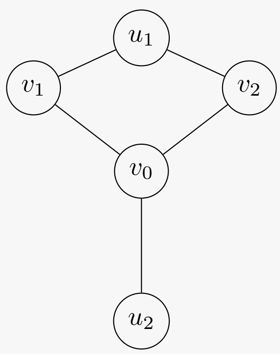

Figure 2. The lotus graph $L(2)$.

Following the construction above, we choose the spanning tree T of $L(2)$ with edge set

$$E(T) = \{v_0v_1,\ v_0v_2,\ v_1u_1,\ v_0u_2\}.$$

The chosen spanning tree T of $L(2)$ is shown in Figure 3.

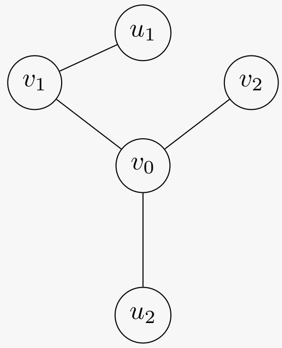

Figure 3. The spanning tree T of $L(2)$.

Using Definition 9, the binary operation $*$ on $V(L(2))$ is obtained from the first vertex next to x on the unique path from x to y in T. Hence, the operation table is given by

$*$	v_0	v_1	v_2	u_1	u_2
v_0	v_0	v_1	v_2	v_1	u_2
v_1	v_0	v_1	v_0	u_1	v_0
v_2	v_0	v_0	v_2	v_0	v_0
u_1	v_1	v_1	v_1	u_1	v_1
u_2	v_0	v_0	v_0	v_0	u_2

Therefore, it follows that $(V(L(2)), *)$ is a concrete realization of a smooth travel groupoid on the selected spanning tree of $L(2)$.

To further illustrate the construction, we also present the case of the lotus graph $L(3)$.

Example 2. For $n = 3$, the lotus graph $L(3)$ has vertex set

$$V(L(3)) = \{v_0, v_1, v_2, v_3, u_1, u_2, u_3\},$$

and edge set

$$E(L(3)) = \{v_1u_1, u_1v_2, v_2u_2, u_2v_3, v_0v_1, v_0v_2, v_0v_3, v_0u_3\}.$$

The lotus graph $L(3)$ is illustrated in Figure 4.

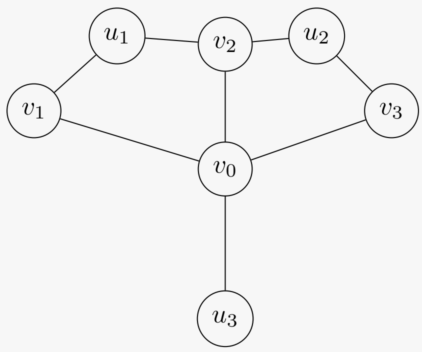

Figure 4. The lotus graph $L(3)$.

Following the construction in Proposition 1, we choose the spanning tree T of $L(3)$ with edge set

$$E(T) = \{v_0v_1, v_0v_2, v_0v_3, v_1u_1, v_2u_2, v_0u_3\}$$

as shown in Figure 5.

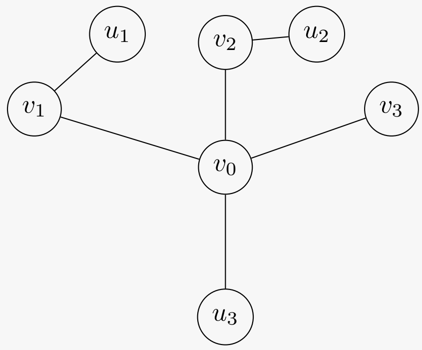

Figure 5. The spanning tree T of $L(3)$.

Using Definition 9, the binary operation $*$ on $V(L(3))$ is determined by the first vertex next to x on the unique path from x to y in T. Thus, the operation table is given by

$*$	v_0	v_1	v_2	v_3	u_1	u_2	u_3
v_0	v_0	v_1	v_2	v_3	v_1	v_2	u_3
v_1	v_0	v_1	v_0	v_0	u_1	v_0	v_0
v_2	v_0	v_0	v_2	v_0	v_0	u_2	v_0
v_3	v_0	v_0	v_0	v_3	v_0	v_0	v_0
u_1	v_1	v_1	v_1	v_1	u_1	v_1	v_1
u_2	v_2	v_2	v_2	v_2	v_2	u_2	v_2
u_3	v_0	v_0	v_0	v_0	v_0	v_0	u_3

Therefore, since $V(T) = V(L(3))$, the binary system $(V(L(3)), *)$ provides another concrete example of a smooth travel groupoid on the selected spanning tree T associated with the lotus graph $L(3)$.

These examples show that the operation is completely determined by the unique paths in the chosen spanning tree. The same idea extends naturally to the selected spanning tree T associated with the general lotus graph $L(n)$. Hence, the construction yields a smooth travel groupoid on T for every $n \geq 2$.

4. Conclusion

In this paper, we have constructed a smooth travel groupoid on a spanning tree associated with lotus graphs. The construction was obtained by choosing a spanning tree T of the lotus graph $L(n)$ and defining a binary operation on its vertex set through the first step along the unique path in T. We proved that the resulting binary system satisfies the axioms of a travel groupoid and also fulfills the smoothness condition. Hence, a smooth travel groupoid on the selected spanning tree T has been obtained for every $n \geq 2$.

As illustrations, the construction was applied to the selected spanning trees associated with $L(2)$ and $L(3)$, and the corresponding operation tables were presented. This result provides an explicit example of a smooth travel groupoid on a tree structure naturally derived from lotus graphs. A possible direction for future research is to investigate whether a different binary operation can be constructed so that the associated graph recovers the full lotus graph $L(n)$.

Author Contributions. **Husnul Khotimah**: Conceptualization, methodology, formal analysis, writing–original draft preparation, writing–review and editing. **Andi Tenri Ajeng Nur**: Methodology, validation, investigation, writing–review and editing. **Putri Nilam Cayo**: Validation, investigation, writing–review and editing.

Acknowledgement. The authors would like to express their sincere appreciation to the editors and reviewers for their careful evaluation, helpful comments, and valuable suggestions, which contributed to the improvement of this paper.

Funding. No external funding was received for this research.

Conflict of interest. The authors declare that there are no conflicts of interest regarding this work.

Data availability. No data are associated with this study.

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