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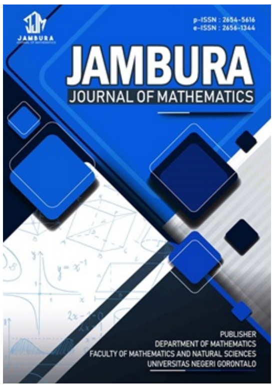

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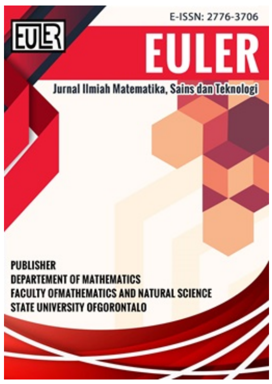

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Research Article

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Joint Life Long-Term Care Insurance: A Semi-Markov Multi-State Model Integrating LTC Prevalence and Mortality Trends

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ABSTRACT. *This study develops a two-person joint-life long-term care (LTC) insurance pricing framework for married couples by incorporating mortality and LTC risks within a semi-Markov-inspired multi-state model. The transition intensity structure accounts for Gompertz mortality, potential common-shock events, and a duration-dependent bereavement effect. LTC-related transition intensities are adjusted using proportional factors, while the transition probabilities are approximated using a matrix-exponential procedure and applied to the calculation of net single premiums. A numerical case study is presented for a married couple aged 62 and 60 under a 20-year coverage period. The alternative benefit designs include mortality protection, LTC annuity protection, and a combined design integrating death and LTC benefits. The results indicate mortality dependence between spouses through the common-shock and post-bereavement mechanisms and show that broader benefit coverage produces a higher net single premium. The main contribution of this study is an actuarial pricing framework for two associated individuals that jointly incorporates spousal mortality dependence, LTC incidence, and combined death and LTC benefits in a unified multi-state structure.*

1. Introduction

Long-term care (LTC) insurance is designed to provide financial protection for individuals who can no longer perform daily tasks independently and require external assistance. Permanent inability to perform essential activities of daily living (ADLs) can significantly impact a person's quality of life [1, 2]. With the increasing prevalence of long-term care needs and a growing elderly population, LTC insurance has emerged as a vital component of the healthcare industry [3].

Various approaches have been adopted by both developed and developing countries to address the growing need for LTC insurance. For instance, Germany and Japan have implemented national long-term care insurance systems [4]. Meanwhile, Denmark has integrated home- and community-based long-term care services into its existing social insurance framework. In contrast, the United States provides public health insurance for the elderly through Medicare [5], but it explicitly excludes long-term care (LTC) coverage [6]. In the United Kingdom, private LTC insurance is almost nonexistent [3]. In developing countries like Indonesia, access to retirement security and LTC coverage remains limited due to the lack of government-backed or private insurance programs. Many individuals do not have retirement insurance, leading to a significant decline in the quality of life for the elderly due to insufficient support. According to [7], Indonesia is entering an aging population phase, with the elderly population reached 18 million (7.56%) in 2010 and is projected to increase significantly, reaching 48.2 million (15.77%) by 2035.

One of the primary barriers to the widespread adoption of LTC insurance is the high premium cost, particularly for stand alone LTC annuities [8]. Over time, LTC insurance has evolved to include various types of products, not only covering long-term care services but also offering additional death benefits [8, 9]. Some insurance products provide benefits linked to joint survival status, such as life annuities designed for married couples [10, 11]. From a couple's perspective, LTC insurance can offer financial protection for both partners living together. This innovation has led to the development of joint LTC insurance policies, which provide a more comprehensive and flexible solution for long-term care planning [12, 13].

The Markov model plays an important role in estimating transition probabilities between various health states, which are subsequently used to calculate additional benefits for married couples [13]. However, a major limitation of the Markov model is its assumption that a spouse's mortality remains constant over time [14]. To address this issue, [15] proposed a semi-Markov model that better represent the dependency between spouses. This dependency arises from shared lifestyles and the possibility of experiencing catastrophic events together [11]. In the semi-Markov framework, transition intensities depend not only by the current state but also by the duration since the last transition.

Ji [10] linked the survival status of joint life insurance policies with the inclusion of long-term care (LTC) coverage, incorporating the concept of *home reversion* contracts. Married couples who live together are more likely to opt for joint LTC insurance policies [12]. Under this insurance arrangement, mortality rates differ significantly between individuals with disabilities and

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those without, depending on gender and age [16]. Furthermore, Zweifel [12] highlighted that in conventional insurance schemes, coverage decisions are often made sequentially, with one spouse typically deciding on joint LTC insurance first. Shang et al. [17] identified six long-term care (LTC) states for elderly couples using a Markov model, demonstrating that those with poorer initial health conditions face a higher risk of requiring LTC and experiencing mortality. Although the Markov assumption is widely used in current research, it may not always adequately capture long-term care risks. Instead, the semi-Markov model is considered a more appropriate approach for modeling risk dynamics [18].

Previous studies have not specifically addressed premium calculation models for long-term care (LTC) insurance for married couples within a joint life insurance framework [10, 12, 15, 17]. This study develops a premium calculation model by integrating long-term care (LTC) insurance and joint life insurance using a semi-Markov approach, building upon the work of [10, 13]. The proposed premium model is adjusted based on the prevalence of home-based LTC utilization and the declining mortality rates observed in IFOA 2005b [19].

The novelty of this study lies in extending the joint-life semi-Markov mortality framework to a two-person LTC insurance pricing model. Unlike previous models that primarily examine mortality dependence between spouses, the proposed eight-state structure also represents individual and joint LTC conditions. The framework integrates bereavement-related mortality, potential common-shock events, and LTC incidence, and applies the resulting transition structure to the valuation of mortality benefits, LTC annuity benefits, and a combined life insurance contract with an LTC annuity rider.

2. Preliminaries

This study applies a semi-Markov framework to model the health dynamics of married couples. Let $\mathcal{S} = \{0, 1, 2, \dots, m\}$ denote the finite state space and the sequence $\{X_n\}_{n\geq 0}$ denotes the state occupied immediately after the n-th transition. Let $\{T_n\}_{n\geq 0}$ denote the corresponding transition times, with $T_0 = 0$. The sojourn time to move to a different state is denoted by $S_n = T_n - T_{n-1}$. The semi-Markov kernel is defined by

$$
\begin{aligned}
Q_{ij}(t) &= P\left(X_{n+1} = j,\, S_n \leq t \mid X_n = i\right) \\
&= \Pr\left(X_{n+1} = j,\, T_{n+1} - T_n \leq t \mid X_n = i\right),
\end{aligned} \tag{1}
$$

for $n \geq 0$, $t \geq 0$, and $i, j \in S$ [20]. Thus $Q_{ij}(t)$ represents the probability that the next state visited by the process is state j and that the sojourn time in state i does not exceed t, conditional on the current state being state i. According to [21], this process models the transition probability of the embedded Markov chain is given by

$$
p^{ij} = \lim_{t\to\infty} Q_{ij}(t) = \Pr\left(X_{n+1} = j \mid X_n = i\right). \tag{2}
$$

For $p^{ij} > 0$, the conditional cumulative distribution function of the sojourn time is defined as $F_{ij}(t) = P\left(S_n \leq t \mid X_n = i,\, X_{n+1} = j\right)$ [21]. Consequently, the semi-Markov kernel can be expressed as

$$
Q_{ij}(t) = p_{ij} F_{ij}(t). \tag{3}
$$

A semi-Markov model with four states is used to represent the health transitions of a married couple, as illustrated in Figure 1.

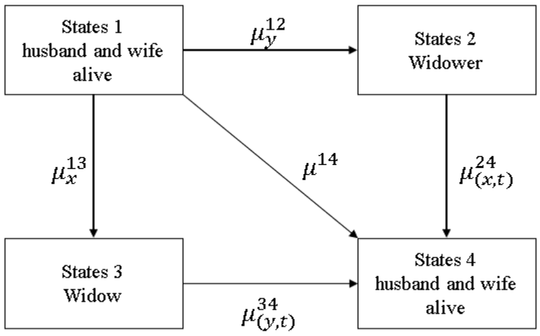

Figure 1. Semi-Markov model for husband and wife.

Let x dan y denote the ages of the husband and wife, respectively. The waiting time spent as a widow or widower is represented by $\mu^{24}(x,t)$ and $\mu^{34}(y,t)$ [15]. In this semi-Markov model, a parametric function is utilized to describe the force of mortality following the death of a spouse $\mu^{24}(x,t) = f_1(t)\left(\mu_{x+t}^{13} + \mu^{14}\right)$ and $\mu^{34}(y,t) = f_2(t)\left(\mu_{y+t}^{12} + \mu^{14}\right)$. where the function $f_i(t)$ is defined as $f_i(t) = A_i e^{-k_i t} + 1$, with parameters $A_1, A_2 > -1$ and $k > 0$ for $i = 1, 2$. The semi-Markov model presented in Figure 2 is applied to joint-life LTC insurance premium calculations, incorporating additional death benefits. Each state is represented by an ordered pair describing the conditions of the husband and wife, respectively. The first letter refers to the husband's condition, whereas the second letter refers to the wife's condition. The symbols a, d, and f denote healthy, requiring long-term care, and deceased, respectively. For example, state 2, denoted by (ad), indicates that the husband is healthy while the wife requires LTC. State 5, denoted by (af), indicates that the husband is healthy while the wife is deceased. State 7 combines the two conditions (df) and (fd), in which one spouse requires LTC and the other spouse is deceased.

The model captures transitions starting from state 1 and progressing toward states 4, 7, and 8, reflecting different stages of long-term care needs and mortality. In the multi-state modeling framework, transition probabilities are formulated based on the Chapman-Kolmogorov equation and are structured within the following transition matrix:

$$
P(z, z+t) = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & 0 & P_{15} & P_{16} & 0 & P_{18} \\
0 & P_{22} & 0 & P_{24} & 0 & 0 & P_{27} & 0 \\
0 & 0 & P_{33} & P_{34} & 0 & 0 & P_{37} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & P_{55} & 0 & P_{57} & P_{58} \\
0 & 0 & 0 & 0 & 0 & P_{66} & P_{67} & P_{68} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \tag{4}
$$

where

$$
\begin{aligned}
&P_{11} = {}_tp_{xy}^{11}, \quad P_{12} = {}_tp_{x:y}^{12}, \quad P_{13} = {}_tp_{x:y}^{13}, \quad P_{15} = {}_tp_{x:y}^{15} \\
&P_{16} = {}_tp_{x:y}^{16}, \quad P_{18} = {}_tp_{x:y}^{18}, \quad P_{22} = {}_tp_{xy}^{22}, \quad P_{24} = {}_tp_{x:y}^{24}, \\
&P_{27} = {}_tp_{x:y}^{27}, \quad P_{33} = {}_tp_{xy}^{33}, \quad P_{34} = {}_tp_{x:y}^{34}, \quad P_{37} = {}_tp_{x:y}^{37} \\
&P_{55} = {}_tp_{x}^{55}, \quad P_{57} = {}_tp_{x}^{57}, \quad P_{58} = {}_tp_{x}^{58}, \quad P_{66} = {}_tp_{y}^{66}, \\
&P_{67} = {}_tp_{y}^{67}, \quad P_{68} = {}_tp_{6}^{68}.
\end{aligned}
$$

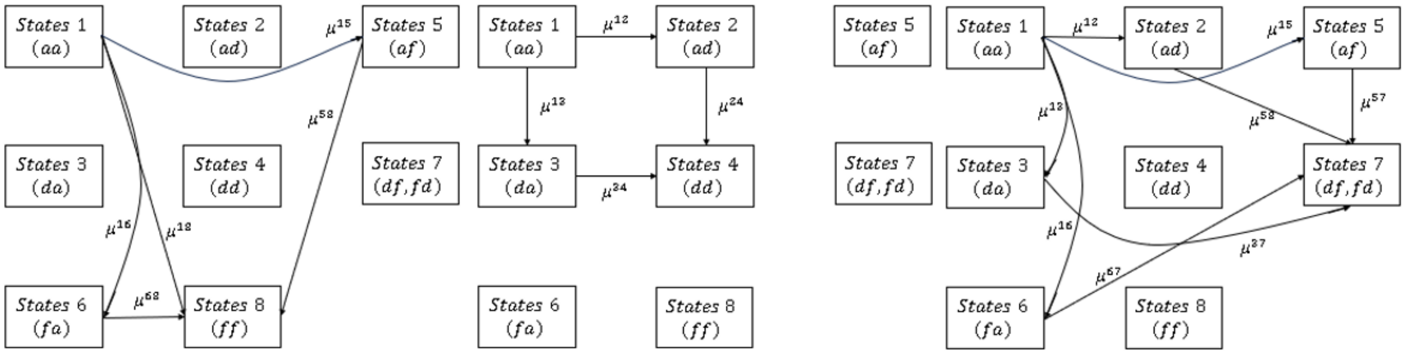

Figure 2. Multistate model for joint LTC insurance.

Let $Z = (X, Y)$ be a random variable representing the ages of a married couple, where $x, y \geq 0$ denote their respective entry ages. If one of the individuals dies while in state i, the notation follows that of individual life actuarial models, namely ${}_tp_x^{ij}$ and ${}_tp_y^{ij}$, which respectively represent the probability that a husband (widower) aged x and a wife (widow) aged y in state i will remain alive for t additional years and be in state j. Assumed that the loss of a spouse affects the transition intensity for states 5 and states 6, which is determined using the semi-Markov functions $f_1(t)$ and $f_2(t)$. The transition intensity values in the model are presented in Table 1, as discussed in [10].

Table 1. Estimated transition intensity for long-term care insurance.

Both husband and wife are healthy
$\mu^{16}_{x:y} = \theta_x \mu^{24}(x,t)$
$\mu^{13}_{x:y} = \rho_x$
$\mu^{15}_{x:y} = \theta_y \mu^{34}(y,t)$
$\mu^{12}_{x:y} = \rho_y$
$\mu^{18}_{x:y} = \mu^{14}$
Requires LTC facility/Death
$\mu^{68}_{y} = f_2(t)(\theta_y \mu^{34}(y,t) + \mu^{14})$
$\mu^{67}_{y} = f_2(t)(\rho_y)$
$\mu^{58}_{x} = f_1(t)(\theta_x \mu^{24}(x,t) + \mu^{14})$
$\mu^{57}_{x} = f_1(t)(\rho_x)$
One or both require LTC facilities
$\mu^{37}_{x:y} = \theta_y \mu^{34}(y,t) + \mu^{14}$
$\mu^{34}_{x:y} = \rho_y$
$\mu^{27}_{x:y} = \theta_x \mu^{24}(x,t) + \mu^{14}$
$\mu^{24}_{x:y} = \rho_x$

Since the primary dataset does not directly record LTC prevalence, age-specific proportional factors are applied to model the transitions . The transition intensity model for long-term care (LTC) status of husbands and wives is defined through ρ_x and ρ_y. The impact of decreasing mortality is modeled under the assumption that the mortality rate in a healthy state is denoted as θ_x and θ_y. The proportional factor used to determine the transition intensity parameters for LTC-required status is based on findings from the Equity Report by [10, 19] and is summarized in Table 2. For ages that are not explicitly reported in Table 2, the proportional adjustment factors are obtained using linear interpolation between the two nearest tabulated ages [10].

Table 2. Proportional adjustment factors of ρ^m, θ^m, ρ^f, and θ^f.

Age	ρ_x	θ_x	ρ_y	θ_y
≤ 60	0.052	0.967	0.062	0.961
65	0.051	0.969	0.078	0.959
70	0.042	0.973	0.069	0.96
75	0.066	0.97	0.18	0.936
80	0.067	0.965	0.191	0.909
85	0.125	0.949	0.314	0.868
90	0.142	0.93	0.312	0.834
95	0.191	0.91	0.409	0.787
≥ 100	0.206	0.895	0.432	0.74

The coverage model for joint LTC insurance is calculated by summing all possible transition paths throughout the coverage period, as illustrated in Figure 2, obtained:

$$A^{(1)}_{x:y:\overline{n}|} = j_1, \tag{5}$$

$$A^{(2)}_{x:y:\overline{n}|} = j_2, \tag{6}$$

$$A^{(3)}_{x:y:\overline{n}|} = j_3, \tag{7}$$

where j_1, j_2, and j_3 denote the net single premiums for the death-benefit design, the LTC annuity design, and the combined life insurance and LTC rider design, respectively.

$$j_1 \approx c \cdot \frac{\delta}{i}\left(1 - \delta t \int_{e=0}^{n} \exp\left(-\delta t\right)({}_tp^{11}_{x:y} + {}_tp^{15}_{x:y} + {}_tp^{16}_{x:y})dt\right), \tag{8}$$

$$j_2 = b'\left(\sum_{e=1}^{n} v^e \cdot {}_{e-1}p^{11}_{x:y} \cdot p^{12}_{x:y+e-1} \cdot \ddot{a}^2_{x:y+e:\overline{n-e}|} + \right. \tag{9}$$

$$\left. \sum_{e=1}^{n} v^e \cdot {}_{e-1}p^{11}_{x:y} \cdot p^{13}_{x:y+e-1} \cdot \ddot{a}^3_{x:y+e:\overline{n-e}|}\right) +$$

$$b\left(\sum_{e=2}^{n} v^e \cdot {}_{e-1}p^{22}_{x:y} \cdot p^{24}_{x:y+e-1} \cdot \ddot{a}^4_{x:y+e:\overline{n-e}|} + \right.$$

$$
\begin{aligned}
&\sum_{e=2}^{n} v^e \cdot {}_{e-1}p^{33}_{x:y} \quad \cdot p^{34}_{x:y+e-1} \cdot \ddot{a}^{4}_{x:y+e:\overline{n-e}|}\Bigg) \\
j_3 = &\Bigg(c'\Bigg(\sum_{e=1}^{n} {}_{e-1}p^{11}_{x:y} \cdot p^{15}_{x:y+e-1} \cdot v^e + \qquad (10)\\
&\sum_{e=1}^{n} {}_{e-1}p^{11}_{x:y} \cdot p^{16}_{x:y+e-1} \cdot v^e\Bigg) + \\
&b'\Bigg(\sum_{e=2}^{n} v^e \cdot {}_{e-1}p^{55}_{x} \cdot p^{57}_{x+e-1} \cdot \ddot{a}^{7}_{x+e:\overline{n-e}|} + \\
&\sum_{e=2}^{n} v^e \cdot {}_{e-1}p^{66}_{y} \cdot p^{67}_{y+e-1} \cdot \ddot{a}^{7}_{y+e:\overline{n-e}|}\Bigg)\Bigg) + \\
&\Bigg(b'\Bigg(\sum_{e=1}^{n} v^e \cdot {}_{e-1}p^{11}_{x:y} \cdot p^{12}_{x:y+e-1}\ddot{a}^{2}_{x:y+e:\overline{n-e}|} + \\
&\sum_{e=1}^{n} v^e \cdot {}_{e-1}p^{11}_{x:y} \cdot p^{13}_{x:y+e-1} \cdot \ddot{a}^{3}_{x:y+e:\overline{n-e}|}\Bigg) + \\
&c'\Bigg(\sum_{e=2}^{n} {}_{e-1}p^{22}_{x:y} \cdot p^{27}_{x:y+e-1} \cdot v^e + \\
&\sum_{e=2}^{n} {}_{e-1}p^{33}_{x:y} \cdot p^{37}_{x:y+e-1} \cdot v^e\Bigg)\Bigg).
\end{aligned}
$$

The possible pathways a married couple may follow during the insurance coverage period. The notations c, c' refer to the lump-sum death benefits, while b, b' represents the annual annuity benefits, which may be paid either fully or partially depending on the insured's condition. Additionally, $\delta = \ln(1 + r)$ denotes the force of interest and $v = (1 + r)^{-1}$ is the annual discount factor, both of which depends on an interest rate of r%. The actuarial present value of the annuity is given by $\ddot{a}^{i\cdot}_{x:y+e:\overline{n-e}|} = \sum_{e=0}^{n-1} v^e {}_k p^{i\cdot}_{xy}$.

3. Results and Discussion

The primary dataset consists of observations on 12170 married couples followed from December 1988 to December 1993. The dataset was developed by Frees et al. with support from the Society of Actuaries and contains joint-life mortality information for husbands and wives [22]. The primary joint-life dataset does not directly contain detailed information on LTC prevalence or LTC incidence. The LTC component is incorporated separately through age-specific proportional adjustment factors obtained from the Equity Release Report. The descriptive statistics of the dataset are summarized in Figure 3.

However, as shown in Figure 3, the dataset does not include information on causes of death among married couples, such as lifestyle factors or economic status. To analyze the effects of common shocks, the data are categorized into several groups based on the semi-Markov joint-life mortality framework proposed by [10]. A five-day operational cut-off is adopted to identify potential common-shock cases, thereby facilitating the estimation of the relevant parameters in the semi-Markov model, as presented in Table 3.

Additionally, Table 4 presents data on the number of deaths following the loss of a spouse, categorized by the time range

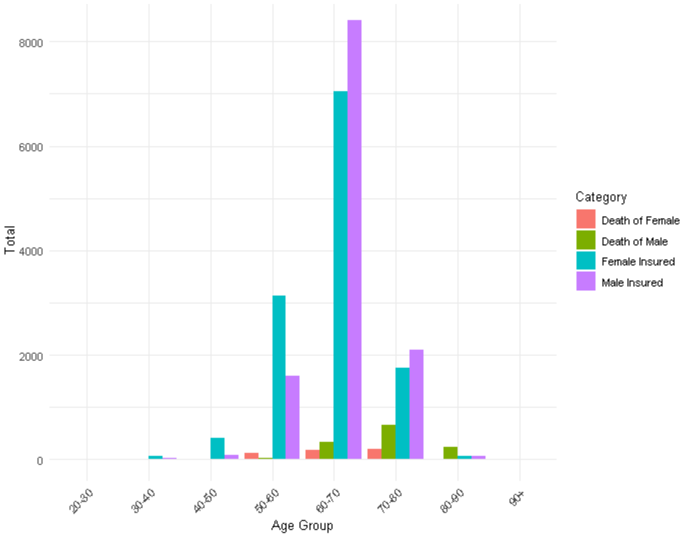

Figure 3. Histogram of the number of insured individuals and deaths by age.

Table 3. Common shock.

Category	Total
≤ 5 after their spouse's death.	53
≥ 5 days after their spouse's death.	1497
Survived until the end of the observation	10620
Total	12170

W (in years) between the bereavement period and death of the surviving spouse. The table indicates that half of the deaths of one spouse occur within the first year.

Table 4. Mortality data based on time range t.

	$0 \leq W \leq 1$	$1 < W \leq 2$	$2 < W \leq 3$	$W > 3$	Total
Husband	82	11	3	7	103
Wife	40	21	14	6	81
Total	122	32	17	13	184

The parameter estimation for the semi-Markov model is supported by the Gompertz model, defined as $\mu_x = BC^x$, where $B > 0$ and $C > 1$, to determine the transition intensity values. Compared to the Markov model, the extension of the semi-Markov joint life model primarily affects mortality after bereavement. The log-likelihood value for the common shock case is determined using information on simultaneous death. Based on the data classification, 53 deaths occurred within five days after losing a spouse, which are considered simultaneous deaths. The parameters of the semi-Markov model are estimated using the maximum likelihood estimation method, as presented in Table 5. The estimated value of the common shock factor (μ^{14}) is 1.4000×10^{-3}, and the force of mortality for husbands and wives aged x and y is given as follows:

$$
\begin{aligned}
\mu^{13}_{x+t} &= B_1 C_1^{x+t} = (2.615031 \times 10^{-5})(1.098900 \times 10^{0})^{x+t} \\
\mu^{12}_{y+t} &= B_2 C_2^{y+t} = (3.062168 \times 10^{-7})(1.133100 \times 10^{0})^{y+t} \\
\mu^{24}(x,t) &= f_1(t)(\mu^{13}_{x+t} + \mu^{14}) \\
&= (1 + 1.105410 \times 10^{+1} e^{-7.906400 \times 10^{+0} t})
\end{aligned}
$$

$$
\begin{aligned}
&((2.618341 \times 10^{-5})(1.098900 \times 10^{0})^{x+t} \\
&+ 1.407000 \times 10^{-3}) \\
\mu^{34}(y,t) &= f_2(t)(\mu^{12}_{y+t} + \mu^{14}) \\
&= (1 + 3.378600 \times 10^{+0} e^{-5.225000 \times 10^{-1} t}) \\
&((4.295736 \times 10^{-7})(1.133100 \times 10^{0})^{y+t} \\
&+ 1.407000 \times 10^{-3}).
\end{aligned}
$$

Table 5. Estimated parameters of semi-markov model.

Parameter	Initials Value	Estimated Value	Std. Error
B^1	2.6220×10^{-5}	2.618341×10^{-5}	1.100100×10^{-20}
C^1	$1.0989 \times 10^{+0}$	$1.098900 \times 10^{+0}$	4.153342×10^{-26}
B^2	9.7410×10^{-7}	4.295736×10^{-7}	2.435939×10^{-18}
C^2	$1.1331 \times 10^{+0}$	$1.133100 \times 10^{+0}$	8.455491×10^{-27}
μ	1.4070×10^{-3}	1.407000×10^{-3}	6.730148×10^{-27}
A_1	$1.1054 \times 10^{+1}$	$1.105410 \times 10^{+1}$	1.148829×10^{-34}
k_1	$7.9064 \times 10^{+0}$	$7.906400 \times 10^{+0}$	5.082315×10^{-33}
A_2	$3.3786 \times 10^{+0}$	$3.378600 \times 10^{+0}$	6.030388×10^{-33}
k_2	5.2250×10^{-1}	5.225000×10^{-1}	2.074172×10^{-34}

According to [11], the force of mortality for surviving individuals is determined solely by their own age and marital status. Based on Figure 4, there is a clear relationship between the number of deaths and time t. The plot of the force of mortality against increasing age for husbands x and wives y indicates that husbands face a higher risk of death than wives, this observation is also supported by Table 4. The plot of the force of mortality after bereavement reveals an initial increase in mortality at the beginning of the mourning period, followed by a subsequent decline. Moreover, the decline in mortality for widows (wives who lost their husbands) occurs more gradually than for widowers (husbands who lost their wives), whose mortality rises more quickly after the loss, before stabilizing again over time

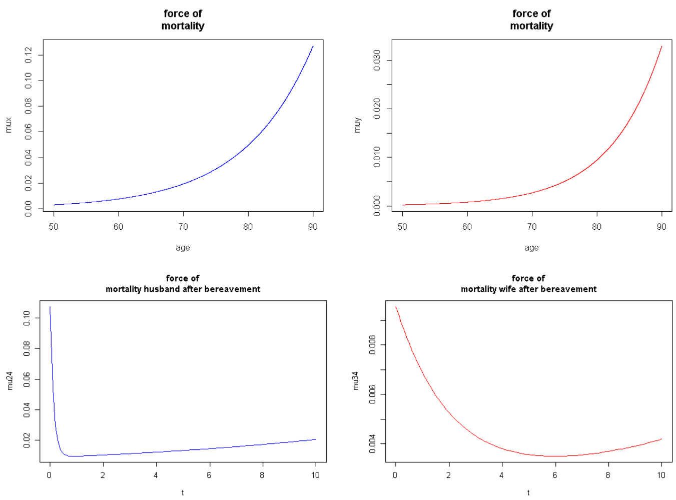

Figure 4. Force of mortality and force of mortality after bereavement age $x, y = 60$ years.

Furthermore, the broken-heart effect semi-Markov model is explained by the impact of a spouse's death on an individual's mortality, which is modeled using a multiplicative factor. It is observed that for the common shock, when $0 < t \leq 1$, it decreases more sharply than when $0 < t \leq 10$. This indicates that the closer the death gap between two individuals, the more likely they are to be affected by the common shock. The factors $f_1(t)$ and $f_2(t)$ are created to represent the force of mortality after bereavement to measure the impact of a spouse's death on mortality [10, 11].

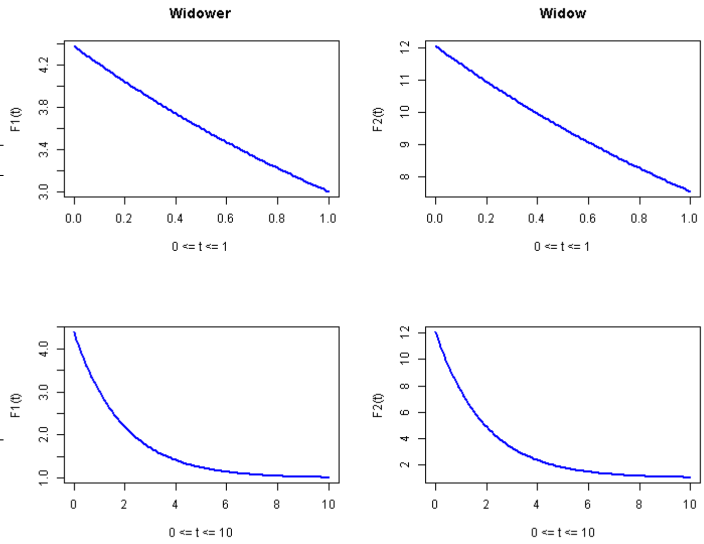

Figure 5. *Force of mortality* $F_1(t)$ and $F_2(t)$ semi-markov model.

Next, a premium calculation scenario is developed for the joint life long-term care (LTC) insurance model for a couple aged $x = 62$ years and $y = 60$ years. Before determining the premium amount for an insurance product, it is necessary to calculate the transition probability matrix were approximated using a matrix-exponential approach as a basis for risk assessment. Let $Q(t)$ denote the transition intensity matrix constructed from the Gompertz mortality intensities. Matrix $Q(t)$ was constructed using the corresponding attained ages and the duration-adjusted mortality intensities. The transition probability matrix was then obtained as $P(t) \approx \exp\{Q(t)t\}$. For a coverage period of 20 years, several transition probability matrices are computed for $n = 1, 5, 10, 15, 20$ years as follows:

$$
p_{62:60} = \begin{bmatrix}
p_{11} & p_{12} & p_{13} & 0 & p_{15} & p_{16} & 0 & p_{18} \\
0 & p_{22} & 0 & p_{24} & 0 & 0 & p_{27} & 0 \\
0 & 0 & p_{33} & p_{34} & 0 & 0 & p_{37} & 0 \\
0 & 0 & 0 & p_{44} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & p_{55} & 0 & p_{57} & p_{58} \\
0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & p_{68} \\
0 & 0 & 0 & 0 & 0 & 0 & p_{77} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88}
\end{bmatrix}.
$$

where

$$
\begin{aligned}
&p_{11} = 0.880685, && p_{12} = 0.056510, && p_{13} = 0.046879, \\
&p_{15} = 0.005739, && p_{16} = 0.008720, && p_{18} = 0.001466, \\
&p_{22} = 0.938663, && p_{24} = 0.050000, && p_{27} = 0.011336, \\
&p_{33} = 0.932669, && p_{34} = 0.059889, && p_{37} = 0.007442, \\
&p_{44} = 1, && p_{55} = 0.938421, && p_{57} = 0.050198, \\
&p_{58} = 0.011381, && p_{66} = 0.811098, && p_{67} = 0.168023,
\end{aligned}
$$

$p_{68} = 0.020879, \quad p_{77} = 1, \quad p_{88} = 1.$

$${}_{5}p_{62:60} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & p_{16} & 0 & p_{18} \\ 0 & p_{22} & 0 & p_{24} & 0 & 0 & p_{27} & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & p_{37} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 & p_{57} & p_{58} \\ 0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & p_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88} \end{bmatrix}.$$

where

$p_{11} = 0.570609, \quad p_{12} = 0.208806, \quad p_{13} = 0.172520,$
$p_{15} = 0.008811, \quad p_{16} = 0.032836, \quad p_{18} = 0.006418,$
$p_{22} = 0.728852, \quad p_{24} = 0.221181, \quad p_{27} = 0.049966,$
$p_{33} = 0.718840, \quad p_{34} = 0.264027, \quad p_{37} = 0.017133,$
$p_{44} = 1, \quad p_{55} = 0.728852, \quad p_{57} = 0.221181,$
$p_{58} = 0.049966, \quad p_{66} = 0.662373, \quad p_{67} = 0.317053,$
$p_{68} = 0.020574, \quad p_{77} = 1, \quad p_{88} = 1.$

$${}_{10}p_{62:60} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & p_{16} & 0 & p_{18} \\ 0 & p_{22} & 0 & p_{24} & 0 & 0 & p_{27} & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & p_{37} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 & p_{57} & p_{58} \\ 0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & p_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88} \end{bmatrix}.$$

where

$p_{11} = 0.358590, \quad p_{12} = 0.311655, \quad p_{13} = 0.256126,$
$p_{15} = 0.010731, \quad p_{16} = 0.050544, \quad p_{18} = 0.012354,$
$p_{22} = 0.531226, \quad p_{24} = 0.382390, \quad p_{27} = 0.086385,$
$p_{33} = 0.519225, \quad p_{34} = 0.454795, \quad p_{37} = 0.025980,$
$p_{44} = 1, \quad p_{55} = 0.531226, \quad p_{57} = 0.382390,$
$p_{58} = 0.086385, \quad p_{66} = 0.513076, \quad p_{67} = 0.460612,$
$p_{68} = 0.026312, \quad p_{77} = 1, \quad p_{88} = 1.$

$${}_{15}p_{62:60} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & p_{16} & 0 & p_{18} \\ 0 & p_{22} & 0 & p_{24} & 0 & 0 & p_{27} & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & p_{37} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 & p_{57} & p_{58} \\ 0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & p_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88} \end{bmatrix}.$$

where

$p_{11} = 0.236587, \quad p_{12} = 0.369710, \quad p_{13} = 0.301756,$
$p_{15} = 0.012519, \quad p_{16} = 0.059895, \quad p_{18} = 0.019533,$
$p_{22} = 0.387185, \quad p_{24} = 0.499887, \quad p_{27} = 0.112928,$
$p_{33} = 0.374338, \quad p_{34} = 0.592172, \quad p_{37} = 0.033490,$
$p_{44} = 1, \quad p_{55} = 0.387185, \quad p_{57} = 0.499887,$
$p_{58} = 0.112928, \quad p_{66} = 0.373848, \quad p_{67} = 0.592636,$
$p_{68} = 0.033516, \quad p_{77} = 1, \quad p_{88} = 1.$

$${}_{20}p_{62:60} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & p_{16} & 0 & p_{18} \\ 0 & p_{22} & 0 & p_{24} & 0 & 0 & p_{27} & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & p_{37} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 & p_{57} & p_{58} \\ 0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & p_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88} \end{bmatrix}.$$

where

$p_{11} = 0.160569, \quad p_{12} = 0.404382, \quad p_{13} = 0.327563,$
$p_{15} = 0.013676, \quad p_{16} = 0.065062, \quad p_{18} = 0.028748,$
$p_{22} = 0.282201, \quad p_{24} = 0.585525, \quad p_{27} = 0.132274,$
$p_{33} = 0.269800, \quad p_{34} = 0.691142, \quad p_{37} = 0.039059,$
$p_{44} = 1, \quad p_{55} = 0.282201, \quad p_{57} = 0.585525,$
$p_{58} = 0.132274, \quad p_{66} = 0.269765, \quad p_{67} = 0.691175,$
$p_{68} = 0.039060, \quad p_{77} = 1, \quad p_{88} = 1.$

The joint life LTC insurance is formulated based on Equations (8-10). The premium calculation is divided into three categories based on the specific needs of the insured couple, classified according to termination types j_1, j_2, and j_3 They represent alternative product structures rather than additive termination components. Consider a joint life LTC insurance product with a coverage period of 20 years and an interest rate of 6%. The benefits provided based on the type of termination are detailed as follows.

- **Termination** j_1: A death benefit of IDR 200 million for the insured couple.
- **Termination** j_2: Annual benefits as follows:
  - IDR 10 million per year for an individual.
  - IDR 20 million per year for a couple if both require long-term care.
- **Termination** j_3:
  - A death benefit of IDR 100 million for an individual.
  - An annuity benefit of IDR 10 million per year.

The single premium for each type of termination is calculated as follows, as presented in Table 6.

Table 6. Single premium for each type of termination.

Type of Termination	Single Premium
j_1	IDR 134.932.859,6
j_2	IDR 183.022.628
j_3	IDR 211.017.882

Based on these termination options, the most suitable insurance product can be selected according to the specific risk profile of the policyholder.

4. Conclusion

This study develops a joint-life long-term care (LTC) insurance valuation framework that incorporates spousal mortality dependence. The proposed multi-state structure is used to evaluate three alternative insurance designs: death-benefit protection, LTC annuity protection, and a combined life insurance contract with an LTC annuity rider.

For the numerical case study involving a husband aged 62 and a wife aged 60 under a 20-year coverage period, the calculated net single premiums are IDR $134.932.859, 6$ for the death-benefit design, IDR $183.022.628$ for the LTC annuity design, and IDR $211.017.882$ for the combined design. The combined design produces the highest premium because it provides protection against both mortality and LTC-related events. These results demonstrate that the premium level increases with the range of benefit-triggering events covered by the insurance contract.

The main contribution of this study is the development of a two-person LTC insurance pricing framework that integrates the health, LTC, and mortality conditions of both spouses within a unified multi-state structure. The framework also provides several benefit alternatives that may be selected according to the protection needs and financial preferences of married couples. Several limitations should be considered. The common-shock classification is based on a five-day operational cut-off because cause-of-death information is unavailable. In addition, the LTC component is incorporated using externally sourced proportional adjustment factors rather than directly observed LTC experience in the joint-life dataset.

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