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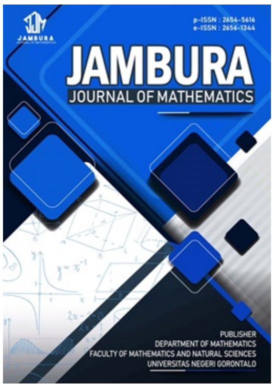

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Research Article

Optimal IDX30 Stock Portfolio Construction Using a Two-Constraint Mean–Variance Model with Robust S-Estimation

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ABSTRACT. *The capital market plays an important role in the economy by providing investment instruments for investors and financing sources for companies. A capital market portfolio consists of a collection of financial assets, such as stocks, constructed to achieve an optimal return while reducing investment risk. Mean–variance portfolio construction is highly sensitive to parameter estimation errors. Therefore, a robust estimation approach is employed to obtain more stable parameter estimates by minimizing the influence of outliers. This study aims to construct an optimal stock portfolio through diversification, determine stock weights using a two-constraint mean–variance model with robust S-estimation, calculate the expected return and risk, and evaluate portfolio performance. The analysis was conducted using the closing prices of stocks included in the IDX30 Index from October 2024 to September 2025. The results identified nine stocks with positive expected returns from five different sectors. Based on the stock selection criteria, two optimal portfolios were constructed. Portfolio 1 consists of ASII, BRPT, INDF, PGAS, and TLKM, whereas Portfolio 2 consists of ASII, ANTM, INDF, PGAS, and TLKM. Portfolio 1 generates an expected return of 0.137% with a risk of 2.226%, while Portfolio 2 generates an expected return of 0.097% with a risk of 1.319%. Based on the Sharpe and Treynor ratios, Portfolio 1 demonstrates relatively better performance than Portfolio 2.*

1. Introduction

Investment is an activity involving the allocation of funds to one or more types of assets over a certain period with the expectation of generating income or increasing the value of the investment in the future [1]. Investment in financial assets is considered an attractive option for investors because it may provide higher returns than investment in real assets. Among the various instruments available in the capital market, stocks are commonly preferred over other instruments, such as bonds and deposits [2]. However, despite their promising return potential, stock investments are also associated with substantial risk because stock prices fluctuate in response to economic conditions, political developments, natural disasters, technological changes, and market speculation.

In investment theory, return and risk have a strong positive relationship, indicating that investments with higher risk are generally expected to provide higher returns, and vice versa [3]. Investors commonly seek to maximize returns while minimizing the risks they must bear. Based on their attitudes toward risk, investors can be classified into three categories: risk seekers, risk-neutral investors, and risk-averse investors. Risk seekers are willing to accept substantial risk, risk-neutral investors consider the balance between return and risk, whereas risk-averse investors prefer investments with lower levels of risk [4]. One strategy for managing investment risk is to avoid allocating all available funds to a single stock and instead distribute the investment across several assets through portfolio diversification.

Portfolio diversification is a strategy for reducing investment risk by distributing funds among several stocks, thereby potentially lowering portfolio risk while maintaining the expected portfolio return [5]. Through diversification, investors can minimize risk for a desired level of return or maximize return for an acceptable level of risk. Portfolio construction is generally performed by selecting efficient stocks issued by different companies. Before being included in a portfolio, each stock must be analyzed to assess its potential future performance. An effective diversification strategy may involve selecting stocks with different characteristics, including stocks from different business sectors, stocks with low correlations, and stocks with positive expected returns [6].

One method commonly employed in stock portfolio management by simultaneously considering return and risk is the mean–variance model. This model enables the construction of an efficient portfolio by assigning appropriate weights to the selected assets. One extension of this approach is the two-constraint mean–variance model, which incorporates two restrictions into the portfolio optimization process. In addition to requiring the total investment weight to be equal to one, the model also imposes a minimum target return expected by the investor [7].

Nevertheless, the estimated inputs used in the mean–variance model, particularly the mean vector and variance–covariance matrix, are often affected by market fluctuations, extreme observations or outliers, and non-normal return distribu-

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tions. Such instability may produce estimation errors in optimal portfolio construction and consequently reduce investment performance [8]. To address this limitation, a robust estimation approach can be applied. Robust estimation provides more stable parameter estimates by minimizing the influence of extreme observations on variance and covariance calculations. In the context of portfolio optimization, a robust portfolio approach aims to reduce errors in estimating the mean vector $\boldsymbol{\mu}$ and the variance–covariance matrix $\boldsymbol{\Sigma}$ used in the mean–variance model [8]. One robust method that can be employed in portfolio construction is robust S-estimation, which was developed by Rousseeuw and Yohai in 1984 [9].

Capital market investment activities in Indonesia are facilitated by the Indonesia Stock Exchange (IDX), which provides trading services for securities such as stocks, bonds, and mutual funds. One of the stock indices listed on the IDX is the IDX30 Index. This index comprises 30 selected stocks evaluated based on their liquidity, market capitalization, and consistency of inclusion in the LQ45 Index over time.

Several previous studies have applied robust mean–variance approaches to optimal portfolio construction. Supandi and Anggara applied K-means clustering to stocks included in the Jakarta Islamic Index (JII) before constructing robust mean–variance portfolios using S-estimation. Their results showed that the multi-objective model generated the best portfolios in both clusters. Cluster 1 performed better under the classical multi-objective model with $\lambda = 10$, whereas Cluster 2 performed better under the robust multi-objective model with $\lambda = 100$, as indicated by the highest Sharpe ratio [9].

Widiawati and Supandi extended this line of research by comparing robust S-estimation and robust constrained M-estimation in portfolios composed of JII stocks grouped using the K-medoids method. Their findings indicated that the robust mean–variance portfolio based on constrained M-estimation achieved better performance than the portfolio based on robust S-estimation, as measured by the Sharpe ratio [8]. Furthermore, Gubu demonstrated that grouping stocks according to business sectors and applying the robust Fast Minimum Covariance Determinant estimation approach produced superior portfolio performance compared with portfolios formed using K-means clustering [10].

Previous studies have generally applied robust portfolio models to stocks included in the JII by combining robust estimation with clustering methods. Earlier research has also primarily focused on comparing several robust estimation techniques to improve portfolio performance. However, the application of the two-constraint mean–variance model with robust S-estimation to IDX30 stocks remains relatively limited. Moreover, only a limited number of studies have incorporated a minimum expected-return target as an optimization constraint in robust portfolio construction.

To address these gaps, this study applies a two-constraint mean–variance model with robust S-estimation to stocks included in the IDX30 Index. The stocks are selected based on sectoral representation using two criteria. The first criterion selects the stock with the highest expected return in each sector, whereas the second selects stocks based on the combination of the highest expected return and the lowest stock risk within each sector. This procedure is expected to generate portfolios that are more stable against the influence of extreme observations.

Accordingly, this study aims to construct an optimal two-constraint mean–variance portfolio using robust S-estimation for stocks that were continuously included in the IDX30 Index from October 2024 to September 2025. The proposed method considers not only the requirement that the total investment weight must be equal to one but also the minimum target return expected by investors. In addition to constructing the portfolios, this study calculates their expected returns and risks using the closing-price data of IDX30 stocks from October 2024 to September 2025.

2. Methods

This study employed a literature review by collecting and examining theoretical foundations related to the two-constraint mean–variance model and robust S-estimation from relevant books and journal articles. The analysis used the daily closing prices of stocks included in the IDX30 Index from October 2024 to September 2025, obtained from https://finance.yahoo.com. The IDX30 Index consists of 30 stocks, while the research sample comprised 25 stocks that were consistently included in the IDX30 Index throughout the observation period. The analytical procedures are described in the following subsections.

2.1. Stock Return Calculation

Return is the gain generated from an initial investment capital a, where $a > 0$, that produces an amount b after one investment period [11]. The return of stock i at time t is calculated using eq. (1).

$$R_{x_i(t)} = \ln\left(\frac{P_{i(t)}}{P_{i(t-1)}}\right), \tag{1}$$

where $R_{x_i(t)}$ denotes the return of stock i in period t, $P_{i(t)}$ denotes the closing price of stock i in period t, and $P_{i(t-1)}$ denotes the closing price of stock i in period $t-1$.

2.2. Expected Return and Stock Risk Calculation

Expected return represents the return anticipated by investors from an investment over a particular period [12]. The expected return of stock i is calculated using eq. (2).

$$E(R_{x_i}) = \frac{\sum_{t=1}^{n} R_{x_i(t)}}{n}. \tag{2}$$

Stock risk represents the deviation between the expected return and the realized return. A larger deviation indicates a higher level of investment risk [13]. The risk of stock i is calculated using eq. (3).

$$\sigma_{x_i} = \sqrt{\frac{\sum_{t=1}^{n}\left[R_{x_i(t)} - E(R_{x_i})\right]^2}{n-1}}, \tag{3}$$

where $E(R_{x_i})$ denotes the expected return of stock i, $R_{x_i(t)}$ denotes the return of stock i at time t, σ_{x_i} denotes the risk of stock i, and n denotes the number of observation periods.

2.3. Selection of Portfolio-Forming Stocks

The stocks included in each portfolio were selected using different criteria. Portfolio 1 was constructed by selecting the stock with the highest expected return in each sector. Portfolio 2 was constructed by selecting stocks with a combination of relatively high expected returns and relatively low risks in each sector. Consequently, the selected stocks were expected to provide optimal returns while maintaining a lower level of risk.

2.4. Descriptive Statistical Analysis

Descriptive statistical analysis was conducted to provide an overview of the characteristics of stock returns included in each portfolio. The statistics examined included the minimum value, maximum value, mean, standard deviation, skewness, and kurtosis.

2.5. Normality Test

A normality test was conducted to determine whether the stock-return data originated from a normally distributed population. The Kolmogorov–Smirnov test was employed for this purpose [14]. The hypotheses were formulated as follows:

H_0 : The stock-return data are normally distributed,

H_1 : The stock-return data are not normally distributed.

The significance level used in this study was $\alpha = 5\%$. The testing criteria were as follows. If the *Asymp. Sig. (2-tailed)* value was greater than α, then H_0 was accepted, indicating that the stock-return data were normally distributed. Conversely, if the *Asymp. Sig. (2-tailed)* value was less than α, then H_0 was rejected, indicating that the stock-return data were not normally distributed.

2.6. Robust S-Estimation of Parameters

A robust portfolio is designed to minimize errors in estimating the mean vector and variance–covariance matrix used in the mean–variance portfolio model. A standard approach to constructing an optimal robust portfolio is to employ a robust estimation method with a high breakdown point. One such method is S-estimation, which was first introduced by Rousseeuw and Yohai in 1984 [9].

The S-estimators of the mean vector and variance–covariance matrix are obtained by minimizing $|\mathbf{\Sigma}|$ subject to the constraint in eq. (4).

$$\frac{1}{n}\sum_{i=1}^{n}\rho\left(\left[(\mathbf{r}_i-\hat{\boldsymbol{\mu}})^T\mathbf{\Sigma}^{-1}(\mathbf{r}_i-\hat{\boldsymbol{\mu}})\right]^{1/2}\right)=b_0, \tag{4}$$

where $|\mathbf{\Sigma}|$ denotes the determinant of the variance–covariance matrix, ρ denotes the loss function, b_0 denotes a constant, $\mathbf{r}_i$ denotes the ith return vector, and $\hat{\boldsymbol{\mu}}$ denotes the estimated mean-return vector.

The constant b_0 must be appropriately specified because it affects the estimation results. When the underlying data distribution is unknown, the constant can be defined as $b_0 = E\{\rho(|\mathbf{r}|)\}$.

The robust S-estimator of the mean-return vector is obtained by solving eq. (5).

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n}u(d_i)\,(\mathbf{r}_i-\hat{\boldsymbol{\mu}}_S)&=0,\\ \hat{\boldsymbol{\mu}}_S&=\frac{\sum_{i=1}^{n}u(d_i)\mathbf{r}_i}{\sum_{i=1}^{n}u(d_i)}.\end{aligned} \tag{5}$$

The robust variance–covariance matrix is subsequently estimated using eq. (6).

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n}\left[\rho u(d_i)(\mathbf{r}_i-\boldsymbol{\mu})(\mathbf{r}_i-\boldsymbol{\mu})^T-v(d_i)\hat{\mathbf{\Sigma}}_S\right]&=0,\\ \hat{\mathbf{\Sigma}}_S&=\frac{\sum_{i=1}^{n}\rho u(d_i)(\mathbf{r}_i-\boldsymbol{\mu})(\mathbf{r}_i-\boldsymbol{\mu})^T}{\sum_{i=1}^{n}v(d_i)}.\end{aligned} \tag{6}$$

The functions used in the S-estimation procedure are defined as follows:

$$\begin{aligned}d_i&=(\mathbf{r}_i-\boldsymbol{\mu})^T\mathbf{\Sigma}^{-1}(\mathbf{r}_i-\hat{\boldsymbol{\mu}}),\\ \psi(d_i)&=\frac{\partial\rho}{\partial d_i},\\ u(d_i)&=\frac{\psi(d_i)}{d_i},\\ v(d_i)&=\psi(d_i)d_i-\rho(d_i)+b_0.\end{aligned}$$

The S-estimation procedure was performed iteratively using eq. (5) and (6), where $\hat{\mathbf{\Sigma}}_S$ denotes the robustly estimated variance–covariance matrix, $\hat{\boldsymbol{\mu}}_S$ denotes the robustly estimated mean vector, $u(d_i)$ denotes the weighting function based on the Mahalanobis distance of observation i, $v(d_i)$ denotes the Mahalanobis-distance function used as a covariance-scale adjustment factor, $\psi(d_i)$ denotes the derivative of the loss function, and d_i denotes the Mahalanobis distance of observation i.

2.7. Portfolio Weight Calculation Using the Two-Constraint Mean–Variance Model

The optimal portfolio weights in the two-constraint mean–variance model are obtained by minimizing the portfolio risk:

$$\min_{\mathbf{w}}\frac{1}{2}\mathbf{w}^T\mathbf{\Sigma}\mathbf{w},$$

subject to the following constraints:

$$\mathbf{w}^T\mathbf{1}_q=1 \qquad \text{and} \qquad \mathbf{w}^T\mathbf{r}=r_0.$$

The first and second constraints are associated with the Lagrange multipliers λ and β, respectively [7]. The corresponding Lagrangian function is presented in eq. (7).

$$L(\mathbf{w},\lambda,\beta)=\frac{1}{2}\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}-\lambda\left(\mathbf{w}^T\mathbf{1}_q-1\right)-\beta\left(\mathbf{w}^T\mathbf{r}-r_0\right). \tag{7}$$

The Lagrangian function is partially differentiated with respect to the weight vector $\mathbf{w}$ and set equal to zero. The optimal portfolio weights under the two-constraint mean–variance model are then obtained using eq. (8).

$$\begin{aligned}
&\frac{\partial L}{\partial \mathbf{w}} = 0,\\
&\mathbf{w} = \boldsymbol{\Sigma}^{-1}\left(\lambda \mathbf{1}_q + \beta \mathbf{r}\right),\\
&\mathbf{w} = \boldsymbol{\Sigma}^{-1}\left[\left(\frac{D - Br_0}{AD - BC}\right)\mathbf{1}_q + \left(\frac{Ar_0 - C}{AD - CB}\right)\mathbf{r}\right].
\end{aligned} \tag{8}$$

The scalar quantities used in eq. (8) are defined as follows:

$$\begin{aligned}
A &= \mathbf{1}_q^T \boldsymbol{\Sigma}^{-1} \mathbf{1}_q,\\
B &= \mathbf{1}_q^T \boldsymbol{\Sigma}^{-1} \mathbf{r},\\
C &= \mathbf{r}^T \boldsymbol{\Sigma}^{-1} \mathbf{1}_q,\\
D &= \mathbf{r}^T \boldsymbol{\Sigma}^{-1} \mathbf{r}.
\end{aligned}$$

These formulations follow the two-constraint mean–variance portfolio model described in [9]. In this formulation, $\mathbf{w}$ denotes the vector of stock proportions in the portfolio, $\mathbf{w}^T$ denotes the transpose of the weight vector, $\boldsymbol{\Sigma}$ denotes the variance–covariance matrix of stock returns, $\boldsymbol{\Sigma}^{-1}$ denotes its inverse, $\mathbf{1}_q$ denotes a vector of ones of size $q \times 1$, $\mathbf{1}_q^T$ denotes its transpose, $\mathbf{r}$ denotes the mean-return vector, $\mathbf{r}^T$ denotes its transpose, r_0 denotes the target return, λ denotes the Lagrange multiplier for the first constraint, and β denotes the Lagrange multiplier for the second constraint.

2.8. Portfolio Return, Risk, and Performance Evaluation

Portfolio return is the weighted average of the returns of the stocks included in the portfolio and is calculated using eq. (9) [15].

$$R_P = \sum_{i=1}^{n} w_i R_i. \tag{9}$$

The expected portfolio return is calculated as the weighted average of the expected returns of the individual stocks in the portfolio. The expected portfolio return is presented in eq. (10) [16].

$$E(R_P) = \sum_{i=1}^{n} w_i E(R_i). \tag{10}$$

In portfolio theory, risk represents the possibility that the realized return differs from the expected return. Portfolio risk is defined as the square root of portfolio variance. Portfolio variance and portfolio risk are calculated using eq. (11) and (12), respectively [17].

$$\sigma_P^2 = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}, \tag{11}$$

$$\sigma_P = \sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}}. \tag{12}$$

Portfolio performance was evaluated using the Sharpe ratio and Treynor ratio. The Sharpe ratio is calculated using eq. (13).

$$S_P = \frac{E(R_P) - R_f}{\sigma_P}, \tag{13}$$

whereas the Treynor ratio is calculated using eq. (14).

$$T_P = \frac{E(R_P) - R_f}{\beta_P}. \tag{14}$$

In eq. (9) to (14), σ_P denotes portfolio risk, R_P denotes portfolio return, $E(R_P)$ denotes expected portfolio return, w_i denotes the portfolio weight of stock i, R_i denotes the return of stock i, $E(R_i)$ denotes the expected return of stock i, n denotes the total number of stocks in the portfolio, σ_P^2 denotes portfolio variance, R_f denotes the average risk-free interest rate, and β_P denotes the portfolio beta coefficient.

3. Results and Discussion

The construction of an optimal two-constraint mean–variance portfolio using robust S-estimation involves several analytical stages. First, the stock returns and risks are analyzed. Subsequently, stocks with positive expected returns from each sector are selected as candidates for portfolio construction.

3.1. Data Description

This study used the daily closing prices of stocks included in the IDX30 Index from October 2024 to September 2025, obtained from https://finance.yahoo.com. Of the 30 stocks listed in the IDX30 Index, 25 stocks were consistently included throughout the observation period. The stocks that were consistently listed in the IDX30 Index are presented in Table 1.

Table 1. Stocks consistently included in the IDX30 Index.

Sector	Stock Code	Company Name
Basic Materials	ANTM	PT Aneka Tambang Tbk
	BRPT	PT Barito Pacific Tbk
	INCO	PT Vale Indonesia Tbk
	INKP	PT Indah Kiat Pulp and Paper Tbk
	MDKA	PT Merdeka Copper Gold Tbk
	SMGR	PT Semen Indonesia (Persero) Tbk
Consumer Non-Cyclicals	AMRT	PT Sumber Alfaria Trijaya Tbk
	CPIN	PT Charoen Pokphand Indonesia Tbk
	ICBP	PT Indofood CBP Sukses Makmur Tbk
	INDF	PT Indofood Sukses Makmur Tbk
	UNVR	PT Unilever Indonesia Tbk
Energy	ADRO	PT Alamtri Resources Indonesia Tbk
	AKRA	PT AKR Corporindo Tbk
	MEDC	PT Medco Energi Internasional Tbk
	PGAS	PT Perusahaan Gas Negara Tbk
	PTBA	PT Bukit Asam Tbk
Industrials	ASII	PT Astra International Tbk
	UNTR	PT United Tractors Tbk
Infrastructure	TLKM	PT Telekomunikasi Indonesia Tbk
Healthcare	KLBF	PT Kalbe Farma Tbk
Financials	BBCA	PT Bank Central Asia Tbk
	BBNI	PT Bank Negara Indonesia (Persero) Tbk
	BBRI	PT Bank Rakyat Indonesia (Persero) Tbk
	BMRI	PT Bank Mandiri (Persero) Tbk
Technology	GOTO	PT GoTo Gojek Tokopedia Tbk

Table 2. Expected returns and risks of the selected stocks.

Stock Code	$E(R_{x_i})$	Risk	Stock Code	$E(R_{x_i})$	Risk
BRPT	0.00539	0.04041	UNVR	-0.00098	0.03324
ANTM	0.00338	0.03391	BBRI	-0.00104	0.02328
PGAS	0.00067	0.02033	AKRA	-0.00105	0.02997
ASII	0.00047	0.01868	BBNI	-0.00109	0.02475
MEDC	0.00032	0.02530	ICBP	-0.00109	0.01754
INCO	0.00026	0.03484	PTBA	-0.00118	0.01860
INDF	0.00023	0.01910	SMGR	-0.00128	0.03164
TLKM	0.00008	0.02212	BBCA	-0.00131	0.01742
CPIN	0.00007	0.02306	KLBF	-0.00189	0.02619
UNTR	-0.00008	0.02159	BMRI	-0.00202	0.02415
INKP	-0.00071	0.03009	AMRT	-0.00227	0.02841
GOTO	-0.00077	0.03269	ADRO	-0.00352	0.03811
MDKA	-0.00086	0.04263			

3.2. Stock Returns, Expected Returns, and Risks

Return represents the gain obtained from an investment after an initial payment has generated a certain amount of profit over one period. The individual stock returns were calculated using eq. (1). Subsequently, the expected return of each stock was calculated using eq. (2). The complete expected-return and risk values are presented in Table 2.

As shown in Table 2, among the 25 stocks analyzed, nine stocks had positive expected returns, whereas 16 stocks had negative expected returns. Stocks with positive expected returns indicate that, on average, they generated gains during the observation period. In contrast, stocks with negative expected returns exhibited a tendency to generate losses when retained in the portfolio.

Therefore, only stocks with positive expected returns were considered in the portfolio-construction stage, whereas stocks with negative expected returns were excluded from the optimization process. This selection was intended to increase the likelihood of constructing portfolios capable of achieving the predetermined minimum expected-return target.

3.3. Selection of Portfolio-Forming Stocks

The nine stocks with positive expected returns represented five different sectors. Subsequently, one stock from each sector was selected according to the portfolio-construction criteria. For Portfolio 1, the stock with the highest expected return in each sector was selected. For Portfolio 2, the stock was selected based on a combination of a relatively high expected return and a relatively low level of risk within each sector. The expected returns and risks of the candidate stocks are presented in Table 3.

The stocks selected for Portfolio 1 were those with the highest expected returns in their respective sectors, namely ASII, BRPT, INDF, PGAS, and TLKM. Meanwhile, Portfolio 2 consisted of stocks with a combination of relatively high expected returns and relatively low risks within their respective sectors, namely ASII, ANTM, INDF, PGAS, and TLKM.

Selecting one stock from each sector was intended to achieve cross-sector diversification so that the portfolio would not be concentrated in a particular sector. The different selection criteria resulted in portfolios with distinct characteristics. Portfolio 1 emphasized the potential for higher returns, whereas Portfolio 2 considered the balance between potential returns and investment risk.

Table 3. Stocks selected as portfolio candidates.

Sector	Stock	$E(R_i)$	Risk
Basic Materials	ANTM	0.003384	0.033977
	BRPT	0.005394	0.040498
	INCO	0.000257	0.034915
Industrials	ASII	0.000471	0.018718
Consumer Non-Cyclicals	CPIN	0.000072	0.023108
	INDF	0.000233	0.019140
Energy	MEDC	0.000322	0.025352
	PGAS	0.000670	0.020369
Infrastructure	TLKM	0.000084	0.022170

3.4. Descriptive Statistical Analysis

The descriptive statistics of the closing prices of the six stocks included in the two portfolios are presented in Table 4. In addition, the stock-price movements, which provide a visual representation of the price patterns of the portfolio-forming stocks, are presented in Figure 1.

As shown in Table 4, ANTM had the largest standard deviation among the selected stocks, indicating the highest level of price fluctuation and investment risk. In contrast, PGAS had the smallest standard deviation, indicating relatively more stable price movements than the other stocks. The differences in volatility demonstrate that each stock had distinct risk characteristics. Therefore, diversification was required to prevent investment risk from being concentrated in a single stock and to produce a more efficient portfolio combination.

Furthermore, all portfolio-forming stocks had mean values greater than their respective median values, indicating that their price distributions tended to be positively skewed. This condition suggests the presence of several relatively high stock-price observations that increased the mean values.

As illustrated in Figure 1, the selected stocks exhibited different price-movement patterns. ASII showed a declining trend from the beginning to the middle of the observation period, fol-

Table 4. Descriptive statistics of the portfolio-forming stocks.

Stock	Minimum	Maximum	Mean	Median	Standard Deviation
ASII	4430	5900	4953.2	4900	320.850
ANTM	1380	3670	2207.65	1650	777.157
BRPT	600	3770	1326.2	972.5	703.230
INDF	6675	8525	7707.6	7700	387.270
PGAS	1440	1885	1618.5	1610	92.252
TLKM	2290	3420	2760	2740	244.200

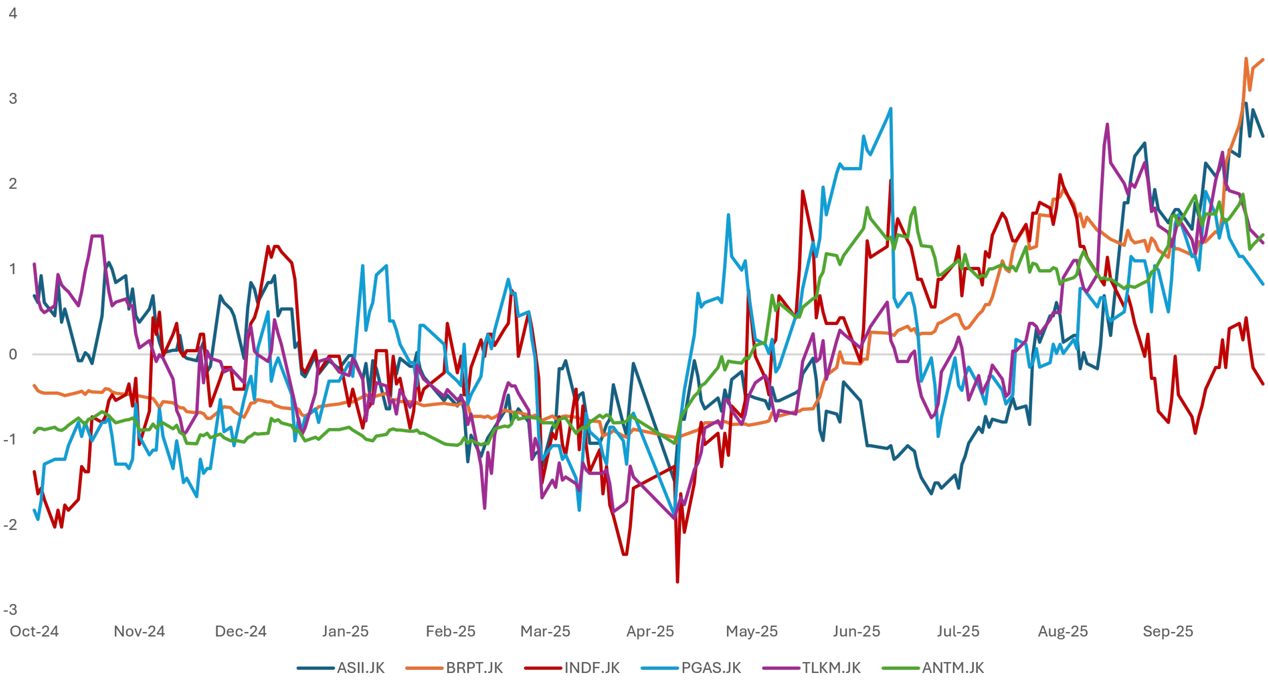

Figure 1. Stock price movement patterns.

lowed by an increase toward the end of the period. BRPT experienced a decline at the beginning of the period before increasing sharply during the second half of the observation period. INDF remained relatively stable with moderate fluctuations throughout the observation period and generally exhibited an upward price trend.

PGAS showed a gradual upward trend from the beginning to the end of the observation period. TLKM experienced a declining trend during the middle of the period, followed by an increase toward the end of the observation period. Meanwhile, ANTM displayed relatively stable movements at the beginning of the observation period. Its price began to increase during the middle of the period and subsequently fluctuated consistently until the end of the observation period.

3.5. Normality Test

The normality of the stock-return data was examined using the One-Sample Kolmogorov–Smirnov test. The decision criterion was based on the *Asymp. Sig. (2-tailed)* value. The return data were considered normally distributed when the *Asymp. Sig. (2-tailed)* value was greater than the significance level of $\alpha = 5\%$. The test results are presented in Table 5.

Based on the decision criteria and the results presented in Table 5, the returns of ASII, ANTM, BRPT, PGAS, and TLKM were not normally distributed. INDF was the only stock whose return data followed a normal distribution. This condition indicates the possible presence of outliers in the stock-return data.

Table 5. Normality test results for the stock-return data.

Stock	*Asymp. Sig. (2-tailed)*
ASII	0.003
ANTM	0.004
BRPT	0.000
INDF	0.051
PGAS	0.000
TLKM	0.025

Non-normality and outliers may reduce the accuracy of the mean-vector and variance–covariance matrix estimates obtained using classical estimation methods, thereby potentially producing suboptimal portfolio weights. Therefore, robust S-estimation was employed to obtain more accurate parameter estimates that are less sensitive to non-normal observations and outliers.

3.6. Robust S-Estimation of Portfolio Parameters

After the portfolio-forming stocks had been selected, the next analytical stage involved calculating the initial parameters required for constructing the two-constraint mean–variance portfolios. Parameter estimation was conducted using robust S-estimation to obtain a mean vector $\hat{\boldsymbol{\mu}}_S$ and variance–covariance matrix $\hat{\boldsymbol{\Sigma}}_S$ that were more resistant to outliers in the stock-return data.

The robust estimates of the mean vector and variance–covariance matrix were obtained using the `rrcov` package. The robust S-estimated mean vector and variance–covariance matrix for Portfolio 1 are presented in Tables 6 and 7, respectively.

Table 6. Robust S-estimated mean vector for Portfolio 1.

Stock	Mean Vector $\hat{\boldsymbol{\mu}}_S$
ASII	-0.000132
BRPT	0.002132
INDF	-0.000278
PGAS	0.000227
TLKM	0.000188

Table 7. Robust S-estimated variance–covariance matrix for Portfolio 1.

Stock	ASII	BRPT	INDF	PGAS	TLKM
ASII	0.000280	0.000095	0.000065	0.000049	0.000082
BRPT	0.000095	0.001121	0.000099	0.000093	0.000171
INDF	0.000065	0.000099	0.000337	0.000043	0.000112
PGAS	0.000049	0.000093	0.000043	0.000340	0.000093
TLKM	0.000082	0.000171	0.000112	0.000093	0.000511

The robust S-estimated mean vector and variance–covariance matrix for Portfolio 2 are presented in Tables 8 and 9, respectively.

Table 8. Robust S-estimated mean vector for Portfolio 2.

Stock	Mean Vector $\hat{\boldsymbol{\mu}}_S$
ASII	0.000184
ANTM	0.002784
INDF	-0.000422
PGAS	0.000386
TLKM	0.000522

Table 9. Robust S-estimated variance–covariance matrix for Portfolio 2.

Stock	ASII	ANTM	INDF	PGAS	TLKM
ASII	0.000277	0.000041	0.000071	0.000041	0.000076
ANTM	0.000041	0.000988	0.000041	0.000098	0.000045
INDF	0.000071	0.000041	0.000318	0.000037	0.000103
PGAS	0.000041	0.000098	0.000037	0.000301	0.000091
TLKM	0.000076	0.000045	0.000103	0.000091	0.000494

The mean vectors and variance–covariance matrices obtained through robust S-estimation were subsequently used as input parameters in the two-constraint mean–variance optimization model.

3.7. Stock Weight Calculation Using the Two-Constraint Mean–Variance Model

The portfolio weights were calculated using eq. (8). The minimum target returns, denoted by r_0, were set at 0.00137 for Portfolio 1 and 0.00096 for Portfolio 2. These target-return values were determined from the average expected returns of the five stocks included in each portfolio. The optimal stock weights for Portfolio 1 and Portfolio 2 are presented in Tables 10 and 11, respectively.

Table 10. Optimal stock weights for Portfolio 1.

Stock	Portfolio Weight
ASII	0.05719
BRPT	0.58376
INDF	-0.11001
PGAS	0.36989
TLKM	0.09915

Table 11. Optimal stock weights for Portfolio 2.

Stock	Portfolio Weight
ASII	0.28210
ANTM	0.26942
INDF	0.03604
PGAS	0.26942
TLKM	0.16142

The optimal weights presented in Tables 10 and 11 indicate the recommended proportions of funds to be allocated to each stock when constructing the optimal portfolios using the two-constraint mean–variance model with robust S-estimation.

In Portfolio 1, BRPT received the largest investment weight of 58.38%, followed by PGAS at 36.99%, TLKM at 9.92%, and ASII at 5.72%. In contrast, INDF received a negative weight of -11.00%. The dominant weight assigned to BRPT indicates that this stock made the largest contribution toward achieving the target portfolio return, which is consistent with its position as the stock with the highest expected return among the constituents of Portfolio 1. The negative weight assigned to INDF indicates that the optimization model recommends a short-selling strategy to improve portfolio efficiency and achieve a more optimal combination of expected return and risk.

Unlike Portfolio 1, the investment weights in Portfolio 2 were more evenly distributed. ASII received the largest weight of 28.21%, followed by ANTM and PGAS, each with a weight of 26.94%, TLKM with 16.14%, and INDF with 3.60%. All stocks in Portfolio 2 received positive weights; therefore, this portfolio did not require a short-selling strategy.

These results indicate that the optimization model produced a more balanced allocation among stocks with relatively high expected returns and lower levels of risk in Portfolio 2. Consequently, Portfolio 2 exhibited more favorable risk characteristics than Portfolio 1.

A negative portfolio weight indicates the implementation of a short-selling strategy. In this strategy, an investor sells borrowed shares at the prevailing market price and subsequently repurchases them when the price declines. The potential profit is

Table 12. Expected return, risk, and performance evaluation of the portfolios.

Portfolio	Expected Return	Portfolio Risk	Sharpe Ratio	Treynor Ratio
Portfolio 1	0.001370	0.022259	-2.445282	-0.100902
Portfolio 2	0.000968	0.013193	-4.155959	-0.219962

obtained from the difference between the higher initial selling price and the lower repurchase price. Short selling is applied when the optimization process assigns a negative allocation to a stock within the portfolio [18].

3.8. Portfolio Expected Return, Portfolio Risk, and Performance Evaluation

Based on the optimal weights obtained, the expected return and risk of each portfolio were subsequently calculated as the basis for evaluating portfolio performance. The expected portfolio return was calculated using eq. (10), whereas portfolio risk was calculated using eq. (12). Portfolio performance was then evaluated using the Sharpe ratio and Treynor ratio to measure the return generated relative to the risk borne by investors, as formulated in eq. (13) and (14). The results of the expected-return, risk, and portfolio-performance calculations are presented in Table 12.

As shown in Table 12, Portfolio 1 generated a higher expected return than Portfolio 2, although it was accompanied by a higher level of risk. This result reflects the risk–return trade-off, in which a higher expected return is generally associated with a greater level of risk borne by investors. In contrast, Portfolio 2 offered a lower level of risk but also generated a lower expected return.

The Sharpe and Treynor ratios of both portfolios were negative, indicating that the returns generated during the observation period were insufficient to compensate for the risks borne by investors relative to the risk-free rate. Nevertheless, Portfolio 1 had higher Sharpe and Treynor ratios than Portfolio 2 because its values were less negative. Therefore, Portfolio 1 demonstrated relatively better risk-adjusted performance than Portfolio 2.

4. Conclusion

The two-constraint mean–variance model with robust S-estimation produced two optimal portfolios based on different stock-selection criteria. Portfolio 1 consisted of ASII, BRPT, INDF, PGAS, and TLKM, whereas Portfolio 2 consisted of ASII, ANTM, INDF, PGAS, and TLKM. The optimization results demonstrated that different stock-selection criteria generated different portfolio compositions and investment weights, which subsequently resulted in distinct expected-return and risk characteristics.

Portfolio 1 generated an expected return of 0.001370 with a risk of 0.022259, whereas Portfolio 2 generated an expected return of 0.000968 with a risk of 0.013193. Based on the Sharpe and Treynor ratios, Portfolio 1 exhibited relatively better performance than Portfolio 2, although both portfolios produced negative risk-adjusted performance measures during the observation period.

These findings demonstrate that the stock-selection criteria used in portfolio construction affect the optimization results and the resulting portfolio performance. Furthermore, robust S-estimation provided parameter estimates that were more representative of stock-return data that did not fully satisfy the normality assumption. Therefore, this approach can serve as an alternative method for constructing an optimal portfolio that simultaneously considers a minimum expected-return target and investment risk.

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