JAMBURA JOURNAL OF PROBABILITY AND STATISTICS
Volume 3 Nomor 2, November 2022
Journal Homepage: http://ejurnal.ung.ac.id/index.php/jps
DOI: https://doi.org/10.34312/jjps.v1i1.15965

THE OPERATING CHARACTERISTICS CURVE OF THE ACCEPTANCE SAMPLING OF TYPE-A BASED ON OUTGOING PERCENT DEFECTIVE LOT

Bahagia Rafika Dewi[1], Fergyanto E Gunawan[2], Firdaus Alamsjah[3]
[1, 2 ,3] Industrial Engineering Department, BINUS Graduate Program - Master of Industrial Engineering, Bina Nusantara University, Jakarta, Indonesia 11480

e-mail: bahagia.dewi@binus.ac.id

Abstract
The common type-A of Operating Characteristics (OC) curve measures the consumer's risk through the incoming quality. However, the proportion of defective can alter after the sampling process; hence, the measure of consumer's risk is better described by the outgoing quality or lot quality of post sampling inspection. A modified OC curve is developed based on the outgoing quality for two applicable cases; returned samples to the lot and non-returned or destructive samples. This research aims to develop the algorithm and evaluate the alternative acceptance sampling plan for isolated lots by outgoing percent defective. For the returned samples, the acceptance sampling requires less sample than that based on the common OC-curve and is seen as an opportunity for sampling size reduction. The number of reduced samples varies depending on the input parameters: Acceptance Quality Level (AQL), Rejectable Quality Limit (RQL), producer's risk (α), consumer's risk (β), and lot size (N). For the non-returned sample, more sample size (n) is required, even more than that of using the Binomial distribution's sample size, which has been considered conservative.
Keywords: Acceptance Sampling, Type-A Operating Characteristics curve, Outgoing Quality, Single Sampling, Isolated Lot, Consumer's Risk, Producer's Risk

1. INTRODUCTION

A critical part of statistical quality control is acceptance sampling. There are several reasons why acceptance sampling is necessary, i.e.; mandatory requirement from the consumer, the process is not in control, regardless of whether process control is in place or not, presence of special cause after process inspection, 100% sam pling is inefficient, yet 0% is risky, 100% inspection does not promote process/product improvement, and tests are destructive (Devore 2009), (Montgomery 2009), (Kreyszig 2009), (Banovac, Pavlovic and Vistica 2012). Acceptance sampling consists of several schemes. At least there are five schemes and procedures that have been standardized by ISO (ISO28590 2017). One of the famous knowns sampling is that for continously lot-by-lot inspection (ISO2859-1 1999, Z1.4 2003 (R2013)). This sampling scheme can require a small sample size depending on the submitted lot size. However, this scheme requires strict conditions to be fulfilled for implementation. Other sampling procedures, the simplest ones, are those based on the Operating Characteristic (OC) curve. A sampling plan by OC curve can be used to evaluate a single lot (Type A) or a process (Type B). The OC curve provides the relationship between the proportion defective and the probability of sampling to accept.
A single-sampling plan for attributes is a simple lot-by-lot acceptance-sampling plan in which one sample of size n is randomly selected from a submitted lot N. Then, the lot will be accepted if the number of non-conforming items in the sample, denoted by O, is less than or equal to an acceptance number c. Otherwise, it will be rejected. (Mostofi and Shirkani 2019)

Important parameters related to consumer expectation/requirement arrive through the OC curve, which are Acceptable Quality Limit (AQL), Rejectable Quality Limit (RQL) or

Received: 2020-08-23 | **Accepted:** 2020-10-02 | **Available Online:** 2020-11-30

Lot Tolerance Percent Defective (LTPD), consumer's risk (beta risk) and producer's risk (alpha risk). In a ideal case where efficient 100% inspection can be implemented, the consumer requirement is reprensented by AQL, a level of quality by defect proportion in the lot that is acceptable by consumer. As acceptance sampling is employed, the AQL can no longer be guaraanteed and there always be a risk of accepting the lot with a level quality worse than AQL. A good sampling should accept a "good" lot and reject a "bad" lot most of the time. Through consent from both parties (producer and consumer), it's defined the level of quality that can be tolerated by consumer above the AQL or a designated high defect that would be unacceptable to the consumer (NIST/SEMATECH 2012), called as RQL. It is also defined the level of probability to accept the lot at AQL and RQL. The level of probability to accept the lot with a defect level equal to RQL (NIST/SEMATECH 2012) is called as beta. This is why the beta is so-called as consumer risk, since the lot should be rejected but acceptance sampling still accept the lot. The level of probability to reject the lot that has a defect level of AQL (NIST/SEMATECH 2012) is called as alpha. As the lot with such quality should be accepted, but acceptance sampling can still reject this, thus alpha is so-called as producer's risk.

Type-A OC Curve based on hypergeometric distribution can unwieldy or imprecisely be calculated by statistical computing packages and is rarely used to calculate sampling plans; hence usually approximated by the binomial or Poisson OC. In many textbooks and papers, for example, (Devore 2009) (Montgomery 2009) (Schilling and Neubauer 2017) (Chukhrova and Johannssen 2019), the OC curve is explained by the Type-B plan giving the relationship between the defect proportion from the process and the probability of acceptance. These OCs reduce, in fact, computational effort but also entail insufficient quality of the numerical results (Chukhrova and Johannssen 2019). With a very low amount of extensive research on hypergeometric distribution, growing utilization of the sampling plan, and supported by the sophistication of computational software, it is then important to have the acceptance sampling of Type-A hypergeometrics.

Some literature describes the Type-A OC curve based on hypergeometric distribution; (Samohyl 2018) (Cavone, Fabbiano and Giaquinto 2009) (Chukhrova and Johannssen 2018) (Al-Nasser 2022) the relationship given is the probability of acceptance with the defective proportion in the incoming lot, where the measure of consumer's risk is defined through the incoming lot. This measure may not be accurate as for the Type A OC curve, the lot size if finite (especially if the lot size is relatively small), the lot size and defective proportion in the lot can be changed after the sampling process. The actual consumer's risk can be either too small or too high than expected. The outgoing quality has been an important parameter since the producers are required to demonstrate that their products are meeting specified quality standard as required by the consumers (Brush, Hoadley and Saperstein 1990). Therefore, it is required to have the sampling plan which represents the final quality level received by customer.

A practical sampling plan which utilizes hypergeometric OC Curve as prime theoretical of single sampling which uses outgoing quality will be very useful in describing the actual risk and quality received by the customer. In the development of this sampling plan, two cases are applicable in using hypergeometric Type-A OC Curve; non-returned sample for destructive inspection and returned sample. The defect proportion in the outgoing lot of destructive inspection will respond differently to returned sample since the number of the outgoing lot will be $N - n$ compared to $N - O$ in returned sample.

2. RESEARCH METHOD

The stages of methodology in this research/study are divided into the following:

2.1.To develop the OC curve based on the outgoing quality.

To present the OC curve by the outgoing lot, it has to take two scenarios in the sampling, either the good samples are returned to the lot, or all samples are destroyed (not returned).

Denote the number of defective parts in the incoming is I and the observed defective is O while the lot size is N, then the outgoing lot quality is $\frac{I-O}{N-O}$ in the returned sample case. Recalling the cumulative distribution function in, the construction of a common Type-A OC curve knowing the sampling plan (n, c), the cumulative distribution function of hypergeometric distribution is used. That's because all probability from zero to the acceptance number $(0,1, \ldots c)$ is attributed onto the same incoming defect proportion.

The construction of the modified Type-A OC curve based on outgoing quality uses the probability mass function (PMF). That is because each case of defect found results in different outgoing quality levels. The probability of each possible results of the OC curve shape will be discussed or evaluated. Two applicable cases that may result differently, which are returned samples to the lot and un-returned samples (destructive), shall be considered.

OC curve is unique by the parameters of n, c & N. The variables in the axis of OC curve are k & K. The variable K starts from 0 (0% defective) to N (100% defective). The variable k starts from 0 to min (c, K).

In the case of returned sample(s), the axis or defective proportion by outgoing quality (post sampling inspection) is:

$$Axis\ (outgoing\ quality) = \frac{K-k}{N-k}, \begin{cases} K = 0,1,2 \ldots N \\ k = 0,1,2 \ldots min(c, K) \end{cases}$$

In the case of non-returned sample(s), the axis by outgoing quality is:

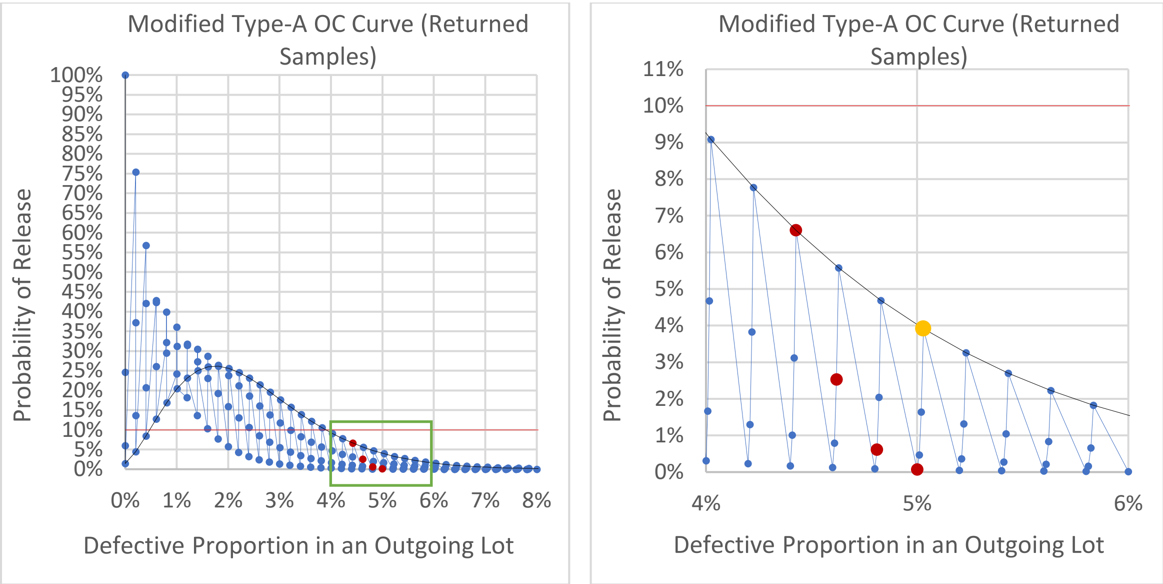
$$Axis\ (outgoing\ quality) = \frac{K-k}{N-n}, \begin{cases} K = 0,1,2 \ldots N \\ k = 0,1,2 \ldots \min(c, K) \end{cases}$$

Both cases of returned and non-returned sample(s) have the same calculation for the ordinate as follows:

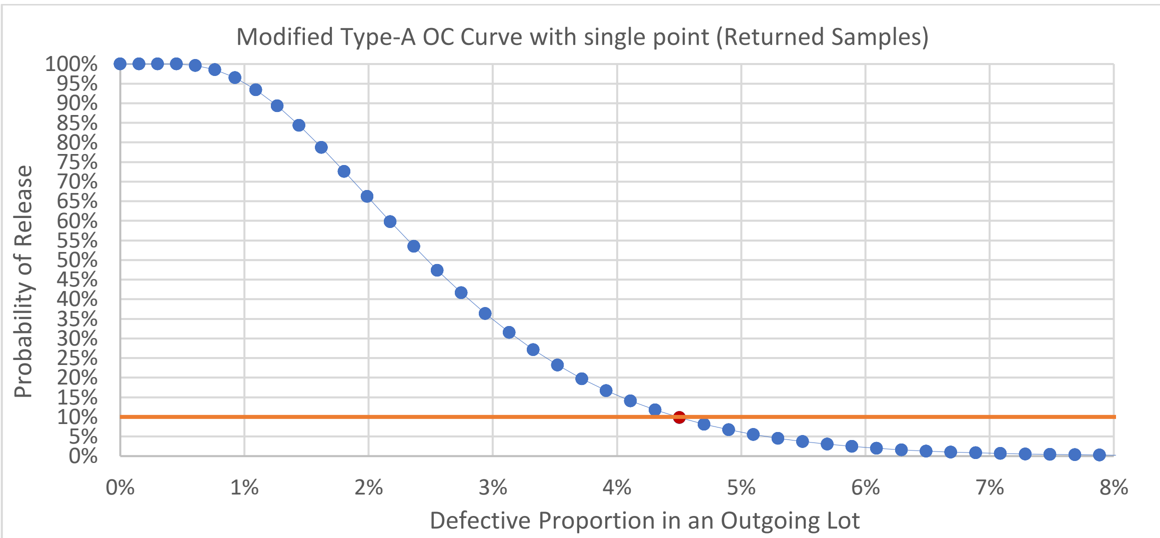
$$Ordinate\ (release\ probability) = h(k, n, K, N)$$

To create the OC curve, the axis ploting starts from the lowest to the maximum value. The lowest value is zero. However, there are as many as c possible ways of zero outgoing defective. Those outgoing lots are from zero incoming defect, one incoming defect that observed during inspection,…, c incoming defect that observed during inspection. Similarly there are c possible ways of outgoing defective with any single outgoing defective units. For the outgoing lotsize that act as the denominator, it starts from the highest one, which is the incoming lot size, and following by one decreased value $(N, N-1, N-2, \ldots, N-c)$.

The OC curve by the outgoing lot is based on acceptance sampling of AQL = 1%, RQL = 5%, α = 5%, β = 10%, n = 123, c = 3, and the graphical result is presented in Figure 1. Every point in the common OC curve is now separated into different points depending on the acceptance number. See the four red dots for incoming quality lots of 5%.



(a)



(b)

Figure 1. (a) Modified Type-A OC curve with returned samples. AQL = 1%, RQL = 5%, Alpha = 5% and Beta = 10%, n = 123, c = 3, **(b)** Modified Type-A OC curve with returned samples with single point outgoing lot. AQL = 1%, RQL = 5%, Alpha = 5% and Beta = 10%, n = 123, c = 3.

Every point in the common OC curve is now separated into different points depending on the acceptance number (c = 3 separated to probable defect observed of c = 0, c = 1, c = 2, and c = 3). See the four red dots which can be compared to incoming quality lots of 5% in the common Type-A OC Curve. Separated points at the outgoing lot from the same single point create additional complications on the OC curve, not to mention that the graph is no longer monotonically decreasing as per the common OC curve. The single point of representative in the outgoing lot needs to be defined. The fair approach may be to average the separated points weighed by each probability of their own. Denoted $HG_T(c, n, I, N)$ as the cumulative distribution function, $HG_F(x, n, I, N)$ as the probability mass function, and x will be the the probable observed defect, then representative outgoing lot is:

$$Lot_O = \frac{\sum_{x=0}^{c} \left(\frac{I-x}{N-x}\right) HG_F(x,n,I,N)}{HG_T(c,n,I,N)}$$

With this approach, the modified OC curve becomes as shown in Figure 1 (b). The incoming lot of 5% moved to 4.5% as the weighted outgoing lot. The outgoing lot at 5% has a probability of release around 5.5%. This indicates that there is room for improvement from the manufacturer's side by reducing the sample size. After developing an algorithm to achieve the acceptance sampling, the comparison of sample size will be addressed in the following discussion.

For un-returned sampling case, like in the destructive test, the calculation of the outgoing lot is similar to that of returned samples, only changing the denominator to be $N - n$. The OC curve results are shown in Figure 2 (a) and Figure 2 (b). As seen, if the incoming lot before sampling inspection is 5% (RQL), the outgoing lot will be higher than 5% whenever all samples are not returned to the lot. This substantiates the need to present the Type-A OC curve based on the outgoing lot, as the normal OC curve presents the quality lot that is lower than what the customer receives.

The beta level at RQL is exceeded in Figure 2 (b), requiring more sample size than what is derived per common Type-A OC curve. However, from Figure 2(b), it is not clear whether the acceptance sampling from the common Type-A OC curve is affected. The comparison of acceptance samplings will be discussed in the next chapter after the algorithms to obtain the acceptance sampling have been developed.

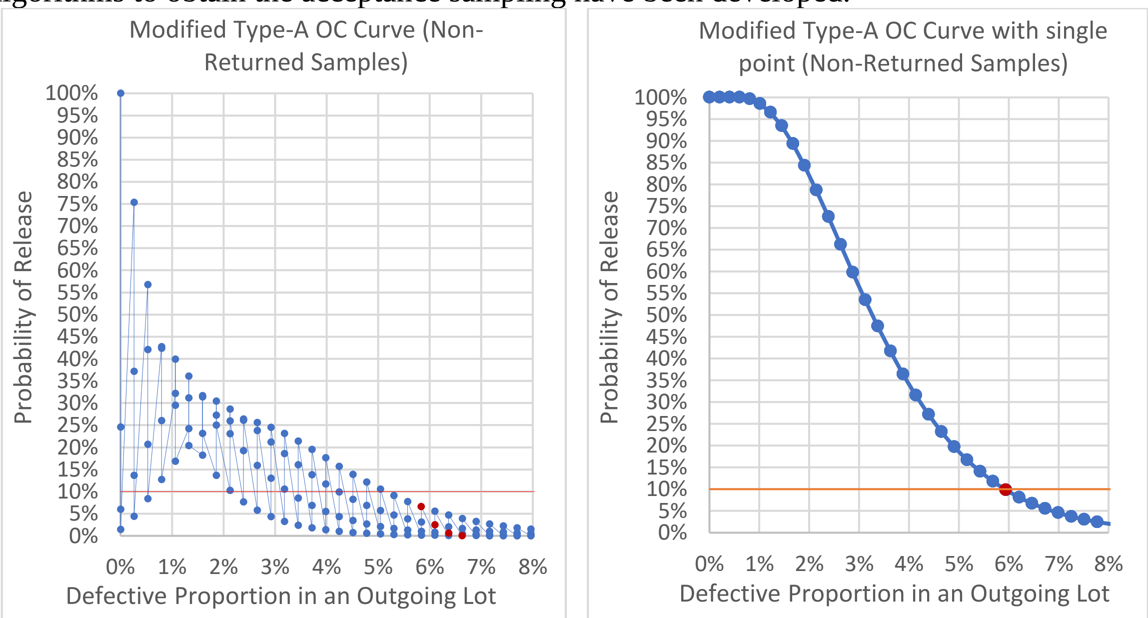


Figure 2. (a) Modified Type-A OC curve with unreturned samples. AQL = 1%, RQL = 5%, Alpha = 5% and Beta = 10%, n = 123, c = 3, **(b)** Modified Type-A OC curve with non-returned samples with single point outgoing lot. AQL = 1%, RQL = 5%, Alpha = 5% and Beta = 10%, n = 123, c = 3.

As seen in the single point representation curve, the outcome on the AQL side of the curve is different between the cases of returned samples and non-returned samples. In returned sample case (Figure 1(b)), the probability of receiving a lot above the AQL is less than 95%. The result came out stricter for returned sample, hence, the algorithm development for the AQL side will be using the incoming/common OC curve calculation for optimized result. For non-returned sample case (Figure 2(b)), result indicates one point above the AQL that has Pa higher than 95%, almost identical as in common OC Curve with the same acceptance sampling, yet still it cannot be concluded that the result around AQL will not change. Thus, it must be considered in the development of the algorithm to find the acceptance sampling for the non-return sample case to calculate the AQL value based on the outgoing lot.

2.2. To propose the alternative acceptance sampling plan by outgoing percent defective lot.

Developing the alternative acceptance sampling was done by developing an algorithm of method. The algorithm development can be implemented using Microsoft Excel through User Defined Function (UDF). To obtain the algorithm for a modified OC curve it is started with developing the algorithm for a common OC curve. There are some behaviors of the OC curve concerning the change in acceptance sampling parameters, especially sample size (n), acceptance number (c), and AQL & RQL. Those are:

1. The probability of acceptance (Pa) decreases as n increases. Pa=HG_T (c,n,I,N).
2. Pa increases as c increases but is less significant in the low region of the Incoming Quality (IQ) level.
3. n increases as AQL & RQL get closer. When Beta is zero, n is maximum or 100% inspection instead of sampling.
4. The Type-A OC curve comes in discrete points of IQ. For specific lot sizes (N), there may not be possible IQ points exactly at AQL and RQL.

The algorithm of iteration can be started from a minimum sampling plan, which is n=1 and c=0. With this starting point, the common OC curve is a line with a gradient of -1, that is, Pa=100% at IQ=0 & Pa=0% at IQ=100%.

The desired sampling plan is when fulfilling three conditions below:

$$Pa_{n,c}(AQL) \geq 1 - \alpha$$

$$Pa_{n,c}(RQL) \leq \beta$$

$$Pa_{n,c+1}(RQL) > \beta$$

If only is the last condition not fulfilled, it means the acceptance number can be increased by one (from c to be $c + 1$).

The three probability parameters above are explored for all possibilities, as shown in Table 1. The three points above cover the OC curve's behaviors from point a to c. To address point d, two types of approaches can be taken when the desired AQL & RQL do not exist in the OC curve: adjusting the AQL & RQL to the nearest lower values; interpolating the AQL & RQL from the two nearest lower and higher points.

Table 1. Exploring all possibilities of three parameters of probability of acceptance

$\boldsymbol{Pa_{n,c+1}(RQL)}$	$\boldsymbol{Pa_{n,c}(AQL)}$	$\boldsymbol{Pa_{n,c}(RQL)}$	**Comments**
$> \beta$	$< 1 - \alpha$	$> \beta$	$increase\ n \ \rightarrow n = n + 1$, run continues
$> \beta$	$< 1 - \alpha$	$\leq \beta$	$increase\ n \ \rightarrow n = n + 1$, run continues

$Pa_{n,c+1}(RQL)$	$Pa_{n,c}(AQL)$	$Pa_{n,c}(RQL)$	**Comments**
$> \beta$	$\geq 1-\alpha$	$> \beta$	c too high, should find the acceptance sampling first
$> \beta$	$\geq 1-\alpha$	$\leq \beta$	**n & c accepted, run stops**
$\leq \beta$	$< 1-\alpha$	$> \beta$	impossible
$\leq \beta$	$< 1-\alpha$	$\leq \beta$	$increase\ c\ \rightarrow c = c+1$, run continues
$\leq \beta$	$\geq 1-\alpha$	$> \beta$	impossible
$\leq \beta$	$\geq 1-\alpha$	$\leq \beta$	$increase\ c\ \rightarrow c = c+1$, run continues

From the above table, three conditions can be taken into the algorithm.:

1. When n & c are accepted
2. When n needs to be increased, which is when $Pa_{n,c+1}(RQL) > \beta$
3. Otherwise c needs to be increased.

The three points above cover the OC curve's behaviors from point a to c. To address point d, two types of approaches can be taken when the desired AQL & RQL do not exist in the OC curve: adjusting the AQL & RQL to the nearest lower values; interpolating the AQL & RQL from the two nearest lower and higher points.

The flowcharts of the algorithms are given in Figure 3(a). Some denotations used have the following meaning:

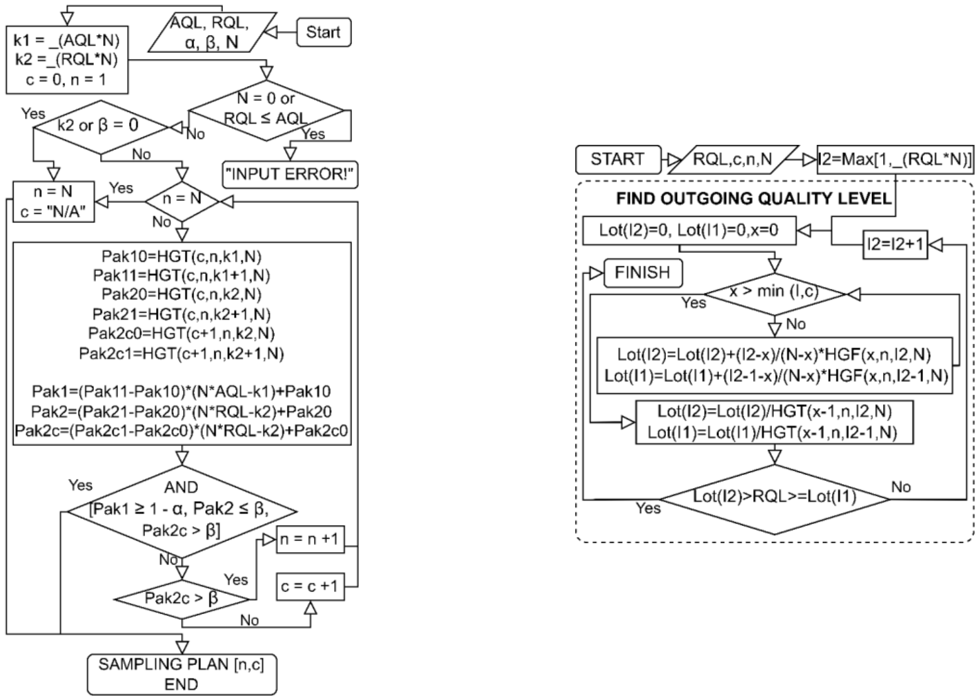
- $\_(AQL * N)$ = rounding down to integer of AQL*LotSize
- $HGT(c, n, k, N)$ = cumulative distribution function of hypergeometric distribution with observed c defective units from n sample size taken from N population size with k defective units. Other nomenclature and notation used in this paper are displayed in below table:

Table 2. Notation and nomenclature used in this paper

Symbol	Meaning
n	sample size
N	lot size
O	observed defect
c	acceptance number
I	total number of defective part in the lot
Pa	probability of acceptance
x	probable observed defect
HG_T	cumulative distribution function of hypergeometric
HG_F	probability mass function of hypergeometrics
HG	algorithm with adjusting to the nearest lower AQL & RQL value
HG_I	algorithm with interpolation
HG_I_RQL	algorithm with linear interpolation at RQL
HG_UO_R	algorithm for case of returned sampling, unified output
HG_UO_NR	algorithm for case of unreturned sampling, unified output
BN	binomial distribution
α	producer’s risk

Symbol	Meaning
AQL	acceptable quality limit
RQL/LTPD	rejectable quality limit/lot tolerance percent defective
β	consumer's risk

Additionally, the algorithms anticipate when the input is incorrect (i.e. $RQL \leq AQL$ and $N = 0$) and when 100% inspection is needed ($k2 = 0, \beta = 0$ & $n = N$). The algorithm development approach will use both interpolations at AQL & RQL.



(a) **(b)**

Figure 3. Algorithm for normal Type-A OC curve**. a).** Two-point linear interpolation at AQL & RQL. **(b)** Algorithm to find the RQL between two outgoing quality lots in the sequence

The algorithm run results are shown in Figure 6 regarding the ratio of a sample size to the lot size. The sampling plan result with a comparison to the Minitab result is also presented in Figure 6. The algorithm results with linear interpolation match those from Minitab, especially at the jump points. The results justify the approach with interpolation as it provides a smooth curve of the results. Based on common OC Curve sampling plan algorithm elaborated above, the sampling plan algorithm for modified OC Curve and returned sample case is developed as following. The calculation in AQL side of the algorithm for returned sample case is based on the incoming/common OC Curve. Thus, the algorithm flowchart is the same for $Pak1$ calculation as Figure 3(a). The part related to $Pak2$ or RQL needs to be thought of.

The first part needs to be determined regarding $Pak2$ is to find the two points of outgoing lots for the interpolation, which are the points where the RQL is in between. The higher point of the two must be the first point above the RQL. Denoting the second point in terms of the defective unit as I_2, the equation below must be satisfied.

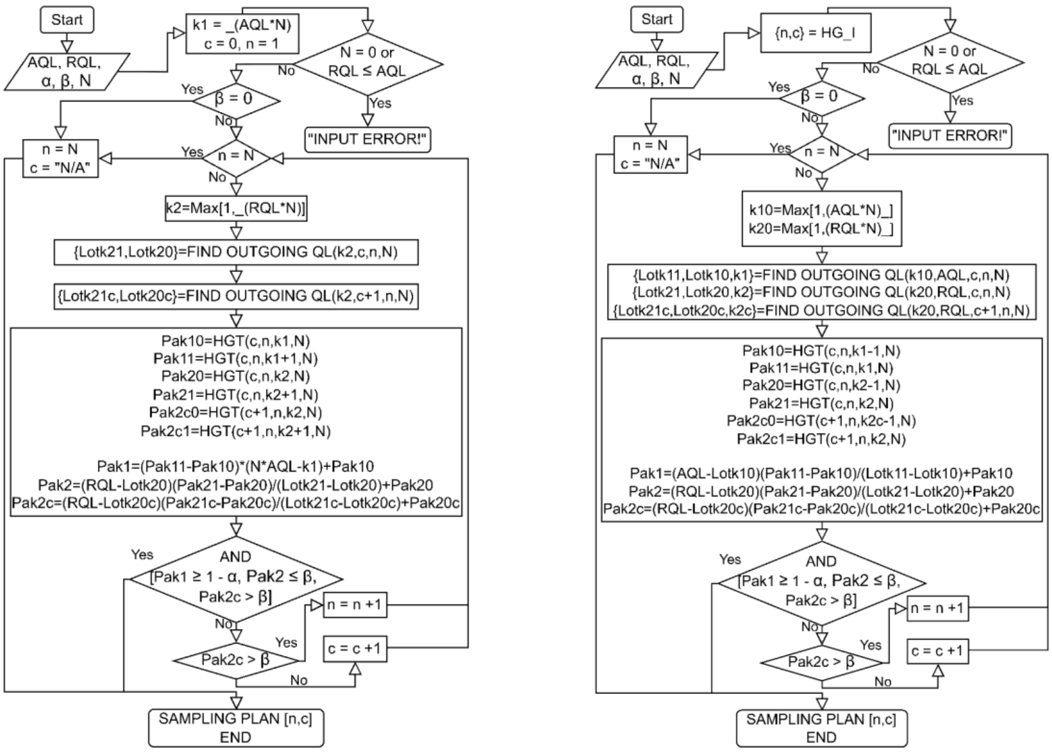
$$\frac{\sum_{x=0}^{c}\left(\frac{I_2 - x}{N - x}\right) HG_F(x, n, I_2, N)}{HG_T(c, n, I_2, N)} > RQL \geq \frac{\sum_{x=0}^{c}\left(\frac{I_2 - 1 - x}{N - x}\right) HG_F(x, n, I_2 - 1, N)}{HG_T(c, n, I_2 - 1, N)}$$

The value I_2 cannot be determined analytically but through the iteration method. As seen from the corresponding defective unit at RQL based on the incoming lot moves to a lower value than the RQL at the outgoing lot; the iteration to find I_2 is started with the corresponding defective unit at RQL. The algorithm is shown in Figure 3(b). This algorithm is then incorporated into the algorithm in Figure 4(a).

The algorithm in Figure 3 (a) also needs to be modified on the calculation of interpolation. If the corresponding quality level for $Pak21$ and $Pak20$ are $lotk21$ and $lotk20$ respectively, then the interpolation formula is:

$$Gradient = \frac{Pak21 - Pak20}{Lotk21 - lotk20} = \frac{Pak2 - Pak20}{RQL - lotk20} \rightarrow Pak2$$
$$= \left(\frac{RQL - lotk20}{lotk21 - lotk20}\right) * (Pak21 - Pak20) + Pak20$$

Thus, the acceptance sampling plan algorithm is given in Figure 4 (a).



(a) **(b)**

Figure 4. (a) Algorithm of acceptance sampling plan for modified Type-A OC curve based on single-outgoing lot with returned samples, and **(b)** with non-returned samples.

For non-returned samples, the algorithm is similar to that in Figure 4(a) with the following modification:

1. The calculation of the probability of acceptance at AQL is based on the single outgoing representative point
2. The iteration points for sample size (n) and acceptance number (c) are started from those derived with ordinary OC curve with interpolation (HG_I). This is to save calculation as the algorithm of non-returned samples requires more computation.

   Thus, the algorithm becomes as in Figure 4 (b).

2.3. To evaluate the proposed alternative/developed acceptance sampling plan.

After the tool (algorithm) to produce the sampling plan is available, a comparison of sampling plan results against the regular sampling plan from a common Type A OC curve can be performed. The comparison will be made by conducting algorithm running by lot number 0 to 1000 for case of AQL = 1%, RQL = 5%, Alpha = 5%, Beta = 10% as baseline. AQL = 1%, RQL = 5% was chosen as it is the common value/common choices (Brush, Cautin and Lewin 1981). The standard sampling plan can be obtained using a Software tool like Minitab. The comparison aims to see any unusual pattern and the conformity from the standard sampling plan and the developed sampling plan. The comparison will also facilitate the evaluation of whether the new results of the sampling plan either reduce or increase the sampling size from the standard sampling plan and by how much the difference is.

3. RESULTS AND DISCUSSION

3.1. Returned Sample Case

The optimum sampling plan is acquired from the algorithm that has been developed. The run results from lot size 1 to 1000 is given in Figure 6 with the ratio of number of sample and lot number as the ordinate. The reason is that the common OC curve has reached the 2nd step in the graph of Figure 6 ([HG_I]) and yet for the modified OC curve. and yet for the modified OC curve. The modified OC curve ([HG_UO_R]) reaches the 2nd step when the lot size is 523, where n = 113 while 123 for the common OC curve. The sample reduction is 8%. When the lot size = 1000, the sample size for both OC curves is 122 and 128, a reduction of 5%; eliminating testing six units can be a significant improvement, depending on the cost of testing, resource, and time.

Taking an example of AQL, RQL, alpha and beta 1%, 5%, 5% & 10% respectively, the acceptance sampling plan based on hypergeometric distribution is $n, c = 123{,}3$ and the normal Type-A OC curve is given in Figure 5 (Pa Returned Mod-OC).

An example of the modified OC curve at lot size = 500 is given in Figure 5. As seen from the graph, at beta (β) risk of 10%, the probability of acceptance reduced from initially (Common OC at 10%) or the sample size for the equal 10% Pa is n=93. The sample size reduction is significant for this case, from n = 123 to n = 93, or a 24% reduction.

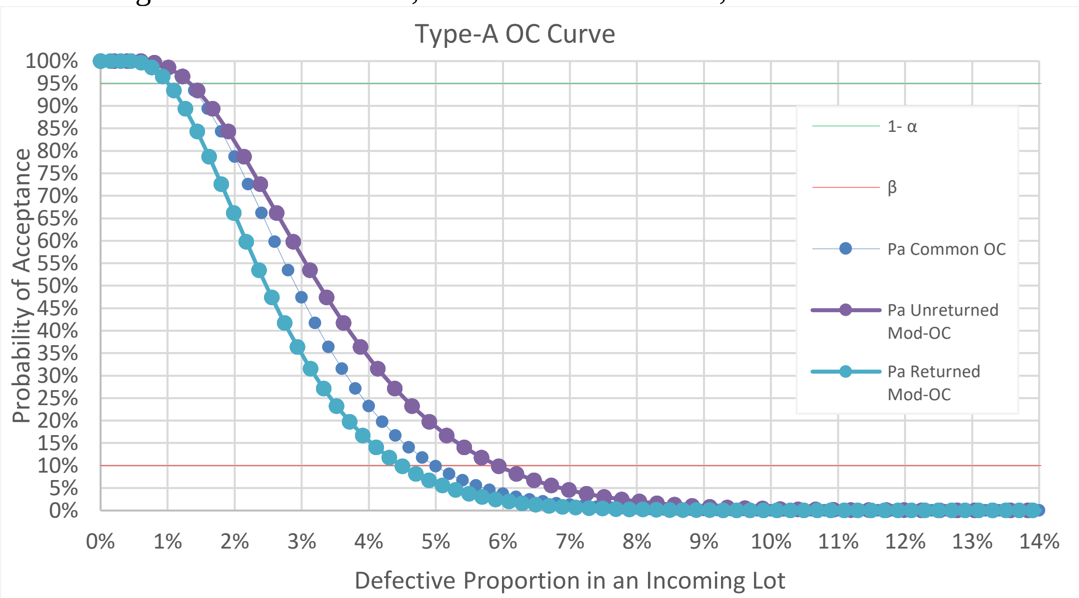


Figure 5. Type-A OC curve at. AQL = 1%, RQL = 5%, Alpha = 5% and Beta = 10% for three cases; Common OC Curve, Modified OC Curve for Returned Case and Modified OC Curve for Unreturned Case

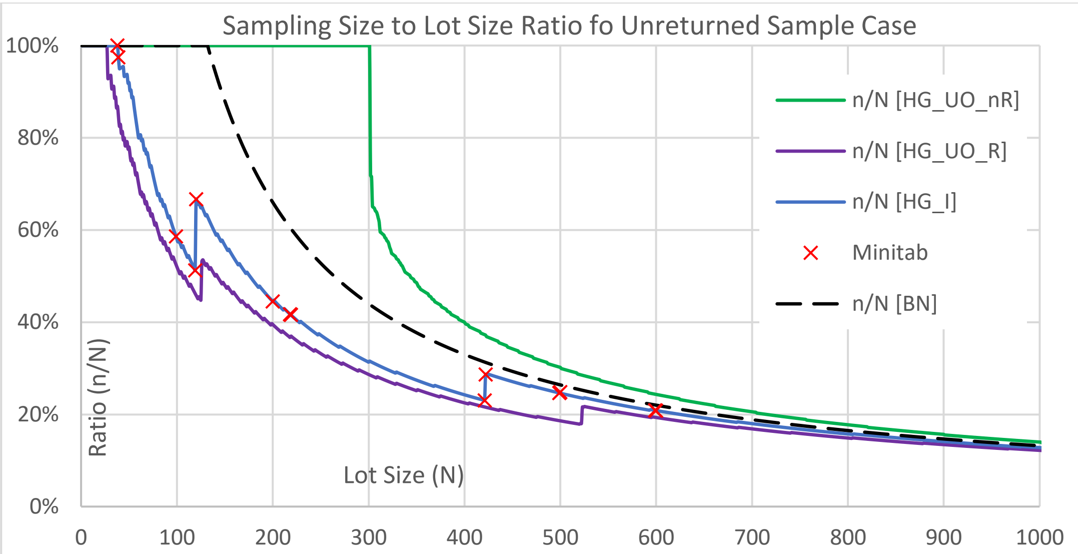


Figure 6. Sampling size to lot size ratio from algorithms per Figure 4(b) (HG_UO_nR), Figure 4(a) (HG_UO_R), Figure 3.(a) (for HG_I), Minitab result (Version 19.2020.1) and Binomial Distribution. A case for AQL = 1%, RQL = 5%, Alpha = 5%, Beta = 10%.

The developed sampling plan that uses a hypergeometric Type-A OC Curve based on the outgoing lot is best to describe the actual risk and quality received by the customer. Incorporating the Type-A OC Curve in the sampling plan will generally results in less complain caused by different quality of received goods and the perceived quality by producer, eventually less complain will also increase the costumer satisfaction.

For the case of returned sample, applying the developed sampling plan will result in the decreasing number of sample to be taken from the lot. Less sample taken will reduce the effort of inspection hence will increase the overall productivity. The number of reduced samples units varies depending on the input parameters (AQL, RQL, Alpha, beta & lot size). Even if the saved quantity is small, cumulative productivity increase can be significant as sampling is routinely performed for every lot.

3.2. Unreturned Sample Case

The unreturned case algorithm was run with 1 to 1000 lot number and resulted in the optimum sampling size as plotted in Figure 6 ([HG_UO_nR]). In addition to, the sample size from Binomial distribution (BN) is included, 132, and fixed for all sample sizes. This sample size represents infinite lot size, which means all the graphs converge onto this line at infinite or huge lot size. An example of the modified OC curve at lot size = 500 is given in Figure 5.

As seen in Figure 5 (Pa Unreturned Mod-OC), the Pa given at 10% of beta (β) risk is higher than initially (Common OC is at 5%), or else at the same Pa of 5%, the number of sample required for lot size of 500 is increasing from n=123 in common OC Curve to n=151 or 19% increase.

The graph for non-returned samples in Figure 6 is above the Binomial graph and at a considerable distance for smaller lot sizes. This pattern means the sample size required for a smaller lot size is larger for a non-returned sample case, which is the only characteristic different from other cases. Considering the sampling cannot return the good parts, the acceptance sampling based on the Type-A OC curve is not favorable for a relatively small lot size. It demands an alternative for an appropriate sampling plan other than the OC curve approach, or the manufacturer may implement a strategy of producing a lot as vast a number as possible or by other means.

As requiring a larger sample size for non-returned sample cases, using Binomial results is no longer a safer approach for the customer. In the previous thought, the Binomial result represents infinite lot size; those using this result mean taking more sample size than required as the lot size must be less than infinite, hence protecting customer more than what is minimally required. This approach usually is acceptable when the sample size result is around 10% of the lot size. Therefore, it is interesting to compare sample requirements for non-returned sample cases at the lot size region more than ten times the sample size from the Binomial result, which in this case, is lot size starts around 1300 which is shown in Table 3.

From the manufacturer's perspective, the required additional sample size may be insignificant and may deviate a little in the OC curve. From the customer's perspective, the OC curve requirement must be met through an accurate sample size which will comes back to an agreement between both parties. Ensuring the compliance to the OC curve is paid through additional scrap for sampling. Both parties may be aware that the manufacturer is always doing their best to ensure the product release is mostly below the AQL; hence there is no significant risk of deviation a little on the RQL.

Table 3. Comparison of Release Probability from Sampling Inspection against Beta level using Acceptance Sampling from Binomial Distribution for non-returned sample inspection, where the lot size is 10x of the sample size.

β(%)	**Pa_UO_nR (n,c_BN) (%)**	**n_UO_nR**	**n_BN**	**Lot Size**	**AQL (%)**	**RQL (%)**	**α (%)**
10	11,7	40	38	380	5	20	5
5	6,0	52	50	500	5	20	5
10	11,78	238	233	2330	5	10	5
5	6,21	305	298	2980	5	10	5
10	11,81	299	292	2920	4	8	5
5	6,15	382	374	3740	4	8	5
10	11,79	180	175	1750	3	8	5
5	6,21	231	224	2240	3	8	5
10	11,67	314	306	3060	2	5	5
5	6,10	396	386	3860	2	5	5
10	11,56	138	132	1320	1	5	5
5	6,01	188	181	1810	1	5	5
10	11,75	403	390	3900	1	3	5
5	6,11	536	521	5210	1	3	5
10	11,65	411	462	4620	0,50	2	5
5	6,11	542	523	5230	0,50	2	5

β(%)	Pa_UO_nR (n,c_BN) (%)	n_UO_nR	n_BN	Lot Size	AQL (%)	RQL (%)	α (%)
10	11,52	554	531	5310	0,10	1	5
5	6,00	656	628	6280	0,10	1	5
10	11,42	407	388	3880	0,065	1	5
5	5,94	497	473	4730	0,065	1	5
10	11,54	1109	1063	10630	0,065	0,50	5
5	6,00	1313	1258	12580	0,065	0,50	5

Applying the modified Type-A OC Curve is not preferable when the goods were manufactured in small batches, the effort may be taken is to produce the goods in a large batches then applying the sampling plan. The discussion of possible sampling plan for small batches will be left open. Incorporating the modified sampling plan for large batches, especially if the lot size is ten times the sample size will be possible with developed Type-A algorithm or Binomial (Type-B) sampling plan. Applying type-B will be manageable since the Pa of binomial mostly deviate no more than 2% from the beta risk. The deviation risk is considered small and acceptable considering the effort to introduce more samples to be taken. The rationale is that the manufacturer is always performing their best to get the quality which mostly passes the acceptance sampling, inferred as the quality lot produced below the AQL. Thus, obtaining acceptance sampling based on Binomial distribution is still recommended when the sampling cannot return the sampled unit to the lot.

CONCLUSION

A study of the Type-A OC curve based on the outgoing lot has been performed. The outgoing lot refers to (means) the quality level of the lot post sampling inspection, which has been altered during the sampling inspection, reducing some defective units as found and the lot sizes. Two scenarios of sampling inspection are covered, the sampling when the good parts are returned to the lot and with all sampled units are not returned to the lot (i.e., destructive test). The quality level of the lot post-inspection represents more appropriately the quality level that goes customer than that of pre-inspection (incoming lot). Thus, it has studied how this modified Type-A OC curve impacts the acceptance sampling from those derived from the common Type-A OC curve.

The presentation of the Type-A OC curve based on the outgoing lot results in the separation of a single point of the incoming quality lot; this requires additional criteria to obtain the acceptance sampling based on the outgoing lot OC curve, which was by averaging the separated outgoing points weighed based on the probability of acceptance. This alternative was proposed based on the consideration that treating independently the separated outgoing lots was not appropriate, as the original customer requirement is only AQL (RQL and Beta arose due to limitations from the acceptance sampling).

The algorithm for the alternative acceptance sampling plan were developed based on this modified Type-A OC curve using a representative single-point outgoing lot to obtain the acceptance sampling for both scenarios mentioned above. Two point of interest in the development of sampling plan for returned and unreturned case was in the AQL and RQL point. For returned sample case, the AQL point will use linear interpolation based on the Common Type-A algorithm, and RQL point will use iteration for two point;above and below RQL, then uses interpolation to determine the exact Pa at RQL. For the unreturned case, both AQL and RQL point uses interpolation for point above and below AQL and RQL then uses then uses interpolation to determine the exact Pa.

For the returned samples to the lots, it was found that the acceptance sampling requires less sample than that based on the standard OC curve. On the other hand, when the sampled units are not returned to the lot, more sample size is required, even more than using the sample size derived from Binomial Distribution. This means the customer requirement is not met even based on the Binomial approach, which has been considered conservative. Nevertheless, the OC curve approach is mostly utilized when the sample size is sufficiently small compared to the lot size when sampled units cannot be returned to the lot. When the lot size is ten times the sample size, using the samples size from Binomial distribution mostly deviates no more than 2%.

REFERENCES

Al-Nasser, A. (2022), "Acceptance sampling plans using hyper-geometric theory for finite population under Q-Weibull distribution.", *Electronic Journal of Applied Statistical Analysis*, pp. 352-366.

Banovac, E., D. Pavlovic, and N. Vistica. (2012), "Analyzing the characteristics of sampling by attributes." *Recent Researches in Circuits, Systems, Multimedia and Automatic Control,* pp. 158–163.

Brush, G.G., B. Hoadley, and B. Saperstein. (1990), "Estimating Outgoing Quality Using the Quality Measurement Plan.", *Technometrics,* pp. 31-41.

Brush, G.G., H. Cautin, and B.R. Lewin. (1981), "Outgoing Quality Distribution for MIL-STD-105D Sampling Plans." *Journal of Quality Technology,* pp. 254-263.

Cavone, Giuseppe, Laura Fabbiano, and Nicola Giaquinto. (2009), "Closed-Form Equations to Design Single Sampling Plans for Isolated Lots." *XIX IMEKO World Congress: Fundamental and Applied Metrology.* Lisbon.

Chukhrova, N., and A. Johannssen. (2018), "Quality Paper : Improved binomial and Poisson approximations to the Type-A operating characteristic curve." *IJQRM,* pp. 620-652, DOI 10.1108/IJQRM-10-2017-0203.

Chukhrova, N.,, and A. Johannssen. (2019), "Improved binomial and Poisson approximations to the Type-A operating characteristic function.", *International Journal of Quality & Reliabilty Management,* pp. 620-652.

Devore, Jay L. (2009), *Probability and Statistics for Engineering and The Sciences.* 7th. Brooks/Cole Cengage Learning.

ISO28590. (2017), "Sampling Procedure for INspection by Attributes - Introduction to the ISO 2859 Series of Standards for Sampling for Inspection by Attributes."

ISO2859-1. (1999), "Sampling Procedures for Inspection by Attributes - Part 1: Sampling Schemes Indexed by Acceptance Quality Limit (AQL) for Lot-by-Lot Inspection." ISO.

Kreyszig, E. (2009), *Advanced Engineering Mathematics, 10th edition,* Wiley.

Montgomery, Douglas C. (2009), *Statistical Quality Control: A Modern Introduction,* John Wiley & Sons (Asia) Pte. Ltd.

Mostofi, A.G., and S. Shirkani. (2019), "Designing a single-sampling plan for attributes in the presence of classification errors." *Communications in Statistics-Simulation and Computation,* pp. 1-15.

NIST/SEMATECH. 2012. *e-Handbook of Statistical Methods.* https://doi.org/10.18434/M32189.

Samohyl, Robert Wayne. (2018), "Acceptance Sampling for Attributes via Hypothesis Testing and the Hypergeometric Distribution." *J. Ind. Eng. Int.,*pp. 395-414. doi:https://doi.org/10.1007/s40092-017-0231-9.

Schilling, Edward G., and Dean V. Neubauer. (2017), *Acceptance Sampling in Quality Control.* 3rd. Taylor & Francis Group, LLC.

Z1.4, ANSI/ASQ. 2003 (R2013). "Sampling Procedures and Tables for Inspection by Attributes."